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WORKING PAPER

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Abstract

For a small open economy where the government is restricted to raise revenue using border taxes only, the optimal structure of border taxes is considered. As a matter of normalization exports and the supply to the market of the primary factor may be assumed to be untaxed, but that the household use of the primary factor and domestic consumption of the export good cannot be taxed is nevertheless a constraint; this insight provides the key to understanding what determines the optimal tariff structure. The optimal border tax structure is derived for both exogenous and endogenous labour supply, and the results are interpreted in the spirit of the Corlett-Hague results for the optimal tax structure in a closed economy and compared with results from CGE models.

Keywords: Border taxes, small open economy, labour supply, Corlett-Hague.

JEL: H21, F13

1 Introduction

The purpose of this paper is to contribute to the understanding of what determines the optimal tariff structure in a small open economy in the case where domestic taxes are not available. Recent papers by Hatta and Ogawa (2003) and Hatta (2004) have contributed to such an understanding. They showed

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that in such an economy the optimal tariff structure is characterized by i) the optimal tariff rate being lower for the import good that is the closer substitute for the export good, and by ii) the stronger the cross-substitutability between imports the closer the optimal tariff structure is to uniformity.

By considering an economy with fixed coefficients rather than a general aggregate production constraint and an endogenous rather than exogenous primary factor supply \(^1\) we are able to further broaden and deepen the analysis by Hatta and Ogawa, explaining the optimal tariff structure as a trade-off between on the one hand the objective of maintaining the pattern of consumption of the import goods similar to that under the first best, and on the other hand the objectives of discouraging the consumption of the export good, and of the primary factor in the household sector. It also provides for an alternative explanation of why in computable general equilibrium (CGE) models the optimal tariff structure is in general close to uniformity.

The paper is structured as follows. In section 2 the general model is presented. Section 3 derives the conditions for an optimal tariff system and Section 4 interprets these conditions when labour supply is fixed and when it is endogenous. It also considers the implications of separability assumptions for the optimal tariff structure. Section 5 concludes.

2 The Model

Consider a small open economy comprised of a representative household, perfectly competitive production sectors and a government. There is one primary factor (labour) indexed \(i = 0\) and three traded commodities indexed \(i = (1, 2, 3)\). Using standard sign conventions household supply of labour is \(-x_0 > 0\), while household demand for the produced goods is \(x = (x_1, x_2, x_3)\). The supply of goods from firms is \(y = (y_1, y_2, y_3)\) and the corresponding demand for labour is \(-y_0 = (-y_{01}, -y_{02}, -y_{03})\). To facilitate the main objective of the analysis - providing a deeper understanding of what determines the optimal tariff structure - the production structure is specified in a very simple manner. The primary factor is the only input and the productivity of the primary factor is constant:

\[
y_i = -\alpha_i y_{0i}, \quad i = 1, 2, 3. \tag{1}
\]

\(^1\)The households endowment of the primary factor is divided between supply to the market and use in the household sector. The use in the household sector is often denoted "leisure", but does in fact represent any untaxed use by the household of the primary factor.
The government is constrained to use border taxes, \( t = (t_1, t_2, t_3) \), for raising revenue. World market prices are \( p = (p_1, p_2, p_3) \), making domestic prices of final commodities equal to \( q = p + t \) (so \( t_i \) is positive for import tariffs and negative for export taxes). Production is determined by comparative advantage, so by assuming 

\[
\frac{q_1}{q_i} > \frac{\alpha_i}{\alpha_1}, \quad i = 2, 3,
\]

we get full specialization in production of good 1 (becoming the export good while goods 2 and 3 become import goods).\(^2\) Obviously then, \( y_{02} = y_{03} = 0 \). The zero profit condition for the production of the export good, thus determines the wage rate 

\[
q_0 = \alpha_1 q_1.
\]

The preferences of the representative household are characterized by the expenditure function, \( E(q_0, q, u) \), defined over domestic prices \((q_0, q)\) and utility \( u \). By Shephard’s lemma, net demands are given by, 

\[
E_i(q_0, q, u) = \frac{\partial E(q_0, q, u)}{\partial q_i}, \quad i = 0, 1, 2, 3,
\]

such that the compensated supply of labour is 

\[
-x_0 = -E_0(q_0, q, u),
\]

while compensated demand functions for the produced goods are 

\[
x_i = E_i(q_0, q, u), \quad i = 1, 2, 3.
\]

Government consumption is \( x^G_0 \) of the primary factor and \( x^G = (x^G_1, x^G_2, x^G_3) \) of the final goods. Net import is \( z = x + x^G - y \) such that with border taxes on the tradable commodities the government budget constraint becomes 

\[
\sum_{j=1}^{3} t_j z_j - q_0 x^G_0 - \sum_{j=1}^{3} q_j x^G_j = 0.
\]

Given the static set-up, equilibrium requires balanced trade at world market prices 

\[
\sum_{j=1}^{3} p_j z_j = 0.
\]

\(^2\)Notice that the condition in equation 2 should hold for all relevant tariff levels. Thus, we simply assume \( \frac{\alpha_i}{\alpha_1} \) to be sufficiently small to insure that equation 2 is satisfied for all the tariffs levels we consider.

\(^3\)In the case of a fixed labour supply \( E(q_0, q, u) \equiv e(q, u) + q_0 x_0 \), thus \( E_i = e_i, i = 1, 2, 3, E_0 = x_0, E_{ij} = e_{ij}, i, j = 1, 2, 3 \) and \( E_{0j} = 0, j = 1, 2, 3, E_{j0} = 0, j = 1, 2, 3, E_{00} = 0. \)
Equilibrium in the labour market (labour being internationally nontradable) requires that the household supply of labour equals the demand for labour by firms and the government

\[-x_0 = -y_{01} + x^G_0.\]  

(8)

Output supplied of the export good can thus be written as

\[y_1 = -\alpha_1(x_0 + x^G_0).\]  

(9)

and foreign net trades as

\[
\begin{align*}
z_1 &= E_1(q_0, q, u) + x^G_1 - y_1 = E_1(q_0, q, u) + x^G_1 + \alpha_1(x_0 + x^G_0) \quad (10a) \\
z_j &= E_j(q_0, q, u) + x^G_j, \quad j = 2, 3. \quad (10b)
\end{align*}
\]

as goods 2 and 3 are not produced domestically.

Substituting for the wage rate by the zero profit condition, \(q_0 = \alpha_1 q_1\), the equilibrium conditions can then be written as (see Diamond and McFadden (1974) and Dixit and Munk (1977))

\[E(\alpha_1 q_1, q, u) = 0 \]  

(11)

\[
\begin{align*}
t_1 & \left[ E_1(\alpha_1 q_1, q, u) + x^G_1 + \alpha_1(E_0(\alpha_1 q_1, q, u) + x^G_0) \right] \\
&+ \sum_{j=2}^3 t_j \left( E_j(\alpha_1 q_1, q, u) + x^G_j \right) - \alpha_1 q_1 x^G_0 - \sum_{j=1}^3 q_j x^G_j = 0 \quad (12)
\end{align*}
\]

\[
\begin{align*}
p_1 & \left[ E_1(\alpha_1 q_1, q, u) + x^G_1 + \alpha_1(E_0(\alpha_1 q_1, q, u) + x^G_0) \right] \\
&+ \sum_{j=2}^3 p_j \left( E_j(\alpha_1 q_1, q, u) + x^G_j \right) = 0. \quad (13)
\end{align*}
\]

By Walras’ Law two of the equilibrium conditions imply the third. Considering equations 11 and 13 it can be verified that an equi-proportionate increase in all three domestic good prices will not change the equilibrium. Hence, we can without loss of generality fix one of the taxes as a normalization rule. We assume that the tax on the export good is zero.\(^4\)

\(^4\)It is in this context important to emphasize the distinction between that exports are untaxed and that the domestic consumption of the export good cannot be taxed. That exports are untaxed may be interpreted as a normalization rule whereas the assumption that the domestic consumption of the export good cannot be taxed is a constraint on the government’s maximization problem. Formulations like "the export good is assumed untaxed as a matter of normalization" or "the export good is an untaxed numeraire" are therefore ambiguous and should be avoided whenever they may lead to misinterpretations.
3 Derivation of Optimal Tax Formulae

With the assumption that \( t_1 \equiv 0 \), the Lagrangian corresponding to the government maximization problem may thus be formulated as

\[
\mathcal{L} = u + \mu \left[ -E(\alpha_1 q_1, q, u) \right] + \lambda \left[ \sum_{j=2}^{3} t_j \left( E_j(\alpha_1 q_1, q, u) + x_j^G \right) - \alpha_1 q_1 x_0^G - \sum_{j=1}^{3} q_j x_j^G \right].
\]  

(14)

The first-order conditions for an optimal solution thus become

\[
-\mu E_i(\alpha_1 q_1, q, u) + \lambda \left[ E_i(\alpha_1 q_1, q, u) + \sum_{j=2}^{3} t_j E_{ij}(\alpha_1 q_1, q, u) \right] = 0, \quad i = 2, 3.
\]

(15)

Solving for the optimal taxes (using the symmetry of the Slutsky matrix, \( E_{ij} = E_{ji} \)) yields

\[
t_2 = \frac{\lambda - \mu}{\lambda} \frac{E_{23}E_3 - E_{33}E_2}{E_{22}E_{33} - E_{23}E_{32}},
\]

(16a)

\[
t_3 = \frac{\lambda - \mu}{\lambda} \frac{E_{32}E_2 - E_{22}E_3}{E_{22}E_{33} - E_{23}E_{32}},
\]

(16b)

or written in terms of compensated demand elasticities, \( \varepsilon_{ij} \equiv \frac{E_{ij} q_i}{q_i} \),

\[
t_2 = \frac{\lambda - \mu}{\lambda} \frac{\varepsilon_{23} - \varepsilon_{33}}{E_{22}\varepsilon_{33} - E_{23}\varepsilon_{32}},
\]

(17a)

\[
t_3 = \frac{\lambda - \mu}{\lambda} \frac{\varepsilon_{32} - \varepsilon_{22}}{E_{22}\varepsilon_{33} - E_{23}\varepsilon_{32}}.
\]

(17b)

By the negative semidefiniteness of the Slutsky matrix, \( \lambda \), the net social marginal value of government income, exceeds \( \mu \), the net social marginal value of private income, (see Diamond and Mirrlees 1971), and the denominator of the

\[5\]The size of the compensated cross-price elasticities can be used to characterize the goods in terms of the degree of complementarity/substitutability. \( \varepsilon_{ij} < 0 \) for goods \( i \) and \( j \) being complements while \( \varepsilon_{ij} > 0 \) for substitutes. Hence, a large value of the compensated cross-price elasticity indicates that the two goods are relatively close substitutes while a small value of the compensated cross-price elasticity indicates that the two goods are more complimentary in consumption.
second factor in equations 17a and 17b is positive. By definition $\varepsilon_{ij} \equiv s_j \sigma_{ij}$, where $\sigma_{ij}$ is the Allan elasticity of substitution between commodity $i$ and commodity $j$, and $s_j$ the share of commodity $j$ in full income. Furthermore, the elasticities of substitution are symmetric, i.e. $\sigma_{23} = \sigma_{32}$. Therefore, as the compensated own price elasticities are negative, $\varepsilon_{ii} < 0$, a sufficient condition for both tariff rates to be positive is that the elasticity of substitution between the two import goods is positive, i.e. that $\sigma_{23} > 0$.\(^6\)

4 Interpretation

In order to identify what determines the optimal tariff structure, we interpret the optimal tax formulae, 17a and 17b, under two alternative assumptions: That the labour supply is fixed, and that it is endogenously determined.

4.1 Fixed Labour Supply

First consider the case where the supply of labour is fixed. This case is entirely analogous to the Corlett-Hague result for an economy with endogenous supply of labour and two final goods, with the domestic consumption of the export good taking the place of the untaxed consumption of leisure.\(^7\)

By homogeneity of degree zero of compensated demand we have in general that $\sum_{j=0}^{3} \varepsilon_{ij} = 0$. In the case of a fixed supply of labour $\varepsilon_{i0} = \varepsilon_{i0} = 0$, so using equations 17a and 17b the optimal tariff structure can be characterized by

$$\frac{t_2}{q_2} = \frac{-\varepsilon_{22} - \varepsilon_{33} - \varepsilon_{21}}{-\varepsilon_{22} - \varepsilon_{33} - \varepsilon_{31}}$$

or

$$\frac{t_2}{q_2} = \frac{\varepsilon_{23} + \varepsilon_{32} + \varepsilon_{31}}{\varepsilon_{23} + \varepsilon_{32} + \varepsilon_{21}}.$$  \hspace{1cm} (18)

Substituting using $\varepsilon_{ij} \equiv s_j \sigma_{ij}$ and $\sigma_{ij} = \sigma_{ji}$, 18 and 19 become, respectively.\(^8\)

\(^6\)Notice, as usual in optimal tax problems, uniqueness of the optimal tax rates is not guaranteed by the first-order conditions (see e.g. Atkinson and Stiglitz (1980)).

\(^7\)Hatta and Ogawa also exploit this analogy, but because they assume an aggregate production constraint the interpretation of their result is complicated by the fact that the supply elasticities enter the tax formulae (even in the case of CRTS) contrary to the Harberger analysis of the Corlett and Hague results and to our formulation.

\(^8\)Notice that since $\sum_{j=1}^{3} \varepsilon_{ij} = 0$, $i = 2, 3$ and $\varepsilon_{ii} < 0$, we must have that at least one of $\sigma_{23}$ and $\sigma_{31}$ are positive (and similarly for $\sigma_{23}$ and $\sigma_{21}$).
\[
\frac{t_2}{q_2} = \frac{-\varepsilon_{22} - \varepsilon_{33} - s_1\sigma_{21}}{-\varepsilon_{22} - \varepsilon_{33} - s_1\sigma_{31}} \quad (20)
\]

and
\[
\frac{t_3}{q_3} = \frac{(s_2 + s_3)\sigma_{23} + s_1\sigma_{31}}{(s_2 + s_3)\sigma_{23} + s_1\sigma_{21}} \quad (21)
\]

That exports are untaxed may, as already mentioned, be interpreted as a normalization rule. However, that the domestic consumption of the export good cannot be taxed constitutes a constraint on the government’s maximization problem which implies that the consumption of the export good is encouraged compared with the first best where the government’s resource requirement is financed by lump sum taxes. This suggest that starting from a uniform tariff structure increasing the tax on the import good most complementary with the domestic consumption of the export good and decreasing it for the other import good, keeping the government tax revenue constant, will increase social welfare by discouraging the consumption of the export good, but at the same time have the opposite effect on social welfare by distorting the first best pattern of consumption of the import goods. This insight provides the key to the understanding of the optimal tariff structure as representing a trade-off between two objectives:

- **Objective 1**: To maintain the first best pattern of consumption of the import goods.
- **Objective 2**: To discourage the untaxed consumption of the export good.

The elasticity of substitution between the two import goods, \(\sigma_{23}\), may be taken as an *indicator of the costs in terms of Objective 1 of differentiating the tariff rates*, whereas the difference in the two elasticities of substitution between the import goods and the export good, \(\sigma_{31} - \sigma_{21}\), may be taken as an *indicator of the potential benefits in terms of Objective 2 of a differentiated tariff structure*. Which import good will be taxed at the highest rate depends entirely on the sign of the indicator for Objective 2, \(\sigma_{31} - \sigma_{21}\); the import good which is the closest complement to the export good will be taxed at the highest rate (see equation 20). For a given positive value of the indicator for Objective 1, \(\sigma_{23}\), the differentiation of the tax rates will be the greater, the greater the numerical value of the indicator for Objective 2, \(\sigma_{31} - \sigma_{21}\), i.e. the greater the potential benefits of a differentiated tax structure. Again assuming \(\sigma_{23} > 0\), for given values of the indicator for Objective 2, \(\sigma_{31} - \sigma_{21}\),
the differentiation in tariff rates is the smaller the larger the indicator for 
Objective 1, $\sigma_{23}$, i.e. the greater the costs of a differentiated tax structure; 
if the import goods are close substitutes, without any of the import goods 
being a close substitute to the export good, then the optimal tariff structure 
will be close to uniformity (see equation 21). It is thus not sufficient for 
the optimal tariff structure to be close to uniformity that the elasticity of 
substitution between the import goods is large. However, if $\sigma_{31} = \sigma_{21}$ then 
the optimal tariff structure is proportional whatever the value of $\sigma_{23}$.

4.2 Endogenous Labour Supply

Consider now the case of endogenous labour supply. The optimal tariff struc-
ture now represents a trade-off between three objectives. In addition to the 
two mentioned above, we now also have

- Objective 3: To discourage the untaxed consumption of leisure.

The optimal tax formulae corresponding to equations 20 and 21 now 
become, respectively,

$$\frac{\ell_2}{q_2} = \frac{-\varepsilon_{22} - \varepsilon_{33} - s_1\sigma_{21} - s_0\sigma_{20}}{-\varepsilon_{22} - \varepsilon_{33} - s_1\sigma_{31} - s_0\sigma_{30}}$$

(22)

and

$$\frac{\ell_3}{q_3} = \frac{(s_2 + s_3)\sigma_{23} + s_1\sigma_{31} + s_0\sigma_{30}}{(s_2 + s_3)\sigma_{23} + s_1\sigma_{21} + s_0\sigma_{20}}.$$

(23)

The optimal tariff structure now reflects the desire to discourage both the 
untaxed consumption of leisure, and the untaxed domestic consumption of 
the export good. Which commodity will be taxed at the highest rate depends 
entirely on the sign of $(s_1\sigma_{31} + s_0\sigma_{30}) - (s_1\sigma_{21} + s_0\sigma_{20})$ (Objective 2 and 3) (see equation 22). For a given positive value of $\sigma_{23}$, the difference in taxes will be 
the greater the greater the numerical value of $(s_1\sigma_{31} + s_0\sigma_{30}) - (s_1\sigma_{21} + s_0\sigma_{20})$, 
and again assuming that $\sigma_{23} > 0$, for a given value of $(s_1\sigma_{31} + s_0\sigma_{30}) - (s_1\sigma_{21} + 
softmax_1\sigma_{20})$ the difference will be the smaller, the greater is $\sigma_{23}$ (Objective 1)(see 
equation 23). Objective 2 and Objective 3 may be conflicting, but if the 
consumption of the same import good is both more complementary to the 
untaxed consumption of the export good and to the , then it will be taxed 
at a higher rate than the other import good. For uniformity it is not only 
necessary for the elasticity of substitution between the import goods to be 
very large relative to the elasticities of substitution between the import goods 
and the export goods when these differs between the the two import goods, it
also needs to be very large relative to the elasticities of substitution between the import goods and the untaxed consumption of primary factor.

The optimal tax formulae are not invariant under renormalization, but as the ranking of commodities in terms of tax rates and the substitution elasticities is invariant under renormalization, the interpretation of what determines the optimal tariff structure is therefore the same if, for example, the tax on exports is fixed at a certain rate rather than at zero.

### 4.3 Separability Assumptions

In the case of independent demands between the import goods the optimal tax structure becomes\(^9\)

\[
\frac{t_2}{q_2} = \frac{\varepsilon_{33}}{\varepsilon_{22}}.
\]  

(24)

which, as Dasgupta and Stiglitz (1974) observe, implies that the second best tariff structure needs not be uniform. This have made several authors wonder how this squares with the experience with CGE models that the welfare loss caused by uniform rather than optimal tariffs is negligible (Dahl et al. (1994) and Mitra (1992)). Hatta and Ogawa (2003) note that CGE models are not based on the unrealistic assumptions of the inverse elasticity rule, and suggest as a solution to this puzzle that a uniform tax structure is close to optimal when the elasticity of substitution between imports is large. However, as there is no compelling reason why the elasticity of substitution between import goods should be larger than that between import goods and export goods (i.e. \(\sigma_{23}\) being large compared to \(\sigma_{21}\) and \(\sigma_{31}\) in equation 21), this seems not to be the correct explanation, not in the case where the supply of labour is fixed, and even less so in the case where the supply of labour is endogenous. A more plausible explanation is that it is due to the (also unrealistic) separability assumptions often employed in CGE models. Sufficient conditions for the optimal tariff structure to be proportional is in the case of exogenous labour supply that the consumption of the import goods is separable from the consumption of the export good, and in the case of endogenous labour supply in addition that the import goods are separable from the consumption of the primary factor in the household sector, and that the sub-utility function of the two import goods is homothetic (so that \(\sigma_{21} = \)

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\(^9\)This "inverse elasticity rule" is typically obtained in partial equilibrium models where the optimal tax rate of a commodity is inversely related to its own-price elasticity of demand, see Atkinson and Stiglitz (1980).
and \( \sigma_{31} = \sigma_{30} \) in equations 22 and 23). For an additively separable utility function (as the CES utility function which is widely used in CGE models), the optimal tariff structure is thus uniform.

5 Concluding Remarks

The optimal tariff structure of a small open economy with a fixed coefficients technology and with an endogenous labour supply has been analyzed, extending previous studies by providing tax formulae which lend themselves easily to interpretation. The analysis has clarified what determines the optimal tariff structure: A trade-off between on the one hand the objective of maintaining the pattern of the import goods at their first best level, and on the other hand the objective of discouraging the consumption of the primary factor in the household sector and of the export good.

Under a general production structure where the imported goods are also produced domestically, the use of border taxes also prevents the government from taxing that part of the consumption of the imported goods which is produced domestically, in which case the optimal tax structure is complicated also by how taxation distorts production. However, since the share of untaxed consumption will remain greater for the export good than for the import goods, the insight gained from the model considered here remain valid also under move general assumptions. For the optimal tax structure to be proportional it is therefore in general not sufficient for the elasticity of substitution between the import goods to be large; in general, it also has to be large relative to the cross-price elasticities of the import goods with the consumption of the export good and the consumption of the primary factor in the household sector.

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