Employment Adjustment and Labor Utilization

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Preliminary

Abstract

Labor adjustment costs affect both the timing and the extent of employment variation in response to exogenous shocks. Most of the models that are used to formalize and assess the impact of labor adjustment costs tend to focus on changes in the number of workers, largely ignoring the variation in the amount that each worker works. Yet, the empirical evidence suggests that firms use variation in work hours extensively to adjust their labor input. The goal of this paper is to develop a structural general equilibrium model that introduces labor adjustment on both intensive and extensive margins. The model includes a theory of a producer with multiple jobs, heterogeneous profitability and hiring that is impeded by search frictions. Firms and workers bargain over wage contracts that specify work hours schedule and compensation. The driving force of the model is idiosyncratic profitability shocks that the firm can accommodate by changing its labor force and work hours of its employees. In the presence of search costs, the employment adjustment is sluggish that creates a dynamic interaction between hours and number of workers that is the focal point of this paper. The empirical analysis in this study is based on a matched employer-employee panel of Danish administrative firm data. The model is calibrated to assess its fit to the data. It appears to be quite successful in capturing the overall characteristics of the data. The model does an outstanding job of reproducing the rich pattern of labor dynamics at the firm level. However, on-the-job search is required to reconcile the theory with the empirical facts on hiring and separation flows. Regarding the changes in work hours, the simulation reproduces the negative comovement between hours and employment, but falls short with respect to the magnitude of the association observed in the data. In addition, it matches the empirical regularities found between firm size, wage and productivity.

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1 Introduction

Labor adjustment costs affect both the timing and the extent of employment variation in response to exogenous shocks, such as demand fluctuations, price variations in input factors and changes in labor market policies. They include among others search costs, such as advertising and screening new employees, the cost of training, and mandated severance pay. A significant part of labor adjustment costs is related to various forms of labor market regulations. In fact, legal impediments to dismissal were often held responsible for the relatively sluggish employment growth and high levels of unemployment that many Europeans countries have experienced in recent decades. That has led economists to promote labor market reforms aimed at easing of labor-market regulations and lowering the costs of dismissals, a policy change that many European countries have considered or implemented.

Most of the existing models that are used to formalize and assess the impact of labor adjustment costs only allow firms to change the number of workers they employ and not the amount that each worker works. Yet, variation in labor utilization is an important channel through which firms can adjust their labor input in response to demand shocks. The aim of this study is to specify a structural model that includes both intensive and extensive margins of labor adjustment in the economy with frictions in the labor market, and subsequently bring it to the firm data.

This paper develops a general equilibrium model that includes a non-trivial theory of a producer with multiple jobs (as opposed to a standard search theory where a firm employs one worker at a time), heterogeneous profitability and hiring that is impeded by search frictions. The driving force of the model is idiosyncratic productivity shocks that the firm can accommodate by changing work hours of it employees beyond varying its labor force. The presence of search costs causes a mismatch in the timing of adjustment in the average hours of work and in employment in response to exogenous shocks. That, in turn, creates a dynamic interaction between the two channels that is the focal point of this paper.

In addition, I introduce on-the-job search to the model that generates different retention and attrition rates across firm types. The fact that firms can vary their workforce through modifying their attrition rates have different implications in terms of adjustment costs. Many of the existing literature essentially ignores this channel by assuming constant quit rates and makes no distinction between net and gross employment changes. To reconcile the theory with the empirical fact that firms face different attrition rates depending on firm size, age, wages etc., this model allows for endogenous and stochastic quits.

The empirical analysis in this paper exploits a matched employer-employee data set that is drawn from a panel of Danish administrative firm data that includes all private businesses for the period of 1999-2006. This data set provides quarterly figures on total work hours (interval variable), payroll costs, and value added, as well as monthly statistics on employment. High-quality longitudinal links ensure that accurate monthly hiring and separation series can be constructed. The empirical evidence suggests that firms use variation in work hours extensively as a way to adjust their labor input in response to shocks. This paper also documents significant worker turnover in terms of job flows and worker flows in the Danish labor market.

The model is calibrated to assess its fit to the Danish firm data and appears to be quite successful in capturing the overall features of the data. The model is able to match the rich patterns of labor adjustment at the firm level, including the relationship between hiring and separation rates by net employment growth, firm size and wage. Regarding the changes in labor utilization, the simulation reproduces the negative comovement between hours and employment, but falls short with respect to the magnitude of the relationship observed in the data. In addition, the model predictions seem to be in line with the empirical facts on the association between employment, wages and productivity.

The next step of this project is to estimate the model applying indirect inference approach. Using the estimated structural parameters, one can perform policy experiments of such changes in labor market policy as mandatory over-time premium, severance pay etc. The estimation part, however, is currently in progress and no results are available yet.

There are several strands of literature related to this paper. This study is obviously linked to the empirical and theoretical literature on the impact of adjustment costs on labor demand (see Hamermesh and Pfann [25] for a comprehensive survey). In general, it is not easy to obtain information on the sources and sizes of adjustment costs because many of these costs are implicit, in that they result in lost output and are thus not measured and reported; therefore, until recently, the empirical work did not attempt to quantify the costs. Recent works of Abowd and Kramarz [3] and Kramarz and Michaud [26] find considerable fixed costs of separation in the data on French manufacturing firms. Rota [36], based on annual firm level data from Italian manufacturing industry, finds that fixed adjustment costs are 15 months of labor costs.

The previous work that accounts for labor utilization in the adjustment cost models includes Caballero, Engel and Haltiwanger [9]. They use variation in hours to identify (unobserved) gap between the actual and target levels of employment that, in turn, affects the hazard rate of employment adjustment[1]. Cooper and Willis [15] estimate a structural model (based on the "gap" methodology of Caballero, Engel and Haltiwanger [9]) where the relationship between variation in hours and employment, and shocks is specified explicitly. The goal of their paper is to characterize plant-level adjustment costs (in particular the presence of non-convexities) to match aggregate observations. Cooper, Haltiwanger and Willis [14] (see also Cooper, Haltiwanger and Willis [16] for partial equilibrium model) examine hours, employment, vacancies and unemployment at micro and macro levels based on a search model with frictions in hiring and firing. They simplify the worker's side of the market by assuming that firms post wage contracts that provide workers with the value of unemployment. This implies an unrealistic assumption that workers get no surplus in equilibrium.

This paper is also related to the literature on labor dynamics at the firm level. Most of the studies do not distinguish between net and gross employment adjustment, assuming exogenous and deterministic quits. In contrast, ample empirical evidence suggests that worker flows and job flows are quite distinct - most employers are simultaneously hiring and facing separations; moreover, quit rates are found to be increasing at contracting firms (see for instance Burgess, Lane and Stevens [8], Davis, Faberman and Haltiwanger [17]). The recent work of Faberman and Nagypal [21] is the only paper I am aware of that has a theory of quits at the firm level. They introduce a notion of replacement hiring; thus, explicitly distinguishing the cost of creating a new job from the cost of replacing a worker.

On many aspects of the methodology, this paper is linked to standard random search models (see for instance Mortensen and Pissarides [31], Mortensen [29]) and more recent work that introduces a theory of multi-worker firms into the equilibrium search model (see for instance, Lentz and Mortensen [28]).

The paper proceeds as follows. Section 2 delivers empirical evidence on employment and hours adjustment using Danish firm data. Section 3 describes the model in details. Section 4 shows the calibration of the model and its fit to the data. Section 5 summarizes the findings and outlines the directions for future work.

[1] As Caballero, Engel and Haltiwanger [9] note, their estimation procedure may yield biased estimates since the error term in the identifying equation is likely to be correlated with changes in hours. That is, a positive shock to profitability may induce the plant to increase both hours and the desired level of employment. They argue that this problem is partially alleviated by looking at period of large adjustments, so that the changes in hours and employment overcome the changes in error.

2 Data

This section documents empirical facts on labor adjustment that provide the motivation for this paper. The key moments pertain to the firm-level employment and hours dynamics. The empirical analysis of this paper is based on a matched employer-employee data set that is drawn from a panel of Danish administrative firm data. It includes all private businesses in the economy for 1999-2006[2]. The data sources and construction of variables is described in details in Appendix A1. First, section 2.1 presents the empirical evidence on the relationship between growth rates of hours and workforce. Second, section 2.2 examines gross and net employment change patterns and differences between them.

The available data come from two major sources. First, a work hours series is derived from firm mandatory pension contribution data collected on a quarterly basis. These pension contributions are paid by a firm for each employee according to whether her weekly hours of work fall into one of the following intervals: less than 9 hours, 9 to 18 hours, 18 to 27 hours and more than 27 hours. Total contributions paid by a firm for all employees are reported. Then, the work hours measure is constructed by dividing the total amount of contributions by a payment norm for a full-time employee and multiplying by 27 hours a week. Given the proportionality of the contributions schedule, the hours measure thus constructed represents a lower bound on weekly hours of work, H_{LB}. The empirical analysis in this paper refers to this hours measure, unless stated otherwise.

Bear in mind that the hours measure used in the analysis below may mask some (or even most) of the variation in actual hours due to its interval nature. This is particularly true for the case of a positive shock since it is not possible to identify overtime. Most of the observed variation in hours comes from employees switching hours intervals. For instance, if some workers switch from part-time to full-time then an increase in labor utilization will be reflected in work hours measure. Therefore, the observed hours variation in the data is thought of as a lower bound on actual hours variation.

The second data source is a matched employer-employee panel that includes all individuals that have paid employment in a given month. Monthly number of workers is obtained as a head count of individuals employed in a given firm. Quarterly employment is derived as an average of three months employment. The particular structure of this data set makes it possible to construct monthly hires and separations series for each firm.

The empirical analysis is carried out based on private companies. The firms that employ less than five employees for six consecutive months are excluded from the analysis (they comprise 53.7% of all firm-quarter observations, but only 6.3% of total employment). The resulting data set has 120,058 firms that are observed in the data for 14 quarters on average[3].

2.1 Hours and Employment

In the presence of labor adjustment costs a firm may opt for varying work hours in response to exogenous shocks instead of (or jointly with) adjusting its workforce. Suppose that hiring a new worker is impeded by search frictions. Then, in case of a positive shock to the demand for its product, a firm raises average hours of work immediately and adjusts employment more slowly. As firm's labor force builds up to its

[2] Due to data limitations, most of the previous empirical literature on labor adjustment models that uses data on work hours is limited to industry-level data (see Hamermesh and Pfann [25] for a survey) or establishment-level data that pertains to the US manufacturing sector for the period of 1972-1980 (see for example Caballero, Engel and Haltiwanger [9] and a more recent paper by Cooper, Haltiwanger, and Willis [14] - they build their analysis on Longitudinal Research Database).

[3] According to FIDA dataset (yearly matched employer-employee data that provides information on establishment level employment), more than 99.9% of firms are one-establishment units; while less than 0.1% of firms have more than one establishment. Therefore, the results in this paper are comparable to previous studies that have used establishment-level micro data.

new desired level, the average hours of work fall. In that case, one would expect to see growth in hours and employment moving in different directions in the data. Moreover, the changes in average hours would precede changes in number of workers, thus leading employment growth. Similarly, a negative shock in combination with mandated advanced layoff notice would produce an immediate hours response and a more sluggish employment drop.

The data set underlying the empirical moments contains quarterly observations of total hours and monthly employment from which quarterly employment (N_t) and hours per employee (H_t) series are constructed. In the analysis below, I focus on the growth rates of hours ($\Delta h_t = \Delta \log H_t$) and employment ($\Delta n_t = \Delta \log N_t$) expressed as first differences of log variables. The results reported here are based on the raw series, as well as employment share-weighted moments. In addition, I abstract from aggregate demand shocks and explore the cross-sectional variation in growth rates of hours and employment. Hence, aggregate time effects are removed from the original growth rates.

Figure 1 depicts the histograms of growth rates of both variables. First, it is worth to mention the incidence of a significant inaction region in both hours and employment. A spike at zero change in average working hours is not surprising given the interval nature of hours variable. The growth of employment, however, is measured more precisely and thus is more informative about the region of zero employment adjustment: about 15% of firms employ the same number of workers in two consecutive quarters, and about 22% of firms have employment change less than 2.5%.

Second, despite the fact that hours are measured in intervals, there is a significant variation in hours growth. In fact, the standard deviation of hours and employment growth is about the same (see Table 1). This finding provides some evidence that firms use both intensive and extensive margins to make adjustments to their labor input. Therefore, hours margin seem to be an important channel through which firms change their labor demand and should not be ignored when modelling employment decisions of firms.

Table 1 presents empirical moments that describe the relationship between hours and employment adjustment. Similarly to the results reported by Cooper, Haltiwanger and Willis [14] for the US labor market, I find a negative correlation between hours and employment growth. Figure 2 shows that hours-employment relationship is monotone and that the negative correlation between the two series is observed for virtually all values of employment growth. Moreover, growth of hours seems to lead growth of employment: there is a positive association between employment growth this period and hours growth last period. Both of these findings are consistent with the predictions of labor adjustment models.

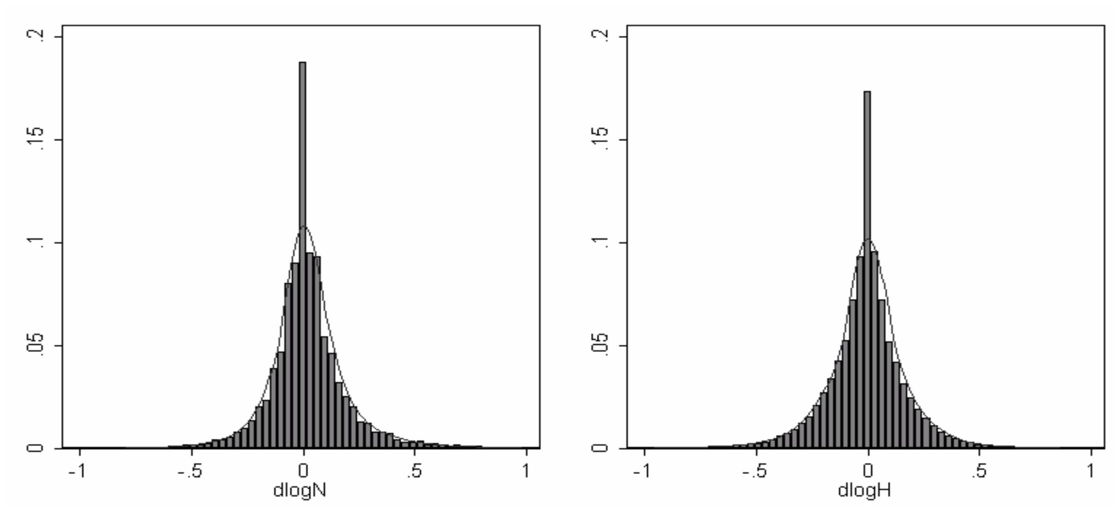
Table 1: Hours and employment adjustment.

	$\sigma(\Delta n_t)$	$\sigma(\Delta h_t)$	$\rho(\Delta h_t, \Delta n_t)$	$\rho(\Delta h_{t-1}, \Delta n_t)$
Non-weighted	0.2421	0.2478	-0.3404	0.0707
Emp. share-weighted	0.2153	0.2218	-0.4635	0.1091
Weighted, no time effects	0.2146	0.2207	-0.4689	0.1151
N obs	1444397	1435330	1435330	1329466

Source: Author's tabulations from the Danish firm data, 1999-2006.

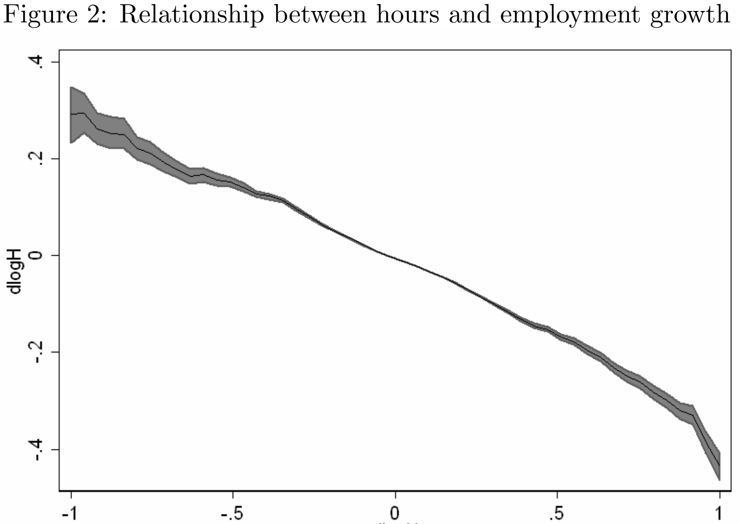
Table 2 reports the correlation between hours and employment growth by broad industry groups (in accordance with the standard Statistical Classification of Economic Activities in the European Union - NACE). The results show that hours-employment relationship is weaker in Hotels and restaurants, Fishing and Construction sectors. These industries are associated with relatively low-skilled labor and presumably with lower adjustment costs. On the other hand, Real estate and business activities, Transport and telecommunication, and Other social and personal services demonstrate stronger association between growth rates of hours and employment.

Figure 1: Growth rate of employment (left panel) and average work hours (right panel)



Note: Vertical axis shows a fraction of firm-quarter observations. Density estimation is based on Uniform kernel with bandwidth of 0.1. Source: Author's calculations from the Danish firm data, 1999-2006.

Figure 2: Relationship between hours and employment growth



Note: Nadaraya-Watson estimator based on Gaussian kernel with bandwidth of 0.025. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 1999-2006.

Table 2: Correlation between growth rate of average hours of work and employment, by industry.

	Non-weighted	Emp.share weighted	Weighted, no time effects	Emp. share,%	N obs
Agriculture	-0.2915	-0.3687	-0.3786	1.47	54545
Fishing	-0.2418	-0.2407	-0.2611	0.08	4716
Mining and quarrying	-0.1994	-0.2895	-0.3025	0.15	2037
Manufacturing	-0.3451	-0.5088	-0.5208	27.22	228782
Electricity, gas and water supply	-0.7758	-0.3647	-0.3614	0.56	7834
Construction	-0.1607	-0.2568	-0.2814	8.90	215092
Wholesale and retail trade	-0.3459	-0.4015	-0.3983	23.80	369562
Hotels and restaurants	-0.1903	-0.1798	-0.1845	3.78	77350
Transport and communication	-0.3393	-0.5486	-0.5513	6.87	91663
Financial intermediation	-0.6202	-0.3396	-0.3320	5.37	16530
Real estate and business activities	-0.4201	-0.5986	-0.5970	14.79	201658
Education, health and social work	-0.4198	-0.3905	-0.3948	2.07	72414
Other social and personal services	-0.5744	-0.5686	-0.5725	4.88	90481
Activity not stated	-0.4028	-0.2804	-0.2847	0.06	2666

Source: Author's tabulations from the Danish firm data, 1999-2006.

Previous studies on labor adjustment costs that used firm-level data pertain to the manufacturing sector only (see Cooper, Haltiwanger and Willis [14]). Therefore, one of the advantages of using Danish firm data is a possibility to compare the manufacturing industry to the overall population of firms. According to Table 2, the manufacturing sector (that comprises 27% of overall employment in Denmark) is characterized by a more negative correlation coefficient than overall economy (-0.52 and -0.47, respectively).

2.2 Worker Flows

It is important to recognize that beyond hires and layoffs, firms can adjust their labor force through modifying their retention and attrition rates. Most of the existing models that formalize the effect of labor adjustment cost of employment dynamics focus at net employment change. Consequently, they do not distinguish between firms raising their hiring rates or devoting more resources to retention rates. Yet, these channels may have different implications in terms of labor adjustment costs, especially if one considers recruiting and training costs. This section examines employment growth at a firm level in a more detailed way.

Here, I follow the existing literature in constructing and analyzing the job and worker flows at the firm-level (see for instance Davis, Faberman and Haltiwanger [17], Burgess, Lane and Stevens [8] and the references therein). Monthly hires are computed as a number of individuals that are working in a given firm during month t but not during month $t-1$. Separation flows are equal to number of workers that are employed in a given firm during month $t-1$ but not during month t. Job flows are defined as a number of jobs created in growing firms (job creation) and a number of jobs destroyed in contracting firms (job destruction) within month t. The corresponding rates are expressed in flows divided by the average employment in month t and $t-1$. This procedure yields growth rates in the interval $[-2, 2]$ with endpoints corresponding to births and deaths (for more details on the properties of this rate measure see Davis, Haltiwanger and Schuh [18]). Job flows are regarded as being driven mostly by the demand-side economic forces (e.g. increase in demand for a product, technological changes etc.); whereas worker flows may be affected by the demand-side influences along with the supply-side events such as movement into and out of labor force, spouse's relocation etc. Both worker and job flows are reported in the statistics below.

The data at hand indicate that there is a fair amount of job and worker mobility in the Danish labor market (see Table 3)[4]. Hiring and separation rates average about 9% of employment. Monthly job destruction and job creation rates are about 5-6% of employment (4% for continuing firms), more than twice the rates in the US labor market (see Davis, Faberman and Haltiwanger [17]). That is, one of every 20 jobs on average is being destroyed from one month to the next.

Table 3: Average monthly job and worker flow rates.

	Non-weighted	Emp. share-weighted	Emp. share-weighted, continuing firms
Hires	0.180	0.097	0.078
Separations	0.152	0.091	0.076
Job Creation	0.158	0.061	0.041
Job Destruction	0.129	0.054	0.039
Net employment change	0.029	0.006	0.002
Churning	0.046	0.074	0.075

Note: Sample includes all private firms. Source: Author's tabulations from the Danish firm data, 1999-2006.

In order to highlight differences between job and worker flows, the churning rates, defined as the sum of hires and separations rates less the absolute value of the net growth rate in employment, are constructed (see Burgess, Lane and Stevens [8] for more details on this measure). The churning rate refers to worker flows in excess of job flows. The fact that firms churn workers indicates that contracting businesses still hire workers and workers leave growing firm. The Danish economy is characterized by quite high average churning rates of 7.5%. On average over the period of 1999-2006, job creation constitutes 32.2% of all (size-weighted) hires; while 30.6% of all separations are associated with job destruction.

Table 4 shows the relationship of monthly worker flows and firm employment adjustment, size-weighted by employment share. The firms are split into five groups according to their net employment growth rate. Firms that represent 50.3% of employment have monthly net employment growth between -2.5% and 2.5%. Contracting firms reduce their labor force mostly through separations; while growing firms increase their employment mostly through hiring. However, even contracting firms are hiring at 4.5% rate. These results appear to be qualitatively similar to those found in the US and Dutch labor markets (see Davis, Faberman and Haltiwanger [17] and Hamermesh, Hassink and van Ours [24], respectively), but are in contrast to the behavior of French firms reported by Abowd, Corbel and Kramarz [2]. The latter paper finds that employment variation in France is made predominantly through the hiring margin; that is, an establishment is changing its labor force primarily by reducing entry and not by changing the separation rates.

Table 4: Average monthly hiring and separation rates, by net employment growth rate.

Net Emp. Growth	Hires	Sep.	Net	Emp. Share, %
Less than -0.10	0.045	0.496	-0.450	10.11
-0.1 to -0.025	0.040	0.093	-0.053	13.72
-0.025 to 0.025	0.035	0.035	0.000	50.31
0.025 to 0.10	0.093	0.040	0.053	14.79
More than 0.10	0.504	0.044	0.460	11.08
Total	0.097	0.091	0.006	100.00

Note: Sample includes all private firms. Source: Author's tabulations from the Danish firm data, 1999-2006.

[4] In order to be able to include firm's entry and exit, I do not restrict the sample to firms with 5 or more employees. In fact, the data set used for this analysis contains all firms in the private sector regardless of their size. Overall, the sample contains more than 10 million firm-month observations.

Compared to the US labor market, there seems to be more labor adjustment in the Danish data. For instance, Cooper, Haltiwanger and Willis [14] document that about 8% of workers are employed at the establishments that had reported net employment growth rate of more than 10% by absolute value. In Denmark, the corresponding figure is 21.2%. Inaction region is larger in the US in terms of employment share of firms that do not change their number of workers from one month to the next (20.6% in Denmark vis-à-vis 32% in the US).

Another question to address regarding employment change in the presence of adjustment cost is temporary layoffs. Presumably it is less costly for a firm to re-hire a worker that has been laid off temporarily than hire a new worker; thus the hours adjustment becomes less important in firms labor demand decisions. In order to examine worker flows during a quarter, I construct two measures of hires and separations. The first measure is derived by contrasting employment in the last month of two consecutive quarters; while the second measure sums monthly flows series over a quarter. Comparing the two measures, it appears that about two fifths of all hires and separations arise in connection with employment relationships lasting less than a quarter. Moreover, about 30% of all individuals hired during a quarter were employed at the same firm during the previous quarter. Hence, the fact that the relationship between employment and hours growth is found to be so strong in the economy with fairly frequent temporary layoffs is reassuring about the importance of hours as a channel for labor adjustment.

The model developed in this paper is motivated by the empirical facts outlined above. First, firms seem to vary their labor input on both extensive and intensive margins, as the standard deviation of employment and average hours growth series is about the same. Second, the movements in hours and employment are negatively related which is consistent with the idea of a fast response of hours to demand shocks and a more sluggish response of employment. Furthermore, hours growth leads employment growth. In addition, this section presents empirical evidence that worker flows and job flows are quite distinct. In fact, only about 30% of monthly hires and separations arise in connection with job creation and destruction, respectively. Different implications of net and gross employment changes in terms of adjustment costs call for a theory that explicitly models hiring and separation decisions of firms.

3 Model

The model is an alternative formulation of Mortensen [30] continuous time matching model of multi-worker firms with heterogenous profitability. This paper modifies the production process by adding an intensive margin and allowing for workers and employers to bargain over a wage contract that specifies hours schedule and compensation. The driving process for the model is firm-specific profitability shocks that a firm can accommodate by adjusting its workforce and number of hours its employees work. Hiring a worker is impeded by search frictions; hence, employment cannot be adjusted instantaneously. Allowing for on-the-job search ensures that the retention and attrition rates vary with firm's profitability.

I start with an overview of the model and then proceed to discuss the components in details.

3.1 Overview

A final good Y is produced by a continuum of intermediate inputs $x(j)$ and is sold by many suppliers in a competitive output market at a price P. Intermediate product suppliers face downward sloping demand for their products, $p(x(j))$. The production technology for the intermediate good is linear in labor input, in particular,

$$x = qhn, \tag{1}$$

where x is the number of units supplied, q is firm's productivity, and hn is total labor input, the product of the number of workers, n, and average work hours, h.

Each firm supplies one intermediate product. Firms differ in their productivity level q that at each point in time is a subject to the shock that arrives at Poisson rate μ. In that case the new productivity is drawn from distribution $\Phi(\cdot)$ defined on support $\left[\underline{q}, \overline{q}\right]$. Existing firms are subject to the exogenous destruction risk and die at rate δ. Market entry rate is exogenous and is equal to η.

Firms and workers are brought together pairwise through a sequential and random matching process. To recruit, firms post vacancies v and incur hiring cost of $c_v(v)$ per unit of time where $c_v(v)$ is assumed to be strictly increasing and convex. Reflecting the search frictions, offer arrival rate and vacancy filling rate are exogenous to workers and firms but are determined in equilibrium. A job separation occurs when a worker quits or is laid off. Firing a worker is assumed to be costless.

There is a continuum of infinitely lived identical workers, with a mass normalized to one, that supply labor to intermediate product firms. Individuals derive utility from consuming the aggregate good and incur disutility from working. Workers' utility function is assumed to be separable in aggregate good consumption and work hours

$$\tilde{u}(y,h) = y - g(h),$$

where y is the amount of the aggregate good consumed, h is hours of work, and $g(\cdot)$ is a strictly increasing convex function with $g(0) = g'(0) = 0$. Worker's consumption flow equals to real wage $\frac{w}{P}$ when employed and equals to b when unemployed. Here, b can be viewed as unemployment insurance benefit that is indexed by the aggregate price level. Alternatively, b can be regarded as the value of home production of the aggregate good less the disutility from producing it. Hours of work and compensation are defined through the bargaining process. Finally, workers search while employed and unemployed.

3.2 Intra-firm Wage Bargaining

Hours of work and compensation are defined through the bargaining process between a firm and its employees. The hiring costs imply that it takes time to replace workers, and therefore employment is considered to be predetermined at the bargaining stage. This setup provides a natural environment for Stole and Zwiebel [38] individual bargaining framework within multi-worker firms, in which firms engage in pairwise negotiations with workers over the current output[5]. The key assumption of their setup is that firms and their employees cannot commit to long-term employment and wage contracts. When a worker joins the firm, wages are renegotiated individually with all workers; therefore, the firm's outside option is not to remain idle, but rather producing with one worker less. Note that in the original Stole-Zwiebel bargaining problem the production function exhibits diminishing returns to scale. Here, the production technology is assumed to be linear in number of workers; however, the assumption of a downward-sloping demand implies decreasing marginal revenue product.

First, a firm chooses work hours schedule that maximizes total surplus for a given number of workers, n, i.e.

$$\max_{h\geq 0}\left\{\frac{R(qnh)}{P} - g(h)\,n\right\} = \max_{h\geq 0}\left\{\frac{p(qnh)}{P}qhn - g(h)\,n\right\}.$$

[5] See also Cahuc, Postel-Vinay and Robin [11], Cahuc and Wasmer [12], and Ebell and Haefke [20] for a similar application of Stole and Zwiebel wage bargaining setup in which firms and their employees bargain over the match surplus instead of the current output. The paper of Cahuc, Marque and Wasmer [10] further extends the original Stole and Zwiebel's problem to account for the case when workers have different bargaining power parameters.

Assuming interior solution, the optimal number of hours, h^*, satisfies

$$\left(p'\left(qnh^*\right)qnh + p\left(qnh^*\right)\right)\frac{q}{P} = g'\left(h^*\right). \tag{2}$$

Next, workers and a firm bargain over wage taking hours schedule as given. Following Hall and Milgrom [23], the outside option of a worker is assumed to be the value of a delay, i.e. value of home production b. Then, based on a generalization of Stole and Zwiebel [38] bargaining problem for continuous employment n and bargaining power of workers $\beta \gtreqless \frac{1}{2}$, a wage contract solves the following problem:

$$\begin{aligned} & \max_{w(n)} \left(\frac{\pi'(n)}{P}\right)^{1-\beta}\left(\frac{w(n)}{P} - g(h(n)) - b\right)^{\beta} \\ = & \max_{w(n)} \left(\frac{R'(n)}{P} - \frac{w'(n)}{P}n - \frac{w(n)}{P}\right)^{1-\beta}\left(\frac{w(n)}{P} - g(h(n)) - b\right)^{\beta}, \end{aligned} \tag{3}$$

subject to the participation constraints for both parties in a sense that the continuation value is no less than that of searching for a new partner. Note that this bargaining problem is equivalent to the one in which workers bargain over both wage and hours of work simultaneously (see Appendix A2 for details).

Solving for the first order condition of (3) leads to a first-order linear differential equation in wage

$$w(n) = \beta R'(n) + (1-\beta)P\left[g(h(n)) + b\right] - \beta w'(n)n, \tag{4}$$

which implies the following eqution for the wage function (see Appendix A2 for details):

$$\frac{w(n)}{P} = n^{-\frac{1}{\beta}}\int_0^n z^{\frac{1-\beta}{\beta}}\left(\frac{R'(z)}{P} + \frac{(1-\beta)}{\beta}g(h(z))\right)dz + (1-\beta)b. \tag{5}$$

The solution has to satisfy the participation constraint for both parties. That is, the value of working is no less than that of quitting and searching for a new employer; moreover, the value of hiring an extra worker for a firm is nonnegative.

Note that if workers and firms bargain over the value of a match then the separations are bilaterally efficient. Here, however, given that the bargaining takes place over the current output, it is not necessarily true. A firm may choose to fire a worker even when worker's value of employment at that firm is higher than the value of unemployment, and vice versa, a worker may quit even though firm's value of employing that worker may be positive. The implicit assumption of this bargaining process is that if the participation constraint binds for one party then there is no renegotiation, instead the match is dissolved.

3.3 CES Production Function

This subsection assumes functional forms for the production and disutility of working functions and uses the results obtained in the previous subsection to derive the optimal hours choice, worker's wage and firm's profit.

The final output is determined by the (Dixit-Stiglitz) CES production function,

$$Y = \left[\int_0^K x(j)^{\frac{\sigma-1}{\sigma}}dj\right]^{\frac{\sigma}{\sigma-1}}, \ \sigma > 0,$$

where $x(j)$ is the quantity of product j, σ represents the elasticity of substitution between any two intermediate goods and K is the total measure of inputs available. The final good is produced by many competitive suppliers; therefore, the profit maximizing demand for each input is given by

$$x(j) = \left(\frac{P}{p(j)}\right)^{\sigma} Y,\ j \in K, \tag{6}$$

where P is the price of a final good Y and $p(j)$ is the price of an input j. Zero-profit condition for final good producers implies that the aggregate price index is derived as

$$P = \left[\int_0^K p(j)^{1-\sigma}\, dj\right]^{\frac{1}{1-\sigma}}$$

The intermediate good is produced according to the linear technology given in (1).

Assuming that the disutility of working, $g(h)$, takes the following functional form

$$g(h) = \chi h^{\xi}, \tag{7}$$

where $\xi > 1$ and $\chi > 0$, and using equation (6) for firm's demand, the optimal number of hours that solves equation (2) is equal to

$$h(q,n) = \left[\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma} Y\frac{q^{\sigma-1}}{n}\right]^{\frac{1}{(\xi-1)\sigma+1}}, \tag{8}$$

which is increasing in firm's productivity, q, and decreasing in number of employees, n.

Firm's revenue at the optimal number of hours expressed in units of aggregate good reads

$$\frac{R(q,n)}{P} = \left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\frac{\sigma-1}{(\xi-1)\sigma+1}} \left(Yq^{\sigma-1}\right)^{\frac{\xi}{(\xi-1)\sigma+1}} n^{\frac{(\xi-1)(\sigma-1)}{(\xi-1)\sigma+1}}. \tag{9}$$

Firm's revenue rises with productivity and with the number of workers if the elasticity of substitution between any too intermediate goods, σ, is higher than one. Hence, the condition $\sigma > 1$ is imposed herein.

Inserting the expression for firm's revenue function (9) and worker's disutility of labor (7) evaluated at the optimal work hours (8) into solution to the bargaining problem (5) leads to the following real wage curve equation

$$\frac{w(q,n)}{P} = \chi\left(1+\beta(\xi-1)\frac{(\xi-1)\sigma+1}{(\xi-1)\sigma+1-\beta\xi}\right)\left[\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma} Y\frac{q^{\sigma-1}}{n}\right]^{\frac{\xi}{(\xi-1)\sigma+1}} + (1-\beta)\,b. \tag{10}$$

Note that $\frac{\partial w(q,n)}{\partial n}$ is negative. Stole and Zwiebel [38] were the first to point out this hiring externality: as the number of workers per firm increases the bargained wage declines. The externality is stronger (in terms of $\frac{\partial w(q,n)}{\partial n}$ being more negative) if workers have higher bargaining power, β. Furthermore, from equation (4) it follows that the wage that a worker gets net of disutility of working is lower than her contribution to the total surplus.

Given the wage curve equation (10), firm's profit is derived as

$$\begin{aligned}\frac{\pi(q,n)}{P} &= \left(\frac{R(q,n)}{P} - \frac{w(q,n)}{P}n\right) \\ &= (1-\beta)\left[\frac{\chi\left((\xi-1)\sigma+1\right)^2}{(\sigma-1)\left((\xi-1)\sigma+1-\beta\xi\right)}\left(\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma} Y\frac{q^{\sigma-1}}{n}\right)^{\frac{\xi}{(\xi-1)\sigma+1}} - b\right] n.\end{aligned} \tag{11}$$

Solving for the first order condition of (11) one finds that the maximum profit is achieved at

$$n^{*}(q)=\left[\frac{((\xi-1)\sigma+1)(\xi-1)}{(\xi-1)\sigma+1-\beta\xi}\frac{\chi}{b}\right]^{\frac{(\xi-1)\sigma+1}{\xi}}\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma}Yq^{\sigma-1} \tag{12}$$

and is equal to

$$\frac{\bar{\pi}(q)}{P}=\frac{(1-\beta)\xi}{(\sigma-1)}\left[\frac{\chi((\xi-1)\sigma+1)}{(\xi-1)\sigma+1-\beta\xi}\right]^{\frac{(\xi-1)\sigma+1}{\xi}}\left(\frac{(\xi-1)}{b}\right)^{\frac{(\xi-1)(\sigma-1)}{\xi}}\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma}Yq^{\sigma-1}. \tag{13}$$

Therefore, firm's profit is a continuous function of employment and productivity, increasing in productivity, and bounded from above for a given q by $\bar{\pi}(q)$.

In Stole and Zwiebel [38] original problem n^{*} is the maximum number of workers the firm would employ. This is a straightforward implication of the bargaining problem in which the firm has an incentive to hire additional workers to decrease their bargaining power up to the point where $\pi'(n)=0$ and workers are paid their reservation wage. Here, however, a firm may choose to employ more than n^{*} workers as long as the value of doing so is positive.

Finally, worker's utility is equal to

$$\begin{aligned}\tilde{u}(q,n) &= \frac{w(q,n)}{P}-g(h(q,n)) \qquad (14)\\ &= \beta(\xi-1)\frac{\chi((\xi-1)\sigma+1)}{(\xi-1)\sigma+1-\beta\xi}\left[\left(\frac{\sigma-1}{\chi\xi\sigma}\right)^{\sigma}Y\frac{q^{\sigma-1}}{n}\right]^{\frac{\xi}{(\xi-1)\sigma+1}}+(1-\beta)b,\end{aligned}$$

that is continuos and decreasing in employment and increasing in productivity q.

Returning to the motivation for this exercise, the driving force of this model is an idiosyncratic shock to firm's productivity q[6]. The hours function defined in equation (8) guarantees an increase in average hours in response to a positive shock to q if there is no change in employment. As the number of workers starts growing the average hours decline. Hence, the model can potentially reproduce the negative relationship between hours and employment growth observed in the data if response in employment is slow enough. The employment decisions of firms and workers are discussed in details in the following two subsections.

3.4 Worker's Problem

Here, I describe the worker's problem taking as given all equilibrium objects that are outside of worker's control, in particular aggregate market tightness, distribution of offers and workers across firm types, as well as optimal employment decisions of firms. Although the value of unemployment and the value of working at a firm with productivity q and employment n depend on the aggregate variables, they are not listed as arguments for notational simplicity.

When unemployed, a worker obtains consumption flow b by means of home production, and she has an option of finding a job. Hence, the value of unemployment expressed in terms of final output, U, solves

[6] Note that I refer to a shock to q as a productivity shock. However, it can be thought of as a firm-specific demand shock or more generally profitability shock. For instance, consider an alternative specification where the aggregate demand function is defined as

$$Y=\left[\int_{0}^{K}\alpha(j)\,x(j)^{\frac{\sigma-1}{\sigma}}dj\right]^{\frac{\sigma}{\sigma-1}},$$

where $\alpha(j)$ is firm-specific demand shock, and production technology for the intermediate good is $x=hn$. This specification is equivalent to the current formulation of the production side of the market if $q=\alpha^{\frac{\sigma}{\sigma-1}}$.

the continuous time Bellman equation

$$rU = b + \lambda(\theta) \int (\max\{W, U\} - U)\, dF(W), \tag{15}$$

where r is the common firm's and worker's discount rate; $\lambda(\theta)$ is the job arrival rate, and θ is market tightness; $F(W)$ is the cumulative distribution function of job vacancies posted by firms that provide workers with employment value of no more than W.

The job arrival rate λ is derived from the matching function that is assumed to be increasing, concave and homogenous of degree one in both arguments, vacancies and job seekers[7]. Given matching function properties, $\lambda(\theta)$ is increasing and concave in market tightness θ, which is the ratio of the aggregate number of vacancies posted to individuals searching for a job, the variable that is determined endogenously in the equilibrium.

The value of employment in the firm with n workers and productivity q, $W_n(q)$, satisfies the following Bellman equation

$$rW_n(q) = \left\{ \begin{array}{c} \tilde{u}_n(q) + (\delta + s_0)(U - W_n(q)) \\ +\lambda(\theta)\kappa \int (\max\{W', W_n(q)\} - W_n(q))\, dF(W') \\ +H_n(q)(\max\{W_{n+1}(q), U\} - W_n(q)) \\ +s_n(q)(n-1)(\max\{W_{n-1}(q), U\} - W_n(q)) \\ +\mu \int\limits_{\underline{q}}^{\overline{q}} \left(\begin{array}{c} \mathbf{1}[f_n(q') > 0]\left(f_n(q')U + (1 - f_n(q'))\max\left\{U, W_{\bar{n}^F(q')}(q')\right\}\right) + \\ \mathbf{1}[f_n(q') = 0]\max\{U, W_n(q')\} - W_n(q) \end{array} \right) \phi(q')dq' \end{array} \right\} \tag{16}$$

where $\tilde{u}_n(q)$ is the utility flow expressed in final output terms as defined in equation (14). The worker becomes unemployed at constant Poisson rate $\delta + s_0$, where s_0 represents the exogenous component of the quit rate and δ refers to the destruction shock. The worker receives an alternative job offer at rate $\lambda(\theta)\kappa$, where $\kappa \geq 0$ represents the search intensity when employed relative to the search intensity when unemployed (if $\kappa = 1$ then workers search with the same intensity regardless of their employment status; $\kappa = 0$ implies no on-the-job search). Hence, the next term in the equation (16) is attributed to the option value of moving to a better employment position. The following two terms are related to the expected change in the value of employment in the event of a firm adjusting its labor force. In particular, at rate $H_n(q)$ the firm hires another worker and at rate $s_n(q)(n-1)$ one of the other $(n-1)$ workers separates from the firm. These rates are determined endogenously in the equilibrium and are defined in the firm's problem below.

Finally, the last term embodies the expected change in the value attributable to the shock to firm's productivity q that arrives at rate μ. The new productivity is drawn from the distribution $\Phi(\cdot)$ with the corresponding density $\phi(\cdot)$. Recall that separations are not necessarily mutually efficient; therefore, one has to take into account that when hit by the productivity shock the firm may find it optimal to fire workers. Let $\bar{n}^F(q)$ be the maximum labor force size that the firm is willing to employ given its current productivity level q (it will be defined more precisely in the firm's problem below). Then, $f_n(q)$ is the firing probability equal to $\frac{n-\bar{n}^F(q)}{n}$ if $n > \bar{n}^F(q)$ and equal to zero otherwise. That is, it is assumed in the model that a firm fires workers randomly if its labor force exceeds its maximum size $\bar{n}^F(q)$.

Proposition 1: A unique continuous solution for the value of employment at the firm with n workers and productivity level q, $W_n(q)$, and value of unemployment, U, exists if hiring and separation rates, $H_n(q)$ and $s_n(q)$, are finite and continuous in q and n.

[7] See Pissarides [35] and Petrongolo and Pissarides [34] for details on the concept of a matching function.

Proof: Equation (16) has a unique solution which is a fixed point of the contracting mapping $[W_n(q), U]_{t+1} = T[W_n(q), U]_t$ defined by equations (15) and (16).Given that $\tilde{u}_n(q)$ is positive and bounded from above by $\tilde{u}_1(q)$, T maps the set of non-negative, continuous, and bounded from above functions into itself. Given this set is compact under the sup norm, one can apply Blackwell's sufficient conditions to show that T is a contraction mapping. Obviously, the mapping is monotone; furthermore, T discounts, i.e.

$$T\begin{bmatrix} W_n(q)+z \\ U+z \end{bmatrix} = T\begin{bmatrix} W_n(q) \\ U \end{bmatrix} + \begin{bmatrix} \frac{\delta+s_0+\lambda(\theta)\kappa+H_n(q)+s_n(q)(n-1)+\mu}{r+\delta+s_0+\lambda(\theta)\kappa+H_n(q)+s_n(q)(n-1)+\mu} \\ \frac{\lambda(\theta)}{r+\lambda(\theta)} \end{bmatrix} z,$$

which completes the proof that T is indeed a contraction. Then by Contraction Mapping Theorem, there exists a unique continuous solution $W_n(q)$ and U.$\blacksquare$

Define $\bar{n}^W(q)$ as the lowest employment level beyond which worker's participation constraint binds, i.e. $W_n(q) \le U$. If search on the job is at least as efficient as when unemployed, i.e. $\kappa \ge 1$, then the participation constraint for the worker never binds for $n \le n^*$. To see it, subtract equation (15) from equation (16) to get

$$W_n(q) - U = \frac{\left\{ \begin{array}{c} \tilde{u}_n(q) - b + \lambda(\theta)\int(\kappa \max\{W' - U, W_n(q) - U\} - \max\{W' - U, 0\})\,dF(W') \\ +H_n(q)\max\{W_{n+1}(q) - U, 0\} + s_n(q)(n-1)\max\{W_{n-1}(q) - U, 0\} \\ +\mu\int_{\underline{q}}^{\overline{q}} \left(\begin{array}{c} \mathbf{1}\left[n > \bar{n}^F(q')\right]\frac{\bar{n}^F(q')}{n}\max\left\{W_{\bar{n}^F(q')}(q') - U, 0\right\} + \\ \mathbf{1}\left[n \le \bar{n}^F(q')\right]\max\{W_n(q') - U, 0\} \end{array} \right)\phi(q')dq' \end{array} \right\}}{r+\delta+s_0+\lambda(\theta)\kappa+H_n(q)+s_n(q)(n-1)+\mu}$$

which is nonnegative for all $\tilde{u}_n(q) \ge b$ due to the option value of a firm getting a positive productivity shock and/or adjusting its labor force. This result implies that as long as worker's current utility is higher or equal to b, she will never quit to unemployment. However, for all $n > n^*(q)$, as well as for other n when $\kappa < 1$, one needs to verify that worker's participation constraint $W_n(q) - U \ge 0$ is satisfied.

Note that the value of working at the firm is not necessarily monotone in its workforce. Wages, on the other hand, are decreasing in employment. Therefore, this model can potentially generate job-to-job transitions that are associated with wage cuts since the change in the value of employment may still be positive.

3.5 Firm's Problem

This subsection provides details on employment changes within a firm. Firms can adjust their labor force by recruiting new workers or firing workers. To hire a worker, the firm needs to post vacancies that are then randomly matched with job seekers. In addition, the employment decision of the firm is affected by the separation rate at which some of the existing workers quit to unemployment or to a better job. Both hiring and separation rates depend on aggregate market tightness, unemployment rate and overall distribution of vacancies and workers across firm types.

For the firm's problem it is useful to write hiring and separating rates explicitly. The rate at which workers separate from the firm with productivity q and employment n is the sum of exogenous quit rate to unemployment and job-to-job transition, i.e.

$$s_n(q) = s_0 + \lambda(\theta)\kappa\left[(1 - F(W_n(q)))\right], \tag{17}$$

where $F(W_n(q))$ is the fraction of vacancies posted by firms that provide workers with the value of employment of at most $W_n(q)$.

The probability that any offer is acceptable to a randomly contacted worker is

$$a_n(q) = \begin{cases} \frac{u+(1-u)\kappa G(W_n(q))}{u+(1-u)\kappa}, & \text{if } W_n(q) - U \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

where u is the fraction of unemployed workers and $G(W_n(q))$ is the fraction of employed workers who get the value of employment of at most $W_n(q)$. Employed job seekers are weighted by their search intensity. If worker's participation constraint is binding then no worker would accept firm's offer. Then, the hiring rate is equal to $H_n(q) = a_n(q)\upsilon_n(q)\omega(\theta)$, where $\omega(\theta)$ is the rate at which vacancies are matched with workers and $\upsilon_n(q)$ is the number of vacancies posted by the firm with n employees and productivity level q. The rate $\omega(\theta)$, that is exogenous to the firm, is derived from the matching function and is equal to $\frac{\lambda(\theta)}{\theta}$.

The value of the firm with productivity q and employment n, $V_n(q)$, expressed in final output terms, solves the following Bellman equation

$$(r+\delta)V_n(q) = \max\left\{ \begin{array}{c} \frac{\pi_n(q)}{P} + \max\limits_{\upsilon \geq 0}\left\{a_n(q)\omega(\theta)\upsilon\left[V_{n+1}(q) - V_n(q)\right] - c_\upsilon(\upsilon)\right\} \\ +s_n(q)n\left[V_{n-1}(q) - V_n(q)\right] \\ +\mu\int\limits_{\underline{q}}^{\overline{q}}\left(V_n(q') - V_n(q)\right)\phi(q')dq' \end{array}, \; (r+\delta)V_{n-1}(q) \right\}, \tag{19}$$

under the assumption that firing a worker is costless and worker's participation constraint is satisfied. If the worker's constraint binds for some $n > \bar{n}^W(q)$ then workers quit randomly and $V_n(q) = V_{\bar{n}^W(q)}(q)$.

The first term on the right in equation (19) is firm's profit flow expressed in final good terms, as defined in equation (11). The second term refers to the capital gain that is obtained from the possibility of hiring an additional worker, given the optimally chosen vacancy posting decision. The third term is the expected capital loss related to the possibility that any worker quits. And the last term accounts for the expected change in the value of the firm caused by the shock to firm's productivity, q.

Proposition 2: Equation (19) has a unique continuous solution, $V_n(q)$, if $c_\upsilon(\upsilon)$ is strictly increasing, convex and $c_\upsilon(0) = c'_\upsilon(0) = 0$ and $a_n(q)$ and $s_n(q)$ are continuous in n and q.

Proof: Equation (19) has a unique solution that is a fixed point of the contracting mapping

$$[TV](q,n) = \max\left\{ \max_{\upsilon\geq 0} \frac{\left(\begin{array}{c} \frac{\pi_n(q)}{P} + a_n(q)\upsilon\omega(\theta)V_{n+1}(q) - c_\upsilon(\upsilon) \\ +s_n(q)nV_{n-1}(q) + \mu\int\limits_{\underline{q}}^{\overline{q}} V_n(q')\phi(q')dq' \end{array}\right)}{r+\delta+a_n(q)\upsilon\omega(\theta)+s_n(q)n+\mu}, V_{n-1}(q) \right\}. \tag{20}$$

Given that profit function is bounded from above by (13), $c_\upsilon(\upsilon)$ is strictly increasing and convex and $c_\upsilon(0) = c'_\upsilon(0) = 0$, and firing a worker is costless, T maps the set of non-negative continuous functions bounded from above by $\bar{V}(q) = \frac{\bar{\pi}(q)/P}{r+\delta+\mu} + \frac{\mu}{r+\delta+\mu}\int\limits_{\underline{q}}^{\overline{q}} \frac{\bar{\pi}(q')/P}{r+\delta}\phi(q')dq'$ into itself. As this set is compact under the sup norm, one needs only to confirm that the map satisfies Blackwell's sufficient conditions for a contraction. First, note that T is monotone. Moreover,

$$T\left[V_n(q)+z\right] = T\left[V_n(q)\right] + \frac{a_n(q)\upsilon_n(q)\omega(\theta)+s_n(q)n+\mu}{r+\delta+a_n(q)\upsilon_n(q)\omega(\theta)+s_n(q)n+\mu}z,$$

where $\upsilon_n(q) = \arg\max_{\upsilon\geq 0}\{a_n(q)\,\omega(\theta)\,\upsilon\,[V_{n+1}(q) - V_n(q)] - c_\upsilon(\upsilon)\}$. The map T discounts if $\upsilon_n(q) < \infty$, a condition that holds under the assumption that $c_\upsilon(\upsilon)$ is strictly increasing and convex and the fact that $0 \leq V_{n+1}(q) - V_n(q) \leq \bar{V}(q)$ due to boundedness and non-negativity of V function and free firing. Hence, T is indeed a contraction mapping. Therefore by Contraction Mapping Theorem, there exists a unique solution to equation (20). ■

Since profit falls without a bound as firm's labor force increases, there exists an upper limit on employment, $\bar{n}^F(q)$, beyond which a firm fires workers. That is, $\bar{n}^F(q)$ is the lowest number of workers for which $V_{\bar{n}^F(q)+1}(q) = V_{\bar{n}^F(q)}(q)$. Note that if worker's participation constraint is binding and $\bar{n}^W(q) < \bar{n}^F(q)$ then workers quit, in which case the maximum labor force $\bar{n}(q)$ is determined by the worker's problem. Therefore, the maximum employment is defined as the minimum of the two threshold values, i.e. $\bar{n}(q) = \min\left\{\bar{n}^F(q), \bar{n}^W(q)\right\}$. Note that $\bar{n}(q)$ is also the lowest number of workers for which firms post zero vacancies (if worker's participation constraint is binding the acceptance probability, $a_n(q)$, is zero which implies zero vacancy posting).

Proposition 3. $V_n(q) - V_{n-1}(q)$ is strictly positive for all $n \leq n^*(q) \leq n^W(q)$, that is $\bar{n}^F(q) \geq n^*(q)$.

Proof. Note that costless firing implies $V_n(q) - V_{n-1}(q) \geq 0$. To show that the difference is strictly positive for $n \leq n^*(q)$, one can apply proof by induction. Differencing equation (**??**) leads to

$$
\begin{aligned}
& (r+\delta+\mu+a_{n-1}(q)\,\omega(\theta)\,\upsilon_{n-1}+s_n(q)\,n)\,(V_n(q)-V_{n-1}(q)) \qquad (21)\\
= \quad & \max\left\{\begin{array}{ll} \frac{\pi_n(q)-\pi_{n-1}(q)}{P} + c_\upsilon(\upsilon_{n-1}) + \max\limits_{\upsilon\geq 0}\{a_n(q)\,\omega(\theta)\,\upsilon\,[V_{n+1}(q)-V_n(q)] - c_\upsilon(\upsilon)\} & \\ +s_{n-1}(q)\,(n-1)\,[V_{n-1}(q)-V_{n-2}(q)] + \mu\int\limits_{\underline{q}}^{\bar{q}}(V_n(q')-V_{n-1}(q'))\,\phi(q')dq' & ,0 \end{array}\right\}.
\end{aligned}
$$

First, note that the assertion holds for $n = 1$; that is, $V_1(q) - V_0(q) > 0$ since $c_\upsilon(\cdot)$ is increasing and $c_\upsilon(0) = 0$, the benefit from posting vacancies is non-negative and $\pi_1(q) > 0$. Then under the assumption that $V_{n-1}(q) - V_{n-2}(q) > 0$, also $V_n(q) - V_{n-1}(q) > 0$ given that profit is increasing in n for all $n \leq n^*(q)$. That completes the induction proof. ■

Proposition 3 shows that, similarly to the original work of Stole and Zwiebel ([38]), firms recruit workers up to the level of employment that maximizes per period profit flow under the assumption that worker's participation constraint is satisfied. Contrary to their work, however, labor hoarding effect may arise in this model if $\bar{n}(q) > n^*(q)$. Intuitively, in the economy with search frictions, a firm might choose to hire workers above profit-maximizing employment level in anticipation of the positive productivity shock.

3.6 Steady State Conditions

(a) **Size distribution**

The firms are identical ex ante and their type is revealed upon the entry, where $\phi(q)$ is the density of potential entrants types. Under the assumption that shocks to q are drawn from the same distribution, $\Phi(\cdot)$, the productivity distribution at entry is preserved also among existing firms. Then the steady state number firms conditional on type is derived by equating market entry and exit

$$\delta K(q) = \eta\phi(q), \tag{22}$$

where η is the aggregate entry rate, $\phi(q)$ is the density of entrants of type q and δ is the proportion of existing firms that become obsolete and exit the market.

Denote by $K_n(q)$ the aggregate number of products supplied by the set of firms of type q with employment n. Steady state mass of firms of type q with employment n is derived by equating inflows into and outflows from $K_n(q)$. First, for all $n \in [1, \bar{n}(q) - 1]$ the following relationship must hold

$$\begin{aligned} & H_{n-1}(q) K_{n-1}(q) + s_{n+1}(q)(n+1) K_{n+1}(q) + \phi(q) \mu \int_{\underline{q}}^{\overline{q}} K_n(q') \, dq' \\ = \; & H_n(q) K_n(q) + s_n(q) n K_n(q) + \delta K_n(q) + \mu K_n(q), \end{aligned} \tag{23}$$

where the first two terms on the left-hand side represent the expected hiring and separation flows, $\phi(q)\mu \int_{\underline{q}}^{\overline{q}} K_n(q') \, dq'$ is the proportion of products that are hit by the idiosyncratic shock and that become $q-$type. The outflow consists of transition flows from n workers to $n+1$ due to new hires and to $n-1$ workers due to quits, firm destruction, $\delta K_n(q)$, and becoming a different type, $\mu K_n(q)$.

For $n = 0$, equation (23) includes an additional term that accounts for entry of new firms of type q, $\eta \phi(q)$. Thus, the steady state relationship for $n = 0$ reads

$$\begin{aligned} & s_1(q) K_1(q) + \phi(q) \mu \int_{\underline{q}}^{\overline{q}} K_0(q') \, dq' + \eta \phi(q) \\ = \; & H_0(q) K_0(q) + \delta K_0(q) + \mu K_0(q). \end{aligned} \tag{24}$$

In addition, equation (23) has to be modified for $n = \bar{n}(q)$ to account for the possibility of firing a worker in case of an adverse productivity shock, i.e.

$$\begin{aligned} & H_{\bar{n}(q)-1}(q) K_{\bar{n}(q)-1}(q) + \phi(q) \mu \int_{\underline{q}}^{\overline{q}} \sum_{n=\bar{n}(q)}^{\bar{n}(q')} K_n(q') \, dq' \\ = \; & s_{\bar{n}(q)}(q) \bar{n}(q) K_{\bar{n}(q)}(q) + \delta K_{\bar{n}(q)}(q) + \mu K_{\bar{n}(q)}(q). \end{aligned} \tag{25}$$

Note that the last equation uses the fact that $\upsilon_{\bar{n}(q)}(q) = 0$ and $K_n(q) = 0$ for $n > \bar{n}(q)$ since there are no firms with employment that exceeds $\bar{n}(q)$. Therefore, $K_n(q) = 0$ for all $n > \bar{n}(q)$ serves as the boundary condition for the second order difference equation defined in $(23) - (25)$.

The steady state distribution of firms of type q is $K(q) = \sum_{n=0}^{\bar{n}(q)} K_n(q)$. Summing equation (23) over $n \in [1, \bar{n}(q) - 1]$ and using equations (24) and (25) one recovers equation (22).

The steady state number of workers employed by all firms of type q is equal to

$$N(q) = \sum_{n=1}^{\bar{n}(q)} n K_n(q). \tag{26}$$

(b) **Aggregate output, vacancies, unemployment and market tightness**

In equilibrium, total output produced by intermediate firms should be equal to aggregate demand, i.e.

$$Y = \left[\int_{\underline{q}}^{\bar{q}} \sum_{n=1}^{\bar{n}(q)} (q h_n(q) n)^{\frac{\sigma-1}{\sigma}} K_n(q) dq \right]^{\frac{\sigma}{\sigma-1}} \tag{27}$$

$$= \left(\frac{\sigma - 1}{\chi \xi \sigma} \right)^{\frac{1}{\xi-1}} \left[\int_{\underline{q}}^{\bar{q}} q^{\frac{\xi(\sigma-1)}{(\xi-1)\sigma+1}} \sum_{n=1}^{\bar{n}(q)} n^{\frac{(\xi-1)(\sigma-1)}{(\xi-1)\sigma+1}} K_n(q) dq \right]^{\frac{(\xi-1)\sigma+1}{(\xi-1)(\sigma-1)}} .$$

Total number of vacancies posted by all firms is

$$\upsilon = \int_{\underline{q}}^{\bar{q}} \sum_{n=0}^{\bar{n}(q)} \upsilon_n (q) K_n(q) dq. \tag{28}$$

The market tightness is defined as the ratio of total vacancies to job seekers

$$\theta = \frac{\upsilon}{u + (1-u)\kappa}. \tag{29}$$

The matching function is assumed to have constant returns to scale in job seekers and vacancies, then the job finding rate is $\lambda(\theta)$ and the worker meeting rate can be expressed as $\omega(\theta) = \frac{\lambda(\theta)}{\theta}$.

The unemployment rate can be derived from the labor market clearing condition that states that labor supplied to the market should be equal to total employment across all firm,

$$1 - u = \int_{\underline{q}}^{\bar{q}} N(q) dq. \tag{30}$$

(*c*) $F(W_n(q))$ **and** $G(W_n(q))$ **distribution functions**

The distribution of job offers $F(W_n(q))$ is merely the proportion of all vacancies that are posted by firms that provide workers with the value of employment of at most $W_n(q)$, i.e.

$$F(W_n(q)) = \frac{\int_{\underline{q}}^{\bar{q}} \sum_{n=0}^{\bar{n}(q')} 1\left[W_{n'}(q') \leq W_n(q)\right] \upsilon_{n'}(q') K_{n'}(q') dq'}{\upsilon}. \tag{31}$$

Similarly, the steady state distribution of workers $G(W_n(q))$ refers to the fraction of total workforce in the economy employed at jobs that guarantee the value of employment of at most $W_n(q)$, i.e.

$$G(W_n(q)) = \frac{\int_{\underline{q}}^{\bar{q}} \sum_{n=0}^{\bar{n}(q')} 1\left[W_{n'}(q') \leq W_n(q)\right] n' K_{n'}(q') dq'}{\int_{\underline{q}}^{\bar{q}} N(q) dq}. \tag{32}$$

Combining equations (23) – (25) with the definitions of $F(W_n(q))$ and $G(W_n(q))$ given in equations (31) and (32) after some manipulations leads to a familiar condition for the steady state unemployment rate that equates flows into and out of unemployment. That is, the steady state unemployment rate solves the following equation:

$$\lambda(\theta)u = (1-u)\left[\delta + s_0 + \mu \int\limits_{\underline{q}}^{\bar{q}} \phi(q) \frac{\int\limits_{\underline{q}}^{\bar{q}} \sum\limits_{n=\bar{n}(q)}^{\bar{n}(q')} (n-\bar{n}(q)) K_n(q')\,dq'}{\int\limits_{\underline{q}}^{\bar{q}} N(q)\,dq} dq\right], \tag{33}$$

where the left-hand side refers to the outflow from unemployment due to finding a job and the right-hand side includes the inflow into the pool of unemployed workers due to three reasons: destruction shock, exogenous quit, and layoff in the case of an adverse productivity shock[8]. The last term is computed as the product of the rate at which productivity shock arrives, μ, and the average proportion of workers that are laid off when the firm's productivity changes.

Denote the expected proportion of laid-off workers by

$$l = \int\limits_{\underline{q}}^{\bar{q}} \phi(q) \frac{\int\limits_{\underline{q}}^{\bar{q}} \sum\limits_{n=\bar{n}(q)}^{\bar{n}(q')} (n-\bar{n}(q)) K_n(q')\,dq'}{\int\limits_{\underline{q}}^{\bar{q}} N(q)\,dq} dq$$

then the steady state unemployment rate can be written as

$$u = \frac{\delta + s_0 + \mu l}{\delta + s_0 + \mu l + \lambda(\theta)}. \tag{34}$$

3.7 Equilibrium

Definition: A steady state market equilibrium is a set of numbers (θ, u, U, Y), a set of functions defined on a state space $(W_n(q), V_n(q), \upsilon_n(q), K_n(q), G(W_n(q)), F(W_n(q))) : [\underline{q}, \overline{q}] \times I_+ \to R_+$, a set of functions defined on firm types $(K(q), N(q)) : [\underline{q}, \overline{q}] \to R_+$, that satisfy equations (15), (16), (19), $(22) - (32)$.

To find a steady state equilibrium, one has to look for a fixed point in the mapping where worker's and firm's problems are solved given the aggregate market tightness, unemployment rate, aggregate demand and distribution functions of vacancies and workers across firm types. Then the aggregate objects and steady state distribution of firms are updated using the optimal employment adjustment decisions of firms. Appendix A3 provides details on steady state equilibrium solution algorithm used in the model simulation procedure described below.

4 Calibration

This section shows the fit of the model to Danish firm data. The focus of the calibration exercise is to demonstrate how well the model can replicate labor adjustment patterns, as well as relationships between wages, employment and productivity observed in the data. The empirical evidence is based on a matched employer-employee data set that is drawn from a panel of Danish administrative firm data. In addition to work hours and employment records described in Section 2, the data contains information on quarterly

[8] Here, a term 'layoff' is used loosely since maximum labor force can be determined by worker's problem, i.e. $\bar{n}(q) = \bar{n}^W(q)$, in which case workers quit.

payroll costs for the period of 1999-2006 and purchases and sales of all VAT-liable businesses for period of 2002-2006 (see Appendix A1 for details).

For the simulation, the vacancy posting costs is parameterized as $c_v(\upsilon) = c_0 \upsilon^{c_1}$, with $c_0 > 0$ and $c_1 > 1$. Parameter c_0 is important for matching job finding rate; whereas c_1 affect the standard deviation of hiring rate. The matching function is assumed to have constant returns to scale in job seekers and vacancies, i.e.

$$M(\upsilon, u + (1-u)\kappa) = m\upsilon^{\zeta}(u - (1-u)\kappa)^{\zeta-1},$$

with $0 < \zeta < 1$ and a scaling parameter $m > 0$ such that $M(\upsilon, u + (1-u)\kappa) \leq \min\{\upsilon, u + (1-u)\kappa\}$. The parameter m represents the efficiency of the matching process. Then the job finding rate can be expressed as $\lambda(\theta) = m\theta^{\zeta}$ and the worker meeting rate as $\omega(\theta) = m\theta^{\zeta-1}$. Note that m and ζ cannot be identified separately. Thus, I set ζ to 0.5.

The model is simulated under the assumption that the economy is in steady state (therefore the aggregate time effects are removed from the data series). In order to obtain the employment path for each firm, I first solve for type conditional equilibrium hiring and separation rates $H_n(q)$ and $s_n(q)$, as well as the maximum labor force size $\bar{n}(q)$. Given Poisson arrival rates for destruction and productivity shocks and hires and separations, the waiting time until the next occurrence of any of the shocks is distributed exponentially with parameter $H_n(q) + s_n(q)n + \delta + \mu$. Then, the employment histories of firms are simulated by drawing randomly from exponential distribution and then aggregated to monthly series (see Appendix A3 for details on solving for a steady state equilibrium, on parameter choice and simulation of the model).

Monthly employment is defined as all individuals who were on payroll in a given month, that includes workers who got hired during the month and those who separated during the same month. Work hours, wages and value added are aggregated to quarterly series to mimic the reporting frequency in the data sources used in this paper. Moreover, the hours measure is constructed according to the pension contribution payment scheme, in parallel to the hours measure observed in the data.

In the calibration exercise below I simulate the economy with 1000 firms for 300 months. In the following subsections, I present some qualitative predictions of the model and compare them with Danish firm data.

4.1 Employment Distribution

A well-established fact in the literature is that the size distribution of firms is highly skewed to the right with a very long right tail. That is, most of the firms in the data are small with a few firms that have much larger than average workforce. In order to replicate the empirical size distribution, the highly skewed distribution for underlying productivity q in required. However, even then the model still falls short in matching the size distribution[9]. It is natural to think of large firms as multi-product entities with separate hiring and separation processes for each product line. Hence, I assume that firms act as a collection of product lines and that each product faces its own hiring and separation process.

Lentz and Mortensen [27] develop a model where the number of product lines for each firm is a result of costly innovation process and product destruction. In their model, more productive firms innovate more frequently and in steady state supply a larger share of the product varieties. Here, I assume for simplification that the distribution of products across firms is exogenous and independent of firm productivity q. In this case, all steady state equilibrium equations hold and one can think of $K_n(q)$ as a distribution of product lines.

[9] Moreover, allowing for firms with greater employment increases the time required to solve the model exponentially.

I use three parameter (location, scale and shape) generalized Pareto distribution where the location parameter is such that the firm with the lowest productivity would hire at least one worker and the scale and shape parameters are set to match the observed dispersion and median to mean ratio in firm size. The number of product lines per firm is drawn randomly from the Poisson distribution at the start of the simulation and evolves as a birth-death process with the birth rate η and death rate δ.

Under these assumptions, the model is able to replicate the employment distribution observed in the data fairly well (see Figure 3). Table 5 shows mean, standard deviation of and median employment for firms in the data and in the model (the empirical moments exclude top one percent of firms).

Table 5: Employment

	$E(N_t)$	$\sigma(N_t)$	$Med(N_t)$
Data	19.1	29.9	9.7
Model	21.6	26.5	14

Source: Author's tabulation from the Danish firm data and simulated data. Empirical moments exclude firms that have than 5 employees for six consecutive months, as well as top one percent of firms.

4.2 Hours and employment adjustment

In the model, a firm respond to a positive shock in profitability by increasing labor utilization and posting more vacancies. However, given search frictions in the labor market, it takes time to recruit new workers. As the vacancies start filling up, hours of work begin to fall. In the case of a negative demand shock the mechanism is slightly different. The initial response of the firm is cut work hours of existing employees and, if firm's productivity falls too low, to fire some workers immediately. However, following the initial cut in employment the firm mainly relies on now higher attrition rate to drive its workforce further down. Thus, the model can capture the mechanism through which firms trade off number of workers and work hours.

There are a few parameters that are crucial for matching the dynamic interaction between employment and hours adjustment. First, more persistent shock process (in terms of a lower arrival rate of the profitability shocks, μ) makes the trade-off between average hours of work and number of workers stronger. Here, I choose a parameter μ such that on average a shock to firm's productivity arrives once in ten months.

Second, higher vacancy cost parameter makes firms post less vacancies. However, it has a countervailing effect of increasing the vacancy filling rate and thus raise the return on vacancies. Thus, the effect of a higher cost on the speed of employment adjustment may be non-monotone. Instead, I pin down vacancy posting costs to match the job finding rate (that implies average duration of unemployment of about five months) and use the efficiency of search parameter m in the matching function to effectively make recruiting process more sluggish.

Third, the elasticity of substitution between intermediate goods, σ, affect the response of the firm to the profitability shock. Intuitively, lower value of σ implies that profit is less sensitive to productivity shock q and thus the incentive to recruit workers is weakened. On the other hand, it also dampens the association between firm's value added and labor productivity that is found to be fairly strong and positive in the data. Here, I choose the value of σ such that the latter relationship is replicated by the model.

Beyond the parameter choice, time aggregation and measurement issues appear to be important when trying to replicate the relationship between growth of hours and employment observed in the data. These will be discussed below. First, I report the variation in employment and average hours growth rates in the model, then I look at the comovement of the two series.

Figure 4 displays the distribution of quarterly labor adjustment series in the simulated data. The model captures the variation in employment growth quite well, but underestimates the variation in average hours growth (see Table 6). Recall that changes in hours are reported in the data only when workers switch between 9-hour intervals. Given that in the model all individuals are identical, they work the same amount of hours; thus, the change in hours will be observed only if all workers switch intervals. Thus, heterogeneity of workers in the data can account for the difference in variation in hours growth in the data and in the model.

Table 6 compares the empirical and simulated moments. In terms of matching the trade-off between hours and workers adjustment, the model does fairy well at monthly frequency; however, when the simulated data are aggregated to quarterly series the correlation between changes in hours and employment increases. The model is more successful in reproducing the feature of the data that hours growth is leading growth in workforce.

Table 6: Hours and employment adjustment.

	Model		Data: Quarterly freq.	
	Monthly freq.	Quarterly freq.	Raw series	Weighted, no time effects
$\sigma(\Delta n_t)$	0.187	0.290	0.242	0.215
$\sigma(\Delta h_t)$	0.148	0.098	0.248	0.221
$\rho(\Delta h_t, \Delta n_t)$	-0.441	-0.097	-0.340	-0.469
$\rho(\Delta h_{t-1}, \Delta n_t)$	0.253	0.069	0.071	0.115

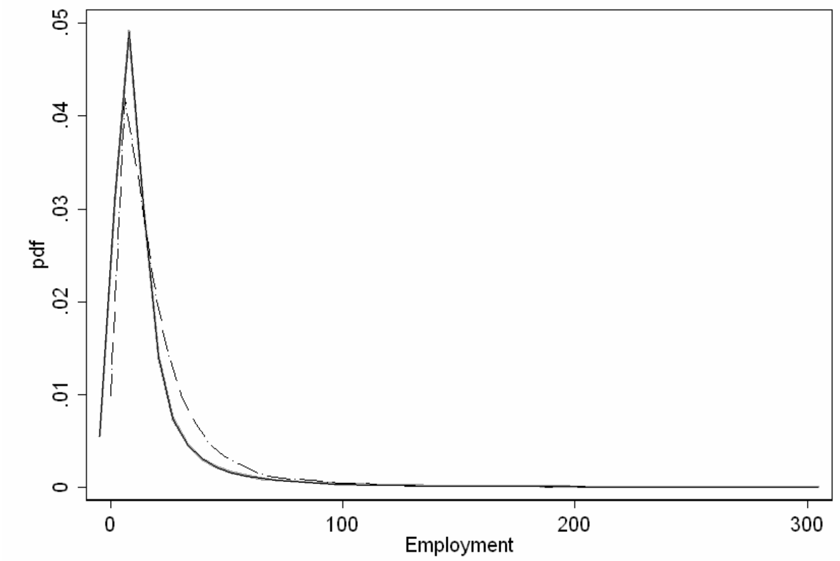
Source: Author's tabulation from the Danish firm data (1999-2006) and the simulated data. Empirical moments exclude firms with less than 5 employees for six consecutive months.

In general, time aggregation and frequency at which the data is reported seem to be important for observing the negative association between labor adjustment on intensive and extensive margins. Note that the correlation increases also in the data although to a much smaller extent (from -0.340 at quarterly frequency to -0.310 and -0.284 at semi-annual and annual frequency, respectively).

According to the results shown above, it appears that in the model most of the adjustment in the workforce is completed within a quarter, so that convex vacancy posting costs and search frictions are not enough to produce enough inactivity in employment. Introducing non-convex labor adjustment cost does remedy the problem. First, fixed firing costs appear to be ineffective since steady state distribution of firms in equilibrium is concentrated on low values of employment; thus, there is a mass of firms for which firing costs would be important is negligible. Moreover, under the assumption that wages and profits are not re-negotiated if worker's or firm's participation constraint is binding, the distinction between layoffs and quits, and correspondingly paying firing costs or not, would be somewhat arbitrary. Adding a fixed component to vacancy posting costs results in inactivity region for all $n > n'(q)$. However, this threshold value seems to increase in q for given parameter values; in that case a positive shock would still raise vacancies and therefore employment.

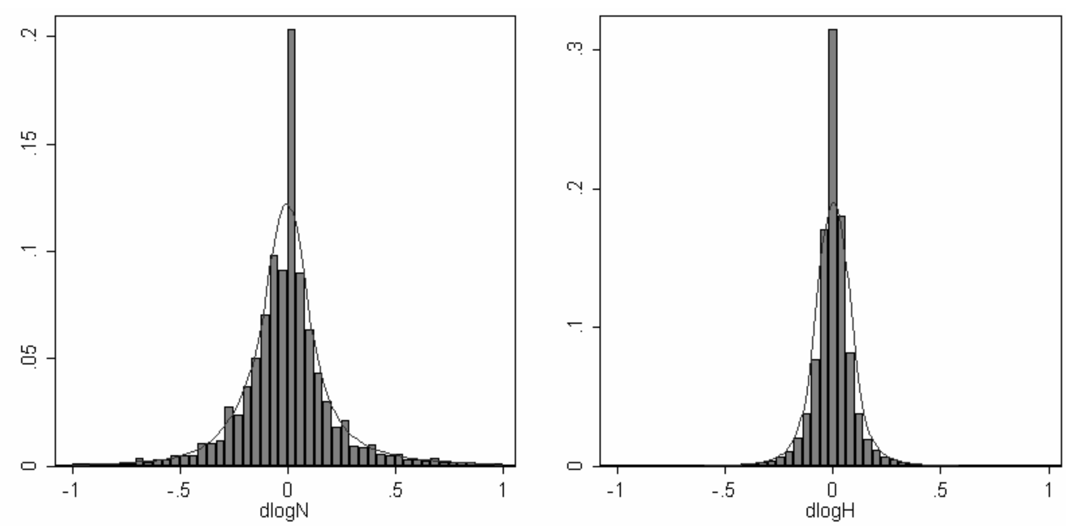
One possible way to slow down the response in employment to shocks is to introduce a recognition lag when the firm is uncertain about how large the shock is (or whether it is temporary or permanent). Adding the recognition lag to the model, however, would greatly complicate it by making value functions of workers and firms time-dependent. Understanding the factors influencing the negative correlation between growth rates of hours and employment, and trying to bring the model closer to the data, remains an area for future work.

Figure 3: Size distribution in the data (solid line) and the model (dashed line)



Note: Density estimation is based on Gaussian kernel with bandwidth of 1. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 1999-2006.

Figure 4: Growth rate of employment (left panel) and average work hours (right panel) in the simulated data



Note: Vertical axis shows a fraction of firm-quarter observations. Density estimation is based on Uniform kernel with bandwidth of 0.1.

4.3 Worker and Job Flows

On-the-job search is a necessary component for the model to capture most of features of the data related to worker and job flows. Allowing workers to search while employed implies that both separation and offer acceptance rates depend on firm's type. More productive firms face higher retention rates and are able to attract workers faster than their less productive counterparts[10]. The model also predicts that a sizable workforce reduction can be brought about through separation in case of an adverse profitability shock.

In the comparison of the empirical and simulated moments that follows, I restrict attention to continuing firms only since the model does not have a rich theory of entry and exit. Table 7 presents the average monthly worker and job flow rates in the data that are matched almost precisely by the model. The assumption that the economy is in steady state implies zero average employment change that imposes symmetry on hires and separations, as well as job creation and job destruction. Also, to fit the average separation rate, the search intensity for employed workers is required to be less than for unemployed workers, that is $\kappa < 1$.

Table 7: Monthly job and worker flow rates in the data and in the model.

	Model	Data
Hires	0.078	0.078
Separations	0.078	0.076
Job Creation	0.045	0.041
Job Destruction	0.044	0.039
Net employment change	0.000	0.002
Churning	0.067	0.075

Note: Empirical moments are size-weighted and include continuing firms only. Source: Author's tabulations from the Danish firm data (1999-2006) and the simulated data.

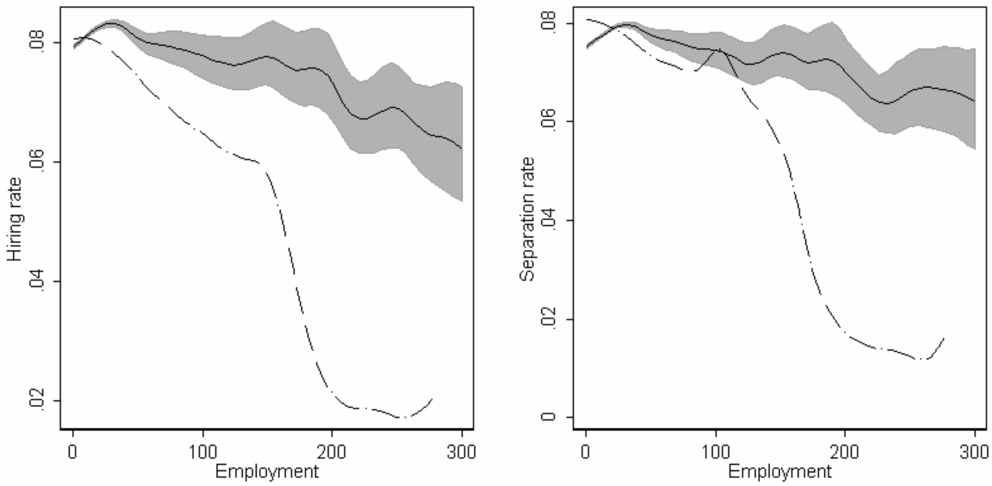
Figure 5 shows the relationship between hiring and separation rates and firm's size. It is apparent that worker turnover is more prominent in smaller firms: both hiring and separation rates decrease as employment rises. The model is capable of replicating this relationship; however, it seems to overestimate the fall in both rates.

Table 8 shows the relationship of monthly worker flows and firm employment adjustment. The model captures employment growth patterns fairly well - about 39.1% of workers are employed at firms with monthly employment change of less than 2.5% by absolute value; whereas in the data the corresponding number is 50.3% (see Table 4). The fact that contracting firms reduce their labor force mostly through separations; while growing firms increase their employment mostly through hiring is consistent with the model predictions. Furthermore, contracting firms exhibit positive hiring rates, albeit lower than observed in the data. In general, the model performs well for firms that adjust their employment by less than 10% , but underestimates the workforce turnover for firms with employment growth of more than 10%.

The previous studies (see for instance Christensen et al [13]) have reported that the separation rate is higher in low-pay jobs. Although in this paper hourly wage per se is not a sufficient statistics for the separation rate, the model admits a negative association between average hourly wages and separations (see Figure 6).

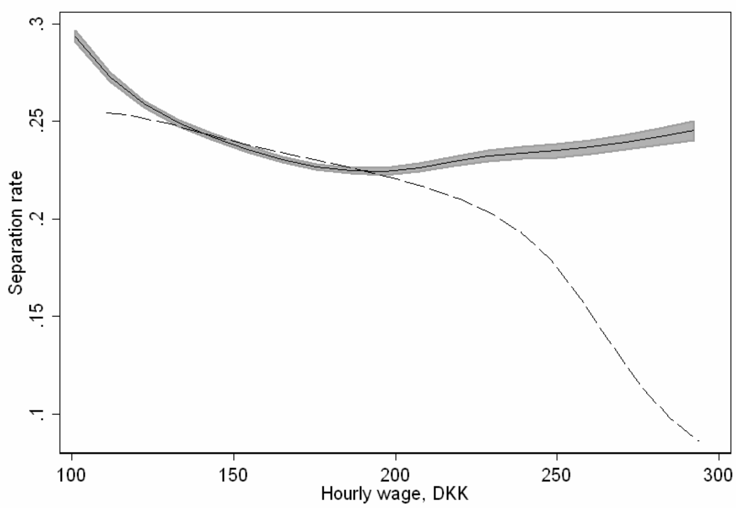
[10] Faberman and Nagypal [21] document that vacancy yield (the number of hires per vacancy) increases in employment growth. This result is consistent with the prediction of the model that more productive firms have higher acceptance rate of their vacancies.

Figure 5: Monthly hiring and separation rates vs average two-month employment in the data (solid line) and in the model (dashed line)



Note: Nadaraya-Watson estimator based on Gaussian kernel with bandwidth of 10. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Based on continuing firms. Source: Author's calculations based on the Danish firm data, 2006.

Figure 6: Quarterly (cumulative) separation rate vs hourly wage rate.



Note: Nadaraya-Watson estimator based on Gaussian kernel with bandwidth of 10. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 1999-2006

Table 8: Simulated average monthly hiring and separation rates, by net employment growth rate.

Net Emp. Growth	Hires	Sep.	Net	Emp. Share, %
Less than -0.10	0.023	0.279	-0.256	9.20
-0.10 to -0.025	0.034	0.089	-0.055	20.43
-0.025 to 0.025	0.045	0.044	0.000	39.09
0.025 to 0.10	0.090	0.036	0.054	21.07
More than 0.10	0.255	0.028	0.227	10.21

Note: Simulated moments are size-weighted by employment share.

4.4 Wages and Hours

The trade-off between changes in hours and employment in firm's labor demand comes from the two sources: labor adjustment costs that halt the adjustment in number of workers and costs of changing hours of work. If variations in hours are inexpensive then firms would make all the adjustment on the intensive margin and not through hires and layoffs. In the model, it is the convexity of disutility of working that ensures that marginal cost of employing a worker for an extra hour is increasing. The existing literature seems to support the claim that variations in hours are expensive in the data. There is a vast empirical evidence for the negative part-time/full-time wage premium documented in the literature (see Blank [7] for a review)[11]. The question of interest then is whether hourly wages are increasing in the Danish data.

The hourly wage measure is constructed as a ratio of total payroll cost paid in a given quarter to total labor input using the lower bound on average hours, i.e. $\frac{W}{NH_{LB}}$. Wages defined in this manner represent the upper bound on actual average hourly wages. The correlation between hourly wages and hours turn out to be slightly negative in the data. One explanation for this negative relationship might be the mismeasurement of wages - if wages are overestimated relatively more for low values of hours then one would expect to see a decline in hourly wages as hours rise. For that reason, I construct an alternative measure on hourly wages based on the upper bound of work hours, H_{UB}. It assumes the right bound of each 9-hour interval for all employees with positive pension contributions and assigns 9 hours to workers with zero contributions (see Appendix A1 for more details on construction of this variable). Then the hourly wages derived by using the upper bound on hours, $\frac{W}{NH_{UB}}$, represent the lower bound on wages. The difference between the two wage measures is more prominent for low values of hours, mainly due to H_{UB} measure including the employees that work less than 9 hours.

Figure 7 displays the relationship between two measures of wages (based on H_{LB} and H_{UB}) and work hours. The upper bound on wages displays a prominent drop in wages for low values of hours. On the contrary, the lower bound of wages is undoubtedly increasing in work hours. It seems that the negative association between wages and hours is attributable mainly to the noise in measurement.

Most importantly, the model is capable of reproducing the empirical relationship between wages and hours. The upper bound on wages (constructed in the same way as in the data) is non-monotone for low values of hours; while the lower bound on wages in increasing for the whole range of hours. In the simulation, the relationship between wages and hours flattens out for high values of hours; whereas in the data it seems to drift upward. Increasing the convexity of disutility of working function might improve the model on that account[12].

[11] Aaronson and French [1], for instance, document a 25% wage penalty for men who cut their work week from 40 to 20 hours; however, they reports no such effect on women.

[12] The parameter that governs the convexity of the utility costs arising from variations in hours, ξ, is set to be equal 2.5, within the range of values estimated by Cooper and Willis [15].

4.5 Wages and Labor Productivity

Figure 8 illustrates that there is a significant dispersion of average hourly wages and labor productivity (constructed as value added per work hour) found among firms. The productivity distribution is significantly dispersed and skewed to the right. The interquantile range to median ratio is 0.86; while the ratio of average value added per labor input for firms in the ninth decile of the productivity distribution relative to the average in the first decile is about 6 to 1. These facts are in line with those found in the other data sets (see Bartelsman and Doms [6] for a review).

The distributions of hourly wages and productivity in the model are shown in Figure 9. The correlation of the two series in the model is very close to one; hence, the simulated distributions follow each other very closely. Although the model can reproduce long right tails observed in the data, the simulated distributions are less skewed to the right than in the data. The interquantile range to median ratio for labor productivity in the model is 0.16.

To find some empirical support for the wage bargaining assumption in the model, I examine the association between wages and labor productivity in the data. Figure 10 depicts non-parametric regression of hourly wages on hourly labor productivity. More productive firms seem to pay higher wages on average, the finding that has been established for other countries and time periods (see for instance Dunne, Foster, Haltiwanger and Troske [19] for labor productivity and Baily, Hulten and Campbell [5] for TFP measure of productivity). This result is consistent with rent-sharing between workers and employers where workers in a more productive firms are able to extract higher wages. The correlation between the two series is 0.36 (0.29 for size-weighted series) in the data. The relationship appears to be concave: wages increase is more pronounced for lower part of productivity distribution. In model, however, the relationship between labor productivity and wages is almost linear with a correlation coefficient close to one.

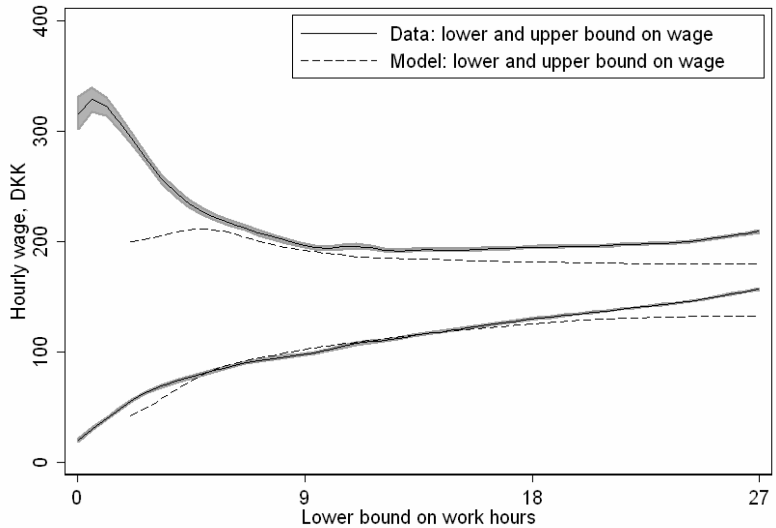
Mean wages appear to be increasing in firm employment, the finding that has been well-documented in many other studies (see for instance Oi and Idson [33], Moscarini and Postel-Vinay [32]). Left panel of Figure 11 shows that hourly wages are higher in larger firms. Despite the fact that Stole and Zwiebel bargaining process implies that wages are decreasing in employment, the model reproduces the increasing relationship between wages and firm size. The reason for that is the change in type composition of firms as their workforce rises: larger firms tend to be more productive on average which offsets a negative effect arising from increasing number of workers. In the data, the wages seem to flatten out as employment increases; in the model, on contrary, the effect is stronger for larger firms. Bear in mind though that there are very few observations in the simulated data with employment greater than 150 workers.

Right panel of the same figure depicts the positive association between firm employment and average work hours - larger firms employ workers for longer hours in the data, as well as in the model. The level of hours measure seems to differ. For the simulation, I chose to match average actual hours of work to 34 hours a week, instead of matching mean of the lower bound on work hours observed in the data.

Positive correlation between output and labor productivity has been previously reported in the literature. Baily, Bartelsman and Haltiwanger [4], for instance, find that the correlation between the two series is 0.29 in the unbalanced panel of US manufacturing firms. Likewise, the relationship between value added and productivity is found to be rather strong and positive in the Danish firm data and more so for size-weighted series (see Table 9). Figure 12 displays the association between the two series in the data (left panel) and in the model (right panel). The model fits the relationship between output and productivity fairly well[13].

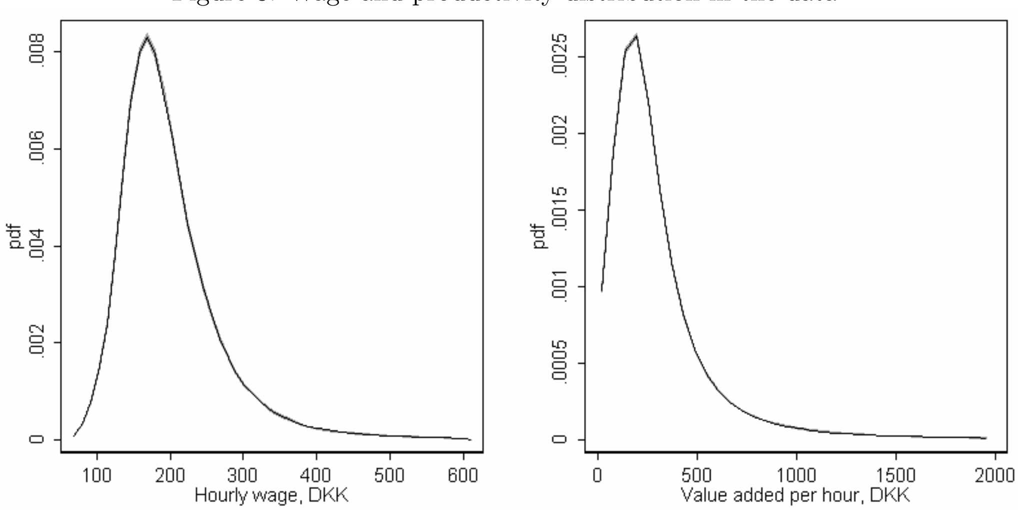
[13] There is a trade-off between matching size-productivity relationship and hours and employment growth relationship. Allowing for a long right tail of the distribution of underlying productivity q weakens the correlation coefficient between employment and hours growth rates. On the other hand, having a skewed to the right productivity distribution is required to replicate a positive relationship between output and labor productivity (so that the positive type composition effect

Figure 7: Hourly wages vs average work hours in the data (solid line) and in the model (dashed line).



Note: Nadaraya-Watson estimator based on Gaussian kernel with bandwidth of 0.5. Shaded area is 90% pointwise bootstrap confidence interval around upper and lower bound on hourly wage (clustered by firm ID). Wages are deflated by the quarterly CPI. Source: Author's calculations from the Danish firm data, 2006.

Figure 8: Wage and productivity distribution in the data



Note: Density estimation is based on Gaussian kernel with bandwidth of 5. Shaded areas are 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 2006.

Table 9: Relationship between average firm productivity and firm size.

	Model	Data: non-weighted	Data: no time effects, emp. share-weighted
	$\frac{R}{NH_{LB}}$	$\frac{R}{NH_{LB}}$	$\frac{R}{NH_{LB}}$
R	0.5168	0.0984	0.2970
N	0.4430	0.0005	0.0012
NH_{LB}	0.4407	0.0008	0.0091

Source: Author's tabulations from the Danish VAT statistics data, 2002-2006.

On the other hand, the model shows some discrepancy with the data when the comovement between productivity and labor input is concerned. As Lentz and Mortensen [27] point out, if technological progress is capital augmenting or neutral then one would expect to see more productive firms employing more people. However, the data seems to be on odds with this prediction as employment in a firm is virtually uncorrelated with firm's productivity (Table 9). The model, on the contrary, predicts a strong positive relationship between labor productivity and number of workers. Some form of labor saving productivity shocks are required to replicate this feature of the data (see Lentz and Mortensen [27]).

5 Conclusion

This paper is motivated by the observation that firms use variation in labor utilization extensively to adjust their labor demand in response to exogenous shocks. The goal of this paper has been to develop a model of labor adjustment that introduces variation in labor resources on both intensive and extensive margins. The model with search frictions in the labor market appears to be a natural candidate for formalizing firms recruiting strategies and factors that hinder employment adjustment.

This paper develops a theory of heterogeneous multi-worker firms that choose their hiring and firing policies optimally in the economy with search frictions. The driving force of the model is idiosyncratic profitability shocks that firms can accommodate by varying work hours of their existing employees or by adjusting their workforce. Wages are determined through bargaining. In addition, allowing for the on-the-job search delivers a rich theory of quits that enables the model to capture most of features of the data regarding worker flows.

The model is calibrated to assess its fit to the Danish firm data and appears to be quite successful in capturing the overall characteristics of the data. The model does an outstanding job of reproducing employment variation at the firm level. It matches hiring and separation rates, the distribution of firms by net employment growth and different retention and attrition rates depending on the firm size and wages paid. In addition, the empirical relationships between firm size, wages and productivity at the firm level are reproduced in the simulated data.

Regarding labor adjustment on the intensive margin, the model underestimates the variation in average work hours. The negative association between changes in number of workers and hours as reported in the data can be replicated in the simulation at the monthly frequency. At the quarterly frequency, however, the correlation rises above the level observed in the data, reflecting the importance of time aggregation for dynamic interaction between hours and employment. It seems that the model requires additional frictions (such as uncertainty on the nature of the shock, whether it is temporary or permanent) to slow down the response of employment. On the other hand, the empirical fact that changes in hours lead changes in employment is consistent with the model predictions.

offsets the negative effect of employment on average revenue). For instance, lognormal distribution was not capable of reproducing the positive output-productivity association.

The next step of this project is to estimate the model using indirect inference approach. Then, using the estimated structural parameters, I can examine the effect of hiring costs and search efficiency on firms optimal employment policies, firms profit and worker welfare. Moreover, I can perform policy experiments of such as changes in mandatory over-time premium, advanced layoff notice, etc. Likewise, I can evaluate the importance of hours channel in labor input adjustment at the firm level by running the counterfactual experiment that shuts down the hours margin completely.

In addition, extending the model to include aggregate shocks may be important for matching observed dynamic interaction between labor adjustment on the intensive and extensive margin. Intuitively, aggregate shocks will have different implications for firm employment strategies than idiosyncratic shocks do. For instance, in the case of a positive shock, in addition to posting more vacancies the firm is able to retain its workers better. However, if all firms experience the increase in their profitability at the same time, the attrition rate remains unchanged; hence, firms have to post more vacancies to reach the desired level of employment.

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Appendix

A1. Data Sources

The empirical analysis in this paper is based on Danish firm data drawn from administrative records for the period of 1999-2006. They come from four major sources. First, a full-time equivalent (FTE) employment series is derived from the firms' mandatory pension contribution data collected on a quarterly basis. These pension contributions are paid by a firm for each 16-66 year old employee according to her weekly hours of work. The rule for the pension contribution (depicted in Figure 13) is as follows:

- full amount of contribution (670.95 DKK in 1999-2005 and 731.70 DKK in 2006 per quarter) is paid for an employee working more than 27 hours a week;
- 2/3 of the full amount is paid for an employee working between 18 and 27 hours a week;
- 1/3 of the full amount is paid for an employee working between 9 and 18 hours a week;
- zero contribution is paid for all employees working less than 9 hours per week.

It is important to note that the mandatory pension contributions are differentiated in accordance with the collective wage agreements - for some employees in the public sector full contributions (A-type) are paid, while for other employees B, C or D contributions are paid. These rates of contributions make up, respectively about 40%, 60% and 48% of the ordinary contributions (further details on pension contribution rule can be found on Statistics Denmark website www.dst.dk and ATP Pension Fund website www.atp.dk). The exact distribution of employees by type of the pension plan is known only on a yearly basis; therefore, if some workers switch to a different plan within a year then the reported FTE measure will be incorrect. Given that B, C, or D contributions are paid primarily in the public sector,the empirical analysis is restricted to private firms. After exclusion of the public sector, there are about 0.6% of firm-quarter observations that report having paid other than A-type contributions. These observations are left out from the analysis to avoid the FTE measurement inaccuracy.

The total amount of pension contributions paid by a firm for all its employees in a given quarter is observed. Then, the FTE measure reported by the Danish Central Statistical Office is constructed as the total amount of quarterly pension contribution divided by the payment norm for a full-time employee (where full-time refers to working more than 27 hours per week). Given the proportionality of the pension contribution schedule, the average weekly hours of work in a firm can be derived by dividing total FTE measure, N^*, by the number of employees, N, and multiplying by 27 hours, i.e.

$$H_{LB} = 27\frac{N^*}{N}. \tag{LB}$$

This approach implicitly assumes the least work hours (i.e. left boundary of each interval) for each employee and therefore represents the lower bound on weekly hours of work.

In order to construct the upper limit of average work hours, I consider the right boundary point of each of 9-hour intervals in Figure 13. The assumption that employees can work at most 36 hours, albeit not very realistic, preserves the proportionality in hours schedule. Also, recall that the FTE measure exclude employees that work less than 9 hours per week. Therefore, if the number of workers in a given firm is higher than the number of full-time employees, I allocate 9 hours of work to those extra workers. In sum, the upper bound on weekly work hours per employee is defined as

$$H_{UB} = \frac{36N^* + 9(N - N^*)\mathbf{1}\left[N > N^*\right]}{N}. \tag{UB}$$

The second data source is a matched employer-employee panel that includes all individuals that have paid employment in a given month. Monthly employment is obtained as a head-count of all individuals employed in a given firm. Quarterly number of employees is derived as the average of three months employment. The particular structure of these data makes it possible to construct hires and separations series for each firm.

The third data set is drawn from the VAT statistics for 2002-2006. It provides information on purchases and sales of all VAT-liable businesses on a quarterly basis, measured in Danish Kroner (DKK). In Denmark a business enterprise must register for VAT if its annual turnover is expected to exceed 50,000 DKK. The VAT declaration frequency depends on the annual turnover: firms report monthly if their annual turnover exceeds 15 million DKK, quarterly if their turnover is between 1 million DKK and 15 million DKK, and semi-annually if it is below 1 million DKK. Hence, the empirical moments on value added and labor productivity in this paper refer to businesses with annual turnover above 1 million DKK (in total, 13.9% of firm-quarter observations are excluded due to missing quarterly information). Lastly, only data on firms with positive value added are used.

The fourth data set contains information on total payroll costs that firms paid in a given quarter. Wages are measured in Danish Kroner (DKK) and are deflated using quarterly CPI with 2001 Q1=100. In Denmark, there is no statutory national minimum wage since legal collective agreements are the main mechanism used for regulating low pay[14]. The hourly wages less than 80 DKK per hour are left out from the analysis. This figure is regarded as an estimate of the effective legal minimum wage. In addition, the

[14] The percentage of employees covered by collectively agreed wages is estimated above 80% (see for instance Danish Confederation of Trade Unions website www.lo.dk).

wage rates of the top one percent of the observed distribution are excluded.

The empirical analysis is carried out based on private firms information. Furthermore, the sample is restricted to firms with at least five employees to reduce the noise in the data, also these firms are largely excluded from the VAT statistics data set. In order to avoid the creation of spurious exit and entry flows into the sample when employment falls below five workers in one quarter and exceeds it in the next quarter, I apply the following sampling rule: the firm is considered to have exited the sample if its employment has been less than 5 employees for six consecutive months. In total, there are 53.7% of firm-quarter observations that fall into the category of 'exiting' firms; however, they comprise only 6.3% of total employment. The resulting data set has 120,058 firms that are observed in the data for 14 quarters on average. In addition, the time effects were removed from hourly wages and labor productivity series.

A2. Wage Bargaining

First, I show that in the Stole and Zwiebel framework the bargaining over hours and wages simultaneously is equivalent to the bargaining over wages when hours schedule is chosen by the employer prior to the bargaining at its optimal level. Second, I provide a solution to the differential equation in wages that arises as the outcome of the bargaining problem.

(a) Let us extend the basic Stole and Zwiebel [38] bargaining problem to include hours of work and continuous labor. Then wage and hours have to solve the following equation:

$$\max_{w_i,h_i}\left\{\begin{array}{c}\left(\frac{R\left(\bar{h}(n+\Delta n)n+h_i\Delta n\right)}{P}-\frac{\bar{w}(n+\Delta n)}{P}n-\frac{w_i}{P}\Delta n-\frac{R\left(\bar{h}(n)n\right)}{P}+\frac{\bar{w}(n)}{P}n\right)^{1-\beta}\times\\ \times\left(\left(\frac{w_i}{P}-g\left(h_i\right)-b\right)\Delta n\right)^{\beta}\end{array}\right\}, \tag{A1}$$

where $\bar{h}(n)$ is average work hours chosen by other workers in the firm with employment n, and $\bar{w}(n)$ is wages paid to other employees if total employment is n. The number of hours for employee i is chosen optimally to maximize the problem above, taking the work hours of other employees as given. The first order condition with respect to wage w_i reads

$$\begin{aligned}&\beta\left(\frac{R\left(\bar{h}\left(n+\Delta n\right)n+h_i\Delta n\right)}{P}-\frac{\bar{w}\left(n+\Delta n\right)}{P}n-\frac{w_i}{P}\Delta n-\frac{R\left(\bar{h}\left(n\right)n\right)}{P}+\frac{\bar{w}\left(n\right)}{P}n\right) \\ = \; &(1-\beta)\left(\frac{w_i}{P}-g\left(h_i\right)-b\right)\Delta n.\end{aligned} \tag{A2}$$

The first order condition with respect to h_i determines the optimal choice of hours as follows

$$\frac{R'\left(\bar{h}\left(n+\Delta n\right)n+h_i\Delta n\right)}{P}=g'\left(h_i\right). \tag{A3}$$

In fact, equation $(A3)$ is equivalent to equation (2), given the symmetry of the bargaining problem for all workers.

Next, note that the following limit can be rewritten as

$$\begin{aligned}
&\lim_{\Delta n\to 0}\frac{R\left(\bar{h}\left(n+\Delta n\right)n+h_i\left(n+\Delta n\right)\Delta n\right)-R\left(\bar{h}\left(n\right)n\right)}{\Delta n}\\
=\ &\lim_{\Delta n\to 0}\frac{R\left(\bar{h}\left(n+\Delta n\right)n+h_i\left(n+\Delta n\right)\Delta n\right)-R\left(\bar{h}\left(n+\Delta n\right)n\right)}{h_i\left(n+\Delta n\right)\Delta n}h_i\left(n+\Delta n\right)\\
&-\lim_{\Delta n\to 0}\frac{R\left(\bar{h}\left(n+\Delta n\right)n\right)-R\left(\bar{h}\left(n\right)n\right)}{\bar{h}\left(n+\Delta n\right)n-\bar{h}\left(n\right)n}\lim_{\Delta n\to 0}\frac{\bar{h}\left(n+\Delta n\right)-\bar{h}\left(n\right)}{\Delta n}n\\
=\ &R'\left(h\left(n\right)n\right)\left(h\left(n\right)+\frac{\partial h\left(n\right)}{n}n\right),
\end{aligned}$$

again, under the symmetry assumption. Likewise,

$$\lim_{\Delta n\to 0}\frac{w\left(n+\Delta n\right)\left(n+\Delta n\right)-w\left(n\right)n}{\Delta n}=w'\left(n\right)n+w\left(n\right).$$

Dividing equation $(A2)$ by Δn and taking limits as $\Delta n\to 0$, one obtains the bargaining problem that is identical to (3).

(b) Solving for the first order condition of (3) leads to a first-order linear differential equation in wage

$$w\left(n\right)=\beta R'\left(n\right)+\left(1-\beta\right)P\left[g(h\left(n\right))+b\right]-\beta w'\left(n\right)n. \tag{A4}$$

The solution of the homogenous equation $w'\left(n\right)+\frac{w(n)}{n\beta}=0$ is equal to

$$w\left(n\right)=An^{-\frac{1}{\beta}}, \tag{A5}$$

where A is a constant of integration of the homogenous equation. Assuming that A is a function of n and substituting $(A5)$ into $(A4)$ one gets

$$A'\left(n\right)=R'\left(n\right)n^{\frac{1-\beta}{\beta}}+\frac{\left(1-\beta\right)}{\beta}P\left[g(h\left(n\right))+b\right]n^{\frac{1-\beta}{\beta}},$$

or, by integration

$$A\left(n\right)=\int_0^n z^{\frac{1-\beta}{\beta}}\left(R'\left(z\right)+\frac{\left(1-\beta\right)}{\beta}Pg\left(h\left(z\right)\right)\right)dz+\left(1-\beta\right)Pbn^{\frac{1}{\beta}}+B,$$

where B is a constant of integration.

The last equation implies that the solution to $(A4)$ is

$$\frac{w\left(n\right)}{P}=n^{-\frac{1}{\beta}}\int_0^n z^{\frac{1-\beta}{\beta}}\left(\frac{R'\left(z\right)}{P}+\frac{\left(1-\beta\right)}{\beta}g\left(h\left(z\right)\right)\right)dz+\left(1-\beta\right)b, \tag{A4}$$

where to pin down B, I assumed as in Stole and Zwiebel [38] that wage is finite when $n\to 0$ which implies $B=0$[15].

A3. Simulation

(a) Finding Equilibrium

Given the model parameters $(r,\ \beta,\ \sigma,\ \mu,\ \delta,\ \eta,\ \chi,\ \xi,\ b,\ s_0,\ m,\ \zeta,\ \kappa,\ c\left(\cdot\right),\Phi\left(\cdot\right))$, the solution algorithm can be desribed as a fixed point search of equilibrium variables (θ,u,Y) and distribution functions

[15] Also, one assumes as in Stole and Zwiebel [38] that the conditions for the existence of the integral in (5) are satisfied.

$F(W_n(q))$ and $G(W_n(q))$ through the mapping defined by equations $(15)-(32)$. I solve for the equilibrium numerically applying iteration on the mapping that turned out to be more stable than looking for a fixed point using minimum distance routines.

Note that all value functions and steady state equilibrium equations can be rewritten in terms of $\hat{q} = Y^{\frac{1}{\sigma-1}}q$. In this way, I reduce the number of equilibrium variables to iterate over. Thus, I can start with the distribution of $\hat{q}$, solve for the equilibrium, and derive the aggregate output as

$$Y = \left[\frac{\sigma-1}{\xi\sigma}\right]^{\frac{\sigma-1}{(\xi-1)\sigma+1}} \int \hat{q}^{\frac{\xi(\sigma-1)}{(\xi-1)\sigma+1}} \sum_{n=0}^{\bar{n}(\hat{q})} n^{\frac{(\xi-1)(\sigma-1)}{(\xi-1)\sigma+1}} K_n(\hat{q})d\hat{q},$$

then, recover the underlying productivity q from $q = \frac{\hat{q}}{Y^{\frac{1}{\sigma-1}}}$.

I discretize the state space in terms of productivity q and use Gaussian quadrature method to approximate the expected value of any function of $\hat{q}$. Here, I assume that productivity $\hat{q}$ follows Generalized Pareto Distribution with density

$$\frac{1}{\sigma_{GPD}}\left(1+k\frac{\hat{q}-\underline{\hat{q}}}{\sigma_{GPD}}\right)^{-\frac{1}{k}-1},$$

where k is a shape parameter, σ_{GPD} is a scale parameter and $\underline{\hat{q}}$ is a location parameter. The mean of the distribution is $\underline{\hat{q}} + \frac{\sigma_{GPD}}{1-k}$ if $k<1$ and variance is $\frac{\sigma_{GPD}^2}{(1-k)^2(1-2k)}$ for $k<1/2$.

First, I compute firm's profit and worker's utility from equations (11) and (14). Then, given the initial guess for distribution functions $F(W_n(\hat{q}))$ and $G(W_n(\hat{q}))$, market tightness θ and unemployment rate u, I construct separation and offer acceptance rates. Value function iteration procedure is applied to find $V_n(\hat{q})$ and $W_n(\hat{q})$. Note that one needs to verify that worker's and firm's participation constraints are satisfied in the bargaining process. The optimal vacancy posting rate, $\upsilon_n(q)$, and maximum labor force, $\bar{n}(q)$, derived from the firm's problem, are used to find steady state distribution of firms across types, $K_n(\hat{q})$. Given the state space is discretized in the solution algorithm, the equations $(23)-(25)$ represent a linear programming problem that can be solved for directly. However, due to numerical approximation issues, iteration over size distribution turned out to perform better. Using equations (29), (33), (31) and (32) I update the initial guess for distribution functions $F(W_n(\hat{q}))$ and $G(W_n(\hat{q}))$, unemployment rate and market tightness and iterate until convergence.

(b) Simulation

For the simulation, I use the following specification of the model. The vacancy posting costs is parameterized as $c_\upsilon(\upsilon) = c_0\upsilon^{c_1}$, with $c_0>0$ and $c_1>1$. Parameter c_0 is important for matching job finding rate; whereas c_1 affect the standard deviation of hiring rate. As is commonly assumed in the literature, the matching function exhibit constant returns to scale in job seekers and vacancies, i.e.

$$M(\upsilon, u+(1-u)\kappa) = m\upsilon^{\zeta}(u-(1-u)\kappa)^{\zeta-1},$$

with $0<\zeta<1$ and a scaling parameter $m>0$ such that $M(\upsilon, u+(1-u)\kappa) \le \min\{\upsilon, u+(1-u)\kappa\}$. The parameter m represents the efficiency of the matching process. Then the job finding rate can be expressed as $\lambda(\theta) = m\theta^{\zeta}$ and the worker meeting rate as $\omega(\theta) = m\theta^{\zeta-1}$.

The equilibrium hiring and separation rates, $H_n(\hat{q})$ and $s_n(\hat{q})$, as well as maximum labor force size $\bar{n}(q)$, are the main variables that determine employment dynamics at the firm level. Given Poisson arrival rates, the waiting time until the next occurrence of any shock is distributed exponentially with parameter $x = \mu + \delta + H_n(\hat{q}) + s_n(\hat{q})n$. Thus, I generate time paths for each simulated firms as a random draw from exponential distribution. Whether it is the destruction, productivity, hiring or separation shock is decided according to the relative probability of each shock. Then, total work hours, revenues and wage bill are constructed for the each time period and then aggregated into monthly observations.

To fit the empirical size distribution of firms, in particular, the long right tail of the distribution, I assume that firms act as a collection of product lines and that each product faces its own hiring and separation process. The number of products per firm is exogenous and independent of firm productivity q. In this case, all steady state equilibrium equations hold and one can think of $K_n(q)$ as a distribution of product lines. The number of product lines for each firm is drawn from the Poisson distribution (with average number of products equal to 2.5) at the start of the simulation; subsequently, it evolves as a birth-death process with the birth rate η and death rate δ.

I simulate 1000 firms for 300 months and discard first 30 months under the assumption that the economy will converge to the steady state equilibrium within the first 30 periods. The simulated monthly employment series includes all workers who were employed in a given month. Quarterly employment is the average of three month employment. The average hours series is constructed according the ATP payment contribution schedule to replicate the (interval) hours measure reported in the data. Wage, revenue and hours variables are aggregated over three months to generate the quarterly series.

Table 10 shows the parameter values used in the simulation. The distribution of underlying productivity is assumed to be Generalized Pareto Distribution. To ensure that the model is able to capture positive relationship between wages and employment, as well as output and productivity, the productivity distribution needs to have a long right tail. The reason for that is that the type composition effect of larger firms being more productive on average has to offset the decreasing effect of employment on wages. Thus, Pareto distribution is preferred over, for instance, lognormal distribution. The scale and shape parameters are chosen to replicate the size distribution observed in the data. Similarly, the elasticity of substitution between intermediate goods, σ, governs the sensitivity of firms profits to profitability shocks. Higher value of σ increases the incentive to hire workers in response to a positive shock and thus magnifies the type composition effect in size-productivity relationship.

The persistence of the shock process (in terms of arrival rate μ) determines the persistence of the labor productivity in the model. I choose a parameter μ such that on average a shock to firm's productivity arrives once in ten months[16]. This value guarantees that the quarterly autocorrelation in labor productivity at the firm level in the model is close to that in the data (0.649 and 0.640, respectively).

The monthly interest rate r is equal to 0.4%. Unemployment benefit, or alternatively home production, b is set to match the unemployment rate of 4.8%, the average unemployment rate during the period of 1999-2006. The vacancy posting cost parameter c_0 is chosen such that job finding rate is about 0.25 that corresponds to the average unemployment duration of 4 months. Note that without the data on vacancies, the matching function parameters m and ζ cannot be identified separately. Then the elasticity of matching function with respect to vacancies, ζ, is set to 0.5 that is within the range of estimates found in the literature (for instance, Shimer [37] reports the estimate of 0.38; while Hall [22] finds the estimate of 0.765). Decreasing m is associated with rise in search frictions and therefore implies stronger trade-off between labor adjustment on intensive and extensive margin. The exact mechanism is working through decreasing the vacancy filling rate for firms (keeping job finding rate the same, the vacancy filling rate declines in m), that in turn decreases the return on vacancies and thus slows down the response in employment adjustment.

Average monthly entry and exit rate in the data over the period of 1999-2006 are above 4%. However, most of the firm turnover is related to small firms that have sporadic zeros in their employment. The employment share of entering firms is 1.4%, while exiting firms employ about 0.8% of total workforce.

[16] The arrival rate μ is equal to 0.04 which together with the average number of products per firm of 2.5 generates the average arrival rate of 0.1.

To fit the exit rate in the data, exogenous destruction rate δ needs to be higher than 2%. That, however, makes it inconsistent with the observed unemployment rate of 4.8% and job finding rate of 0.25. Moreover, higher destruction rate dampens that hours-employment growth comovement; thus, I set δ to 0.001 to get better fit on the latter relationship. The entry rate parameter η determines the total mass of product lines, and through it, the average employment per product line.

The worker and job flow data identify the parameters that govern job-to-job transitions in the model. The exogenous quit rate s_0 is set to be consistent with the job finding rate and unemployment rate based on equation (34) . Note that exogenous quits alone are insufficient to generate separation rates observed in the data. Thus, the model requires to have job-to-job transitions. The relative search intensity, κ, is set to be less than one in order to match average hiring rate in the data. The convexity of vacancy posting costs parameter c_1 determines sensitivity of hires to firm size. Moreover, lower c_1 implies higher standard deviation of hiring rate.

The parameters that govern worker's problem are worker's bargaining power parameter, β, and utility costs from variation in working hours parameters, χ and ξ. The bargaining power of workers affect the amount of rent-sharing in the model and is set to match the labor share in total revenues. In the data the ratio of total wage bill to total value added is found to be about a half. The scale parameter on disutility from working χ is chosen to match average actual work hours to 34 hours a week[17]. The curvature of utility cost, ξ, has a prominent role for the trade-off between changes in hours and employment in firm's labor demand by influencing the cost of varying hours of work. I set $\xi = 2.5$ that is within the range of values estimated by Cooper and Willis [15]. That value ensures that the correlation between hourly wages and hours predicted by the model is close to that observed in the data (the size-weighted correlation coefficient is 0.209 in the data and 0.240 in the model based on H_{UB} measure; recall that H_{LB} measure implies negative wage-hours relationship).

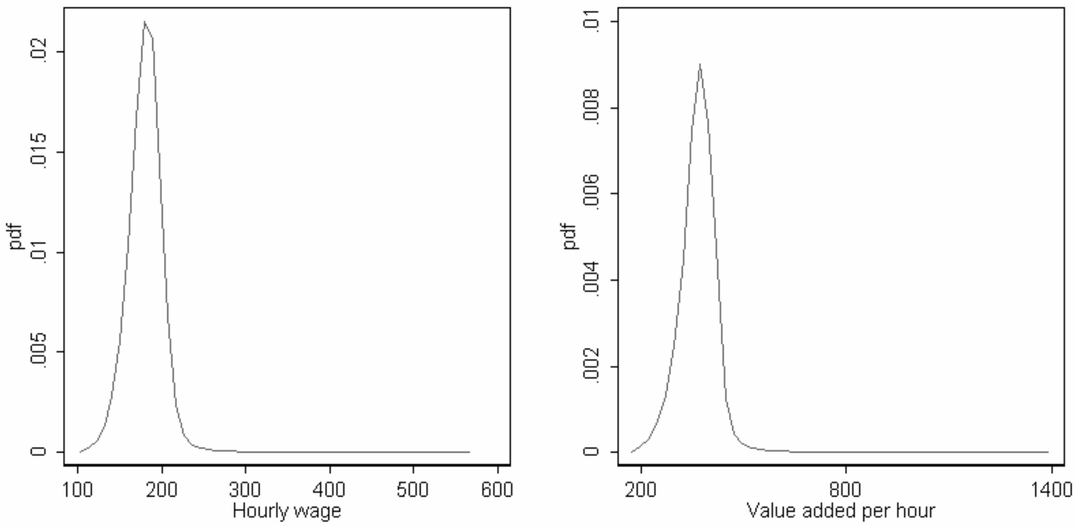
The aggregate price level P equates mean hourly wages in the model to its mean in the data.

Table 10: Parameter values used in the simulation.

Parameter	**Value**
Monthly interest rate, r	0.04
Destruction rate, δ	0.001
Entry rate, η	1.3×10^{-4}
Elasticity of substitution between intermediate goods, σ	2.3
Value of home production, b	12
Disutility of working, $g(h) = \chi h^{\xi}$, (χ, ξ)	$(6 \times 10^{-5}, 2.5)$
Worker bargaining power, β	0.35
Vacancy posting cost, $c_v(v) = c_0 v^{c_1}$, (c_0, c_1)	$(10, 1.5)$
Exogenous quit rate, s_0	1.5×10^{-3}
Relative search intensity, κ	0.7
Matching function, $M(v,u) = m v^{\zeta} u^{1-\zeta}$, (m, ζ)	$(0.4, 0.5)$
Productivity distribution, $\Phi(\hat{q})$, Generalized Pareto Distr. $(k, \sigma_{GPD}, \underline{\hat{q}})$	$(0.45, 24, 10)$

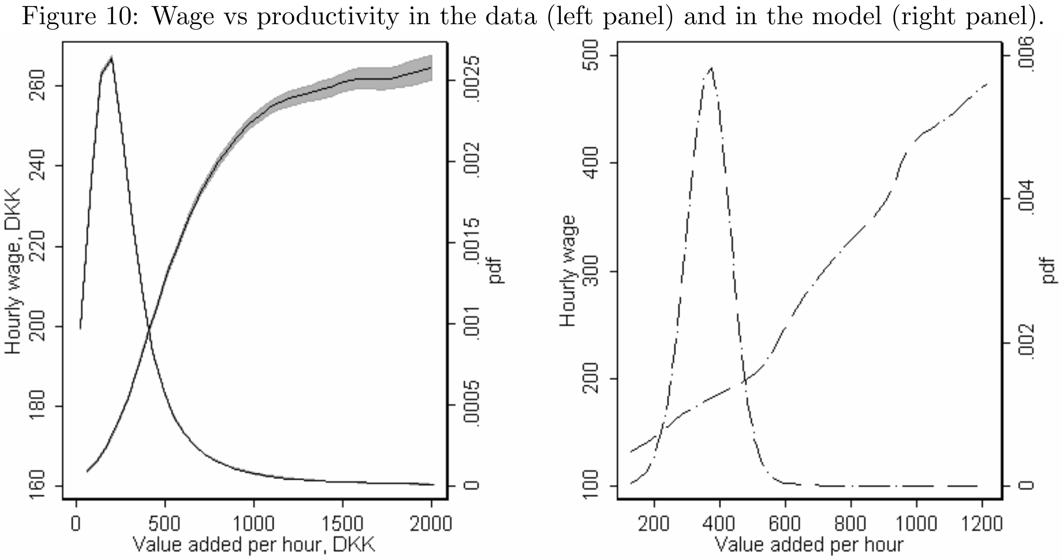
[17] OECD Economic outlook (2007) reports that average annual hours (defined as the total numbers of hours worked over the year divided by the average numbers of people in employment) was 1559 in Denmark over the period of 1999-2006. To get weekly hours, I then divide it by 46 weeks assuming there are 6 weeks of vacation.

Figure 9: Wage and productivity distribution in the model



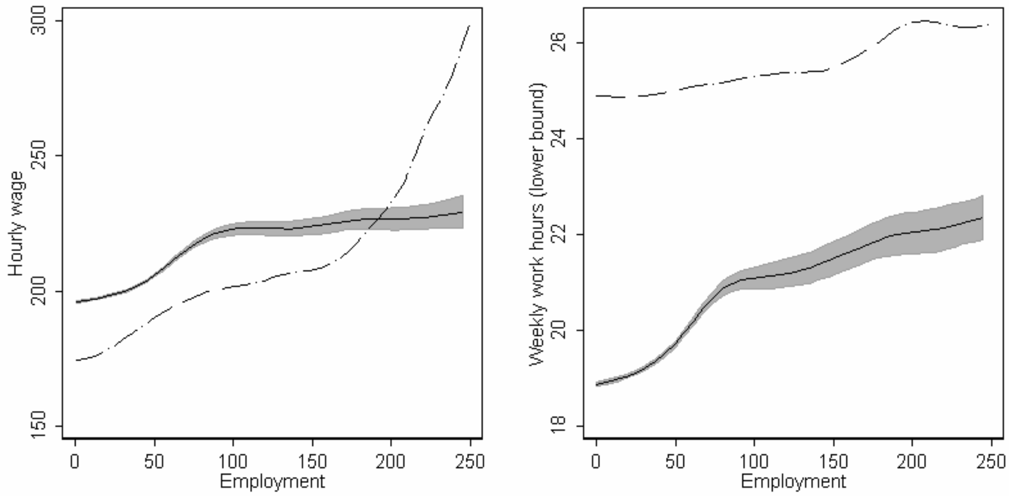
Note: Density estimation is based on Gaussian kernel with bandwidth of 5.

Figure 10: Wage vs productivity in the data (left panel) and in the model (right panel).



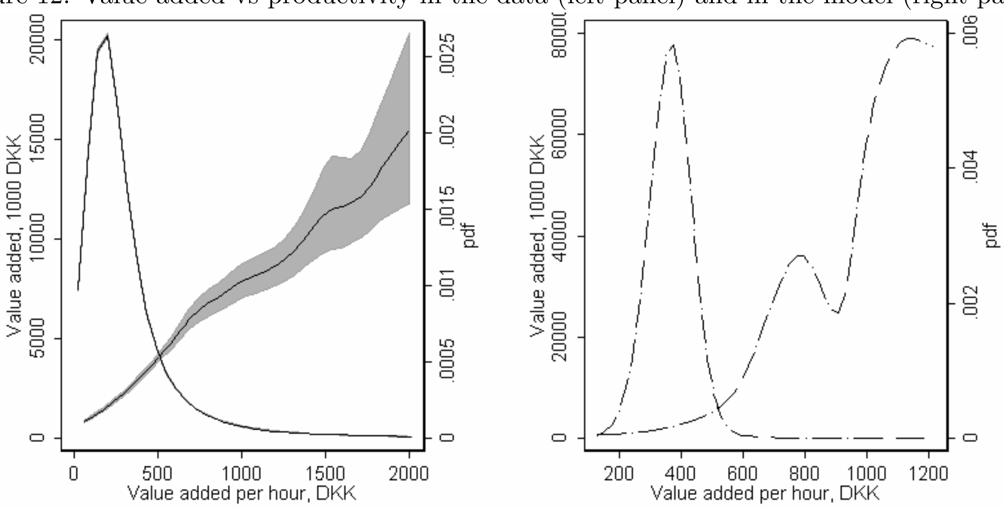
Note: Nadaraya-Watson estimate using Gaussian kernel with bandwidth of 100. Shaded area is 90% pointwise bootstrap confidence intervals. Source: Author's calculations based on the Danish firm data, 2002-2006

Figure 11: Hourly wages and hours vs employment in the data (solid line) and in the model (dashed line).



Note: Nadaraya-Watson estimate using Gaussian kernel with bandwidth of 20. Shaded area is 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 2006

Figure 12: Value added vs productivity in the data (left panel) and in the model (right panel).



Note: Nadaraya-Watson estimate using Gaussian kernel with bandwidth of 100. Shaded area is 90% pointwise bootstrap confidence intervals (clustered by firm ID). Source: Author's calculations based on the Danish firm data, 2002-2006

Figure 13: Mandatory pension contribution scheme.