A Directed Search Model of Ranking by Unemployment Duration*

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Abstract

Empirical evidence shows that longer spells of unemployment are associated with fewer job offer arrivals, lower job-finding rates and wage offers. This paper sets up a directed search model based on informational stigma to replicate these facts. Firms imperfectly test for the applicants’ productivity. Unemployment duration is informative about the applicants’ expected productivity: if high-skilled workers perform better at the recruiting processes, then longer unemployment durations signal lower expected productivity. The intertemporal link leads to constrained inefficiency and makes information be very sensitive to business cycle fluctuations. Hazard rates and exit wages distributions over duration hence show different trends over the cycle. In particular, consistent with the empirical evidence from CPS wages show small negative correlation with duration during bad times, and decline faster during good times.

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1 Introduction

Long-term unemployed workers have been a major focus of labor market policies in most OECD countries due to the steady deterioration of job prospects as unemployment progresses.¹ Negative duration dependence on hazard rates and exit wages is well documented. For example, Machin and Manning (1999) show that a 10% increase in duration reduces the US hazard rate by 4.5%.² Keane and Wolpin (1997) in turn estimate that blue-collar workers experience a wage drop by 9.6% after one year of unemployment, whereas white-collar workers’ wages are reduced by 36.5%. Less known are the decline in the arrival rate of job offers with duration as well as the effect of unemployment duration on employers’ recruiting strategies. Using European panel data, Addison, Portugal, and Centeno (2004) estimate the marginal effect of duration on the arrival rate at -0.131. Furthermore, Oberholzer-Gee (2008) carries out a field experiment and a survey showing that firms discriminate candidates according to their unemployment duration. More precisely, firms report to infer a lower success rate at past interviews the longer the unemployment duration as the main reason to discriminate against candidates of longer unemployment spells, an effect we will refer to as informational stigma. Finally, using CPS data, we report that the correlation between both hazard rates and accepted wages and unemployment duration is quite sensitive to the business cycle fluctuations. In particular, the negative correlation of accepted wages with duration over busts is multiplied by a factor of 4 over booms, -0.0729 vs. -0.2895.

This paper studies a model based on informational stigma to replicate these facts. The stigma mechanism is formalized in a directed search environment with incomplete information and imperfect testing, where the matching process, the wage offers and the unemployment distribution by duration are determined endogenously. As information from the candidates-screening process is not fully captured by the testing firms, unemployment duration is informative about the expected productivity of applicants: longer unemployment durations signal lower expected productivity provided that high-skilled workers are more likely to succeed in the recruiting processes. As a result,

¹According to Heckman, Lalonde, and Smith (1999), Active Labor Market Policies account for 0.16% of the US GDP in the US, and 2.18% in Sweden.
²Two non-mutually exclusive factors have been considered: a genuine and a spurious duration dependence. The latter component is based on heterogeneity of workers that is observable to economic agents, but unobservable to the econometrician. Over the course of unemployment the composition of workers changes in favor of those with lower hazard rates. Genuine duration dependence in contrast results from characteristics, such as loss of skills, search discouragement or stigmatization that can change during unemployment.
job-seekers with longer unemployment spells are discriminated against. However, information does not always remain at a constant level; on the contrary, it is very sensitive to business cycle fluctuations leading hazard rates and exit wages distributions over duration to show different trends over the cycle.

To be more specific, we consider an OLG economy where a continuum of workers are born every period and firms’ entry is free. Firms commit to a job offer specifying a menu of wages, possibly contingent on unemployment durations. Search is directed in the sense that workers know the terms of trade from each vacancy prior to choosing where to submit an application. There is incomplete information on workers’ productivity. Firms have access to an imperfect screening technology, however. Since meetings are multilateral, screening may be used to select one of the applicants out. The test results are private information to the testing firms, and not tradable. Period t firms’ screening has two impacts of different sign on future potential employers. On one hand, it worsens the average quality of the remaining pool of candidates provided that skilled workers are more likely to pass the test. On the other, future firms will freely benefit from period t’s screening as unemployment duration will be informative on job-seekers’ productivity. Thus, due to the informational externality, stigma arises: firms optimally rank candidates by unemployment duration. The model is hence consistent with the first two stylized facts, and wage dispersion arises in equilibrium as identical workers are paid differently because of distinct unemployment histories. The agents’ values and optimal decisions depend crucially on the distribution of workers over duration. Existence of equilibrium is proved by imposing a finite dimension on the state variable space.

The model presented here relates to and benefits from several strands of the literature. This paper is not the first in posing informational stigma in the literature. Whereas Vishwanath (1989) find persistent negative effects of unemployment rate at high-school graduation for Japanese men, whereas only temporary effects for US workers. The Japanese school-based hiring system along with its strong employment protection legislation are pointed to be the rationale behind this result as such a combination encourages recruiting firms to optimally discriminate in favor of newly graduated high-school workers, and against those who graduated in past years, but remained unemployed. The stigma mechanism is thus similar to the one posed in this paper.

Notice that this mechanism is not limited to labor markets. It may apply to other markets as well where it is difficult to evaluate quality and time on the market is informative. For example, searchers in the housing market are on average less interested in ads that have been posted for a long time.
is a partial equilibrium model, Lockwood (1991) sets up a general equilibrium model, where search is random and firms imperfectly test the worker they are assigned to. Hazard rates decline with duration, whereas wages do not. The negative duration dependence result in job-finding rates relies on two key assumptions: First, the unskilled workers’ leisure value is above their market productivity. And, second, workers’ search is costless. On the contrary, our model is robust for these two obvious extensions. Furthermore, unlike in his model, the declining exit rate from unemployment of the average worker is due to both unobserved heterogeneity and state dependence.

Second, ranking of job applicants by unemployment duration has first been introduced by Blanchard and Diamond (1994). In contrast to our paper, they study a random search model where firms rank job candidates by assumption. The search process is relevant because they face the valid criticism that those workers who are discriminated against may optimally direct their search to different jobs. In our directed search framework instead, no firms’ deviation targeting a subset of workers is profitable as firms offer a menu of wages. Hall and Krueger (2008) report that 84% of white, male, non-college job-seekers had at least a “pretty good idea” about the wage their current job was going to pay prior to the first interview. Moreover, Blanchard and Diamond (1994) undertake a comparative statics analysis to get insights on the effects of business cycle fluctuations on the equilibrium outcome, whereas the different trends of the hazard rates and exit wages distributions over the cycle are a central focus.

Third, on the directed search literature, this paper is formally close to Shimer (2005), who introduces two-sided heterogeneity into a static model. In contrast to the literature based on two-sided homogeneity, firms play with wages both at the extensive margin (to attract more applicants), and at the intensive margin (to attract candidates of higher expected productivity). Beyond the differences in goals and the dynamic vs. static feature, his paper assumes an exogenous productivity distribution, whereas it is determined endogenously in our paper. This additional structure also enables us to analyze how wages are negatively related to productivity, a relationship which remains

\footnote{The first assumption ensures that the pool of candidates deteriorates over time, whereas the second implies the labor market participation. Notice that if search were costly, job-seekers would not participate in the labor market as their expectations on wages would rationally be their leisure value, there would be no job-seekers as the participation constraint would not be satisfied because of lack of commitment on the part of the firm. In our paper, endogenizing costly search effort would generate search discouragement effects because of the ranking mechanism. CHECK WHAT IS THE WAGE-SETTING IN HIS CASE}

\footnote{Moen 1999 uses a similar model where job applicants are ranked by skill to show the possibility of overinvestment in human capital.}
ambiguous in Shimer’s setting. Furthermore, it is also responsible for the business cycle fluctuations in the labor market outcomes.

Moreover, other mechanisms of duration dependence have been analyzed in the literature. Gonzalez and Shi (2009) study a competitive search framework with a search discouragement effect. Workers learn about their idiosyncratic job finding abilities and adjust their search intensity over the unemployment spell accordingly.\(^7\)

In the applied literature concerning duration dependence other mechanisms have been pointed out: mainly, the depreciation of human capital and search discouragement over the unemployment spell (see e.g. Pissarides (1982) and Ljungqvist and Sargent). We claim that the skill attrition channel, along with search discouragement, is observationally equivalent to the one of informational stigma. We show this result by taking as exogenous the endogenous productivity distribution that arises in equilibrium in our model. Average productivity declines over duration are then no longer caused by compositional changes (implied by testing), but are caused by productivity changes of individual workers coming from exogenous skill attrition. In steady state, the same allocation can therefore be supported by both mechanisms, implying an identification problem.

The paper proceeds as follows: First we set-up a directed search model with overlapping generations of unemployed workers (section 2), define equilibrium, and then study its properties (section ). Next, we establish that the equilibrium is inefficient and construct a simple policy that restores efficiency (section ). We carry out a calibration exercise to quantitatively assess the welfare implications of some policies, namely unemployment benefits and wage subsidies. Finally, we replace the stigma mechanism with an exogenous process of human capital depreciation, and we conclude. All proofs are in the appendix.

\section{Data}

This section describes the data. The data source is the Current Population Survey (CPS), a monthly US survey published by the Bureau Labor Statistics. Its suitability relies on the two variables we will pay special attention to: namely, empirical hazard rates and exit wages by unemployment duration. Sampled individuals are in the survey for four consecutive months, then eight out, and again four in. This short longitudinal element allows us to keep track of the employment status of

\(^7\)Note, that in contrast to our model productivities are identical across workers, so that all decisions are independent of the distribution of unemployment durations.
the interviewed agents as well as to learn what the exit wage is. The sample period under analysis is 1994-2008, except for a few months in 1995 due to a methodological change.\footnote{More precisely, the methodological change does not allow us to trace the household identifiers between June and September 1995. Moreover, other changes before 1994 make it difficult the time comparison. For example, looking at averages, weekly earnings are hourly wages by a factor of 40 in the 1994-2008 period, whereas hourly wages are almost three times larger than weekly earnings for the 1989-1993 years. Further, its standard deviation also rises by a factor of 90.}

The final pool of observations is obtained after several restrictions of the original sample. First, we focus on individuals of age 18 to 65 who were unemployed at period $t-1$, and report to be either employed or seeking a job one month later. Thus, those individuals employed or not actively seeking a job at period $t-1$ are removed from the sample. Moreover, those unemployed on recall are also suppressed as not only their job-seeking incentives differ from the remaining pool, but also their exit wages are strongly linked to their prior-to-temporary-layoff earnings.\footnote{Katz and Meyer (1990) estimate the mean weekly income loss at 14.44\% upon a job switch, and at 5.73\% after a recall.} Further, when looking at wages, those newly employed by armed forces or public administration are also removed for obvious reasons. Eventually we work with 69190 observations of individuals’ employment status and 16997 observations for hourly wages conditional on being seeking one month earlier. The main reason for the different data sets is due to the fact that weekly earnings and hourly wages are only asked at the fourth and eighth individual’s interviews. Furthermore, unemployment spells are measured in weeks, and reported by the interviewees with no further control.

First, we describe the hazard rate analysis. We take the 1994-2008 observations conditional on being unemployed on the previous period, and, using the final weights, produce one time series of newly employed and another one of total job-seekers for any unemployment duration. We use the Census program X-12-ARIMA to seasonally adjust those series.\footnote{Missing values are a problem. The way X-12-ARIMA deals with them is by treating them as outliers. Thus, we replace the missing values by -99999, which is the coding that the Census program uses to identify outliers. Unfortunately, for many months the seasonal adjustment is not possible due to too many missing values. That is the reason why Figure 4 plots hazard rates for up to 22 weeks.} Finally, we add up all the observations for any given duration to calculate the empirical hazard rate, $h(\tau)$. That is,

$$h(\tau) \equiv \frac{\sum_t e(t, \tau)}{\sum_t s(t, \tau)}$$

where $e(t, \tau)$ and $s(t, \tau)$ refer to the weighted and seasonally adjusted mass of newly employed and
job-seekers who had been unemployed for \( \tau \) weeks at time \( t \). Figure 1a? shows the empirical hazard rate distribution. It steadily declines as unemployment progresses, although the decline is fairly smooth after three months indicating that the reemployment prospects deteriorate fairly soon.\(^{11}\)

Regarding earnings, CPS has historically distinguished between hourly wages and weekly earnings. The latter refers to before-tax-and-deductions earnings, including overtime pay and tips, and hence also includes a working time component. We focus on hourly wages to rule out the working time effect, although the qualitative results appear to be robust when considering weekly earnings instead. After adjusting by monthly CPI, we adopt a log-linear specification using final weights. We regress log hourly wages on a number of observables: monthly dummies to seasonally adjust, and the age and its square and other dummies related to education, sex, race, marital status, state and major industry and occupation. The coefficient of determination \( R^2 \) is 0.3476. Hereafter we will refer to the regression residuals as the accepted wages upon exiting unemployment. The correlation between accepted wages and unemployment duration is -0.4258 as Table 1 indicates implying that wages decline fairly more slowly than hazard rates, yet real wages show neither a convexity nor a concavity clear pattern.

Furthermore, we aim to highlight any distinguishable pattern among time periods with clearly different aggregate economic performance. We sort months out according to whether the difference of the seasonally adjusted detrended unemployment rate with trend is at least half of the maximum distance, i.e 0.4 percentage points in absolute terms. Each group is formed by over 1000 observations.

\(^{11}\)The correlation with unemployment duration goes from -0.9223 during the first fifteen weeks down to -0.3024 for the remaining spells.
There is no appreciable difference in the empirical job-finding probabilities beyond the expected fact that it is easier to find a job during booms than busts as Figure 1 shows. Regarding wages, as the number of observations turns out to be excessively low at some long durations, we will focus on the first 32 weeks of unemployment. The overall correlation with unemployment duration is higher (in absolute value) during good times (-0.2895 vs. -0.0729), both highly significant.

A final note regarding the cyclicality of real wages is worthwhile. Abraham and Haltiwanger (1995) summarizes the literature based both on aggregated and panel data concluding that real wages are slightly procyclical, at least for the mid 1960s to 1980s period analyzed. They also estimate that the composition effect that could bias earlier studies using aggregate data may not be quantitatively relevant. Studies using longitudinal data sets work mostly with either PSID or NLS. The use of monthly CPS data for this work is thus one difference. Yet the main difference relies on the focus on exit wages from unemployment. We also find that real hourly wages are procyclical for the 1994-2008 period. More specifically, if the unemployment rate is the cyclicality indicator and after controlling for unemployment duration and the dummies mentioned above, then one percentage point rise in the unemployment rate leads to a 1.3 percent drop in real hourly wages.

3 Model Set-up

This section presents an overlapping generations, directed search model, where there is incomplete information on workers’ productivity and imperfect testing by firms.

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Table 1: Correlation with Unemployment Duration

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Overall</th>
<th>Boom</th>
<th>Bust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hazard rate</td>
<td>-0.9029 (0.0000)</td>
<td>-0.8695 (0.0000)</td>
<td>-0.8598 (0.0000)</td>
</tr>
<tr>
<td>Log hourly wage</td>
<td>-0.4258 (0.0000)</td>
<td>-0.2895 (0.0000)</td>
<td>-0.0729 (0.0155)</td>
</tr>
</tbody>
</table>

The significance level is reported within parenthesis.
3.1 Environment

There is a measure one of newborn (unemployed) workers every period, who live for T periods. The economy is populated by a measure T of workers and a continuum of firms determined by free entry in every period. All agents are risk neutral and discount future payoffs at a common rate $\beta$. Unemployed workers derive utility from home production $b$.

Since the focus of this section is on the steady-state, the time index will be suppressed. Workers are identified by a pair $(\tau,i)$. $\tau \in \{1,2,\ldots,T\}$ denotes their age or, alternatively, unemployment elapsed if jobless. $i \in \{l,h\}$ in turn stands for their ability or productivity level $y_i$, with $b = y_l < y_h$. The type $i$ is drawn at birth by Nature with probability $\mu$ for type $h$. Unemployment duration is assumed to be public information. In contrast, there is incomplete information on worker’s actual productivity: it is unobservable to both the worker herself and potential employers, but imperfectly testable by the latter.

Let $u_i(\tau)$ denote the non-normalized measure of unemployed workers of type $(\tau,i)$ at the beginning of period $t$, which is endogenously determined. Hence a mass $u_h(1) = \mu$ of newborn workers are high skilled due to the Law of Large Numbers. The two-dimensional unemployment distribution is the state variable in this economy.

At the beginning of every period, workers can be either employed or unemployed. The unemployed seek job opportunities, whereas employment is an absorbing state. Firms in turn can be either active (matched to a worker and actively producing) or vacant. We identify one job with one firm. Posting a vacancy is a costly activity, while testing applicants is costless: firms incur a recruitment cost $k$.

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15 We will refer to type $l$ workers as low-ability, low-skilled or unskilled indistinctly, and analogously for types $h$.
16 This is a simplifying assumption to rule out the scenario in which hiring firms benefit from the information conveyed by the worker’s past (employment and unemployment) history. It can be relaxed, e.g. allowing firms to post offers contingent on the actual productivity, which is realized at the production time, or to dismiss newly hired workers (limited commitment). If $y_l \geq b$, and wages can be made contingent on the actual productivity, firms have no reason to dismiss the worker. This extension does not change the equilibrium and results appreciably. Otherwise, if a unskilled worker’s productivity is above her leisure value, but optimally dismissed, the worker would learn his type instantaneously, and a new submarket for unskilled would be created.
17 Having screening costless is an assumption to make our life easier. Otherwise, if the screening cost function satisfies the Inada conditions, then we can show that firms (and the social planner) have incentives to invest in screening, and the main results do not change.
Timing Before describing the remaining details, the following outlines the timing of events within each period:

1. Unit mass of workers are born, and join the pool of unemployed.

2. Firms post job offers (contracts) at a cost $k$.

3. Workers direct their search and submit at most one application.

4. Matching process takes place:
   
   (a) Workers take the test.
   
   (b) Firms select one worker among the pool of applications received.

5. Successful job-seekers start producing according to their productivity, which is revealed upon hiring.

6. Unmatched workers remain unemployed, whereas unfilled vacancies disappear.

Screening At no cost, firms may imperfectly screen job candidates. The unskilled applicant passes the test with probability $\lambda_l = \lambda \in (0, 1)$, whereas the skilled one always succeeds, $\lambda_h = 1$.\(^{18}\) If $\lambda = 0$, then testing would be perfect. Test information is not verifiable and cannot be traded. Thus workers do not have access to the test results, and only know the result of the hiring decision. This prevents job-seekers from learning their type instantaneously, whereas recurring application failures form a sequence of signals on their (expected) type. For later use, it is convenient to denote the expected productivity conditional on passing the test of any given cohort $\tau$, and on the distribution $u$, by

$$y_{\tau}(u) = \frac{y_h u_h(\tau) + y_l \lambda u_l(\tau)}{u_h(\tau) + \lambda u_l(\tau)}$$  \((1)\)

Matching Workers and firms come together via search. Dealing with a continuum of job-seekers rules out coordinating strategies, and leads to coordination frictions that guarantees the coexistence of unemployed and vacancies. To be more specific, the matching process relies on the urn-ball scheme. Meetings are multilateral: some urns (i.e. posted offers) may receive many balls (i.e.

\(^{18}\)The normalization is a simplification. The relevant assumption is that type $h$ workers are more likely to pass the test than type $l$ workers.
applications), whereas some other will receive none. Finally, it can only be selected one ball per urn. As a result, some workers and firms are matched, and the remaining seekers stay unemployed and vacant, respectively.

In addition to a wage offer \( w \), firms announce a hiring criterium. Obviously, employers seek to recruit the most profitable applicant. This position has two consequences. First, an additional friction: passing the test is a necessary condition for a worker to be shortlisted. Second, firms commit to a hiring rule \( \sigma \), which is a permutation of the set \( \{1, \ldots, T\} \) that reorders age indexes according to their expected profitability.\(^{19}\) If two different cohorts had the same expected productivity, then the younger cohort would be ranked ahead. That is, for any \( \tau, \tau' \in \{1, \ldots, T\} \),

\[
\sigma(\tau; u) > \sigma(\tau'; u) \text{ iff } J_\tau(w; u) > J_{\tau'}(w; u) \text{ or } J_\tau(w; u) = J_{\tau'}(w; u) \text{ and } \tau < \tau'
\]  

(2)

, where \( J_\tau(w; u) \) stands for the firm’s expected discounted value upon filling the vacancy with a worker of age \( \tau \). It is worth emphasizing that, as it is apparent from the definition, the permutation depends on the state variable, which pins down the expected productivity distribution as in (1). Given the state variable \( u \), the permutation is thus bijective. We will refer to \( s \equiv \sigma(\tau; u) \) as the position in the queue of the representative agent of age \( \tau \).

There is no restriction on the contracting space. Let \( w = (w_\tau) \) be an offered contract, which possibly treats applicants differently according to the observables and rational expectations. Workers of same age use identical mixed-application strategies, which is a natural assumption when looking at symmetric equilibrium with a continuum of agents. Let \( v(w) \) be the measure of vacant firms offering \( w \). Then, each of those firms expects \( q_s(w) \equiv \sum_i \lambda_i u_i(\sigma^{-1}(s)) \frac{1}{v(w)} \) type \( \sigma^{-1}(s) \) workers.\(^{20}\) Moreover \( q^s(w) \equiv (q_1(w), q_2(w), \ldots, q_s(w)) \).

Now, as applications are submitted independently over time and across individuals, the probability that no worker better placed than a \( s \) candidate either applies to a firm offering \( \omega \) or, if

\(^{19}\)The permutation \( \sigma \) takes into account both the expected productivity and the candidates’ remaining lifetime.

\(^{20}\)As an abuse of notation, \( \sigma^{-1}(\cdot) \) denotes the inverse function of \( \sigma(\cdot; u) \).
applied, performs correctly in the test is

\[ P_s(q^{s-1}(w)) = e^{-\sum_{s' < s} q_{s'}(w)} \]  

(3)

The probability of filling a job with a type \( s \) worker a firm posting wage \( \omega \) faces is

\[ \eta_s(q^s(w)) = P_s(q^{s-1}(w)) \left( 1 - e^{-q_s(w)} \right) \]  

(4)

Note that this expression is not only due to firms’ ranking strategy, but also based on the fact that unskilled workers are not profitable and hence never hired. This result stems from the normalization of labor productivity to the leisure value for the sake of simplicity. Although the main results would not change, the algebra would get complicated otherwise since firms would randomize among test-failing workers if couldn’t find a successful candidate.

Since it must be the case that the measure of matched workers and firms coincide, we have

\[ \nu_s(q^s(w))q_s(w) = \eta_s(q^s(w)) \]

Therefore, the job-finding probability for a given type \( s \) worker conditional on applying to job \( \omega \) and passing the test is

\[ \nu_s(q^s(w)) = P_s(q^{s-1}(w)) \frac{1 - e^{-q_s(w)}}{q_s(w)} \]  

(5)

**Value Functions** Let us proceed with the value functions of workers and firms. As employment is an absorbing state, an employed worker derives utility from wages till retirement:

\[ E_T(w) = w_T \frac{1 - \beta^{T-\tau+1}}{1 - \beta} \]  

(6)

An unemployed worker of age \( \tau \) in turn will apply costlessly to any posted job offer that maximizes her utility. The applicant gets employed with probability \( \nu_{\sigma(\tau;u)}(q^{\sigma(\tau;u)}(w)) \) conditional on succeeding in the test, remaining unemployed one more period otherwise. Thus, her value function is:

\[ U_\tau(u) = b + \max_w \left\{ p_\tau(u) \nu_{\sigma(\tau;u)}(q^{\sigma(\tau;u)}(w)) (E_T(w) - b - \beta U_{\tau+1}(u)) \right\} + \beta U_{\tau+1}(u) \]  

(7)

\footnote{This comes out from taking the limit to the below expression to the large game, where \( \rho_{(s',i)}(w) \) is the probability assigned to contract \( w \) by type \( i \) workers of age \( \sigma^{-1}(s') \).

\[ \lim_{s' < s} \prod_{s' < s} \left( 1 - \rho_{(s',i)}(w) \right)^{u_k(s^{-1}(s'))} (1 - \lambda \rho_{(s',i)}(w))^{u_i(s^{-1}(s'))} = P_s(w) \]}

12
where \( p_\tau(u) = \lambda + \frac{u_h(\tau)}{u_h(\tau) + u_l(\tau)}(1 - \lambda) \) is the probability of passing the test conditional on age \( \tau \).

Notice that this last expression requires to take into account the state variable \( u \), or, equivalently, we are assuming that job-seekers keep records of the economy’s history from their birth.

Now, a firm that has offered contract \( w \) and has filled its vacancy with a type \( \tau \) worker receives worker’s productivity net of wages till worker retires:

\[
J_\tau(w; u) = (\bar{y}_\tau(u) - w_\tau) \frac{1 - \beta^{T - \tau + 1}}{1 - \beta} \tag{8}
\]

Like workers, also firms need to know the state variable \( u \) in order to compute expected productivity, \( \bar{y}_\tau(u) \), of applicants. The corresponding value function of a vacant can be written as:

\[
V(w; u) = -k + \sum_{s=1}^{T} \eta_s(q^s(w)) J_{\sigma^{-1}(s)}(w; u) \tag{9}
\]

A firm posting a vacancy for one period incurs a recruitment cost \( k \), and gets the job filled in with a type \( s \) candidate with probability \( \eta_s(q^s(w)) \). That is, they start testing the candidates with the largest expected productivity, and continue testing until they find a successful applicant.

### 3.2 Equilibrium

Now, we define the symmetric recursive equilibrium in the steady state. Notice that the aggregate state is endogenous: it is a result of the history of agents’ equilibrium decisions.

**Definition 1** A steady state symmetric recursive equilibrium consists of value functions \( V(\cdot; u) \), \( J_\tau(\cdot; u) \), \( E_\tau(\cdot) \): \([0, y_h]^T \to \mathbb{R}_+ \), and \( U(u) \in \mathbb{R}_+^T \), a distribution for each type \( i \) of workers over durations \( u_i : \{1, ..., T\} \to [0, 1] \), a hiring rule \( \sigma : \{1, ..., T\} \to \{1, ..., T\} \), a menu of contracts \( \omega \in [0, y_h]^T \), and expected queue lengths \( Q : [0, y_h]^T \to \mathbb{R}_+^T \) such that:

i) The value functions satisfy the above Bellman equations.

ii) Profit maximization and zero-profit condition:

- Given \( u \) and \( \omega \), the hiring rule \( \sigma \) satisfies condition (2).

- Given \( U \), \( Q \), \( u \), and \( \sigma \), \( \omega \) is the profit-maximizing contract, and profits become null.

\[
V(w; u) \leq 0, \text{ with equality for } w = \omega \tag{10}
\]
iii) Workers direct their search:

\[ U_\tau(u) \geq b + p_\tau(u)\nu_{\sigma(\tau;u)}(Q^{\sigma(\tau;u)}(w))(E_\tau(w) - b - \beta U_{\tau+1}(u)) + \beta U_{\tau+1}(u) \text{ and } Q_{\sigma(\tau;u)}(w) \geq 0 \quad \forall w \in [0, \frac{1}{4}] \]

with complementary slackness, where the value function \( E \) is defined by (6).

iv) Recursivity condition: The distribution of workers recursively satisfies:

\[ u_i(\tau + 1) = u_i(\tau) \left(1 - \lambda_i\nu_{\sigma(\tau;u)}(q^{\sigma(\tau;u)}(\omega))\right) \quad \text{for } i \in \{l, h\} \tag{12} \]

provided that \( q_{\sigma(\tau;u)} > 0 \). Otherwise, \( u_i(\tau + 1) = u_i(\tau) \).

v) Vacancies condition:

\[ Q_{\sigma(\tau;u)}(\omega) = Q_1(\omega) \frac{u_h(\tau) + u_l(\tau)\lambda}{\mu + (1 - \mu)\lambda} \tag{13} \]

Firms maximize profits, which drop down to zero in equilibrium because of free entry. The third equilibrium condition is required to pin down rational expectations on queue lengths out of the equilibrium. The Law of Large Numbers ensures that if the job-finding probability of a skilled worker is \( \nu(q) \), then \( u\nu(q) \) skilled workers will leave unemployment at the end of the period. Thus, the fourth condition determines the law of motion of the state variable. Finally, the Vacancies condition implies that the ratio of effective job-seekers to the queue length does not change across positions. This must be the case since the measure of vacancies is not contingent on type.

3.3 Characterization of Equilibrium

Now, we turn to characterize the equilibrium. The following result is the generalization of a standard result in the literature. The equilibrium allocation can be expressed as a fixed point of a correspondence \( f \). Let \( f : [0, 1]^{2T} \times \mathbb{R}_+^T \to [0, 1]^{2T} \times \mathbb{R}_+^T \) be defined as the composite correspondence \( f \equiv \phi \circ \psi \), where \( \phi \) and \( \psi \) are defined as follows.

First, given \( u \) and \( U' \), \( \psi(u, U') \) is defined as the set of triples \((U, \omega, q)\) that satisfy the Vacancies condition and solve the dual to the firm’s profit maximization program:

\[
\begin{align*}
\max_{w,q} & \quad \nu_1(q)(E_1(w_1) - b - \beta U_2') \\
\text{s. to} & \quad \sum_s \eta_s(q)J_{\sigma^{-1}(s)}(w) = k \\
& \quad b + p_\tau(u)\nu_{\sigma(\tau;u)}(q^{\sigma(\tau;u)})(E_\tau(w) - b - \beta U_{\tau+1}') + \beta U_{\tau+1}' \geq U_\tau \quad \forall \tau
\end{align*}
\]

\[ 22 \text{By definition, no deviation tailoring type } s \text{ workers is profitable.} \]
It can be shown that the firm’s maximization problem has a unique solution. Secondly, let \( \phi \) be a function defined as \( \phi(U, \omega, q) \equiv (\tilde{u}, \tilde{U}) \), where \( \tilde{u} \) is uniquely determined by the equilibrium \textit{Recursivity condition} and \( \tilde{U} \equiv U \).

The next proposition characterizes the equilibrium.

**Proposition 3.1** Any equilibrium allocation \((u, \omega, U, q)\) is a fixed point of correspondence \(f\); and conversely, a fixed point of \(f\) is an equilibrium allocation.

It can be shown that \(f\) is a continuous function. Further, note that the finite lifetime of workers implies that the domain of \(f\) is of finite dimension. Hence Brower’s Fixed Point Theorem applies to show existence of a fixed point and thereby existence of equilibrium.

**Corollary 3.2** There exists a symmetric steady state equilibrium, which is characterized by the Recursivity and Vacancies conditions along with the following set of conditions:

\[
\sum_{\tau=1}^{T} (\beta(\tau)\bar{y}_\tau(u) - b - \beta U_{\tau+1}) e^{-\sum_{s'=1}^{\sigma(\tau;u)-1} q_{s'}} \left( e^{-q_{s}(\tau;u)} (1 + \sum_{s'=1}^{\sigma(\tau;u)} q_{s'}) - 1 - \sum_{s'=1}^{\sigma(\tau;u)-1} q_{s'} \right) = -k \tag{15}
\]

For \( \tau \in \{1, \ldots, T\} \),

\[
U_{\tau} = b + \beta U_{\tau+1} + \frac{p_\tau(u)}{e^{\sum_{s'=1}^{\sigma(\tau;u)} q_{s'}}} \left( \beta(\tau)\bar{y}_\tau(u) - b - \beta U_{\tau+1} - \sum_{s'=s+1}^{T} e^{-\sum_{s''=s+1}^{\sigma(s');u} q_{s''}} (1 - e^{-q_{s'}}) (\beta(s')\bar{y}_{s'-1}(u) - b - \beta U_{s'+1}) \right) \tag{16}
\]

\[
\beta(s)w_s = \frac{q_{s} e^{-q_{s}}}{1-e^{-q_{s}}} \left( \beta(s)\bar{y}_{s-1}(u) - b - \beta U_{s+1} - \sum_{s'=s+1}^{T} e^{-\sum_{s''=s+1}^{\sigma(s'';u)} q_{s''}} (1 - e^{-q_{s'}}) (\beta(s')\bar{y}_{s'-1}(u) - b - \beta U_{s'+1}) \right) + b \tag{17}
\]

Uniqueness of equilibrium is not guaranteed. The second equilibrium condition corresponds to the first order condition with respect to the queue lengths in the firms problem. By plugging \( \phi \) into the zero-profit condition (26) and the unemployed workers’ value function (27), the first equilibrium condition and the equilibrium wages are derived, respectively. The equilibrium wages condition (17) deserves some comments. The fraction in the first term is the probability of being her the only type \( \tau \) applicant conditional on the firms receiving at least one application of that
type. It multiplies the marginal output obtained by hiring this worker net of the unemployment value, and the firm’s continuation value, which is the value of filling the job with a type \( \tau' \) worker with probability \( e^{-\sum_{\tau''=1}^{\tau'} q_{\tau''}} (1 - e^{-q_{\tau'}}) \). It is tempting to look at the equilibrium wage equation using a surplus-splitting interpretation. We refer to the term between parenthesis as the surplus of the relationship despite not being properly it. Then, the type \( \tau \) worker’s share of the surplus would be denoted by her marginal probability. Notice that if there were another type \( \tau \) applicant, the marginal value of the worker would be zero. The first term of the surplus stands for the discounted expected productivity if a type \( \tau \) worker is hired, i.e. the joint discounted value of the active job-worker pair. The second term is the continuation value of the type \( \tau \) worker if unemployed one more period. Whereas the last term is the threat point of the firm in the understanding that it will take all the surplus from any relationship with workers with longer unemployment spells as their marginal value would be zero.

### 3.4 Properties of Equilibrium

First, it is worth noticing that the information from the past screening processes will be used by period \( t \) firms to infer applicants’ expected productivity, as unemployment duration may be indicative of several failures of previous applications.\(^\text{23}\) The following lemma states that the expected productivity falls with unemployment duration regardless of what recruiting strategies (other than testing) firms choose. The underlying intuition is that, for any given cohort, the pool of job-seekers deteriorates over time as better candidates are more likely to be hired. However, this result is not immediate, and relies on the focus on the steady state as will see later in Section 4.

**Lemma 3.3** Given a distribution \( u \), \( \bar{y}_\tau(u) \) is a decreasing function of \( \tau \) provided that \( \lambda_1 < \lambda_h \), and passing the test is required for the job.

Given the OLG structure of the economy, this result suggests that employers will end up discriminating against applicants with longer unemployment spells and hence ranking candidates by duration. This intuitive result will be verified by Lemma 3.4, which states that ranking recruiting strategies are optimal in equilibrium by showing that the most productive applicant is indeed the most profitable one. Furthermore, both conditional on passing the test and unconditional job-finding rates also decline with duration.

\(^{23}\)Notice that if multiple employment spells were allowed, recruiting firms would take into account the whole employment history.
Lemma 3.4 The value of an active firm, $J_\tau(\omega)$, and the (conditional and unconditional) job finding rates decline with duration in equilibrium. Therefore, $\sigma(\tau) = \tau$ in the steady state equilibrium.

Workers with longer unemployment spells have a harder time seeking a job relative to those with shorter ones. This equilibrium feature is mainly due to the assumption of skilled workers being more successful on the test. The ranking recruiting strategies also exacerbates the negative duration dependence on hazard rates. Indeed, ranking is the state dependence component of it. Notice that if firms were not discriminating according to unemployment duration, then negative duration dependence would be fully caused by unobserved heterogeneity: Each type of workers would face a constant hazard rate over their lifetime, being higher for skilled ones.

Shimer (2005) shows that wages need not decline given any exogenous declining productivity distribution, as they are set to pay for the marginal value of the worker. If there were two consecutive types with almost identical productivity (with the next type with much lower productivity) and of equal measure, then the less productive worker would be promised a higher wage. That is, some degree of convexity is required to obtain declining wages. In contrast, the expected productivity distribution is endogenously determined here. Sufficient conditions for declining wages imply some degree of convexity. Nonetheless, we have not been able to prove that such conditions hold in equilibrium. Despite that, simulations depict a U-shaped wage scheme

... extensive vs. intensive margin: firms want to post higher wages, not only to attract more candidates, but also to attract better candidates. i.e. not only to fill the job in, but also to fill it with a better worker. That concerns the STU, so that’s another reason why we should expect wages decline

$\Delta(\tau) = \bar{y}_\tau(u) - \bar{y}_{\tau+1}(u)$ is declining with $\tau$. Still need to be proved. Also there are some other elements that matter in the equil. wage equation.

3.5 Simulated Data

For a given set of parameters, we have simulated the equilibrium distributions of wages, and job-finding rates. The qualitative results appear to be robust over different sets of parameter values. The parameters are set as follows: $\mu = 0.5$, $\beta = \ldots$, $T = 100$ periods....

Figure 1 shows the wage distribution by unemployment duration. As expected, it shows a declining trend, and a mild rise after some point. The wage increase is due to the finite horizon framework of the OLG economy as described by the equilibrium wage equation: at the very last
period of life, the marginal value of a candidate exceeds her value at $T - 1$, what remains true for a number of periods going backwards. Figure 2 in turn depicts the distribution of conditional job-finding rates.

3.6 Efficiency Properties of Equilibrium

A number of public policies have been implemented worldwide to help the long term unemployed. Policies are of two types: redistributive and market failure-targetting. In this section we claim that a benevolent social planner could improve upon the decentralized equilibrium, and thus public intervention is needed to maximize social welfare.

The centralized economy is set in the Appendix. As is standard in the literature with risk neutrality, a benevolent dictator posts a number of vacancies to maximize total output net of recruitment costs. The planner is also subject to the same constraints specified above for the decentralized economy: namely, the anonymity constraint and the imperfect $\lambda$-testing technology. The latter constraint ensures that the planner has no access to better screening technologies than firms do in the decentralized economy. The former in turn limits the ability of the planner to assign workers to jobs: Each vacancy receives a number of applications, identified by the age of the applicants, and the planner is free to choose any among them.
Proposition 3.5 The steady state equilibrium is not constrained efficient provided the test is informative. A tax on firms’ profits or on the vacancy-posting activity leads the equilibrium to attain efficiency.

We think the $T = 2, b = 0$ case provides the insights for the inefficiency result. Along with the vacancies conditions, the equilibrium and constrained efficiency conditions are, respectively:

\[-k \frac{p(1)}{q_1^t} = p(1) \left( \frac{dp_1(q^1_1)}{dq_1} \beta_1 \bar{y}(1) + \frac{\bar{y}(2)}{q_1^t} \left( q_1 q_2 \frac{\partial \nu_2(q^2)}{\partial q_1} + q_2^2 \frac{\partial \nu_2(q^2)}{\partial q_2} - \beta q_1^2 p(2) \frac{\partial \eta_2(q^2)}{\partial q_2} \frac{dp_1(q^1_1)}{dq_1} \right) \right)\]

\[-k \frac{p(1)}{q_1^t} - A = p(1) \left( \frac{dp_1(q^1_1)}{dq_1} \beta_1 \bar{y}(1) + \frac{\bar{y}(2)}{q_1^t} \left( q_1 q_2 \frac{\partial \nu_2(q^2)}{\partial q_1} + q_2^2 \frac{\partial \nu_2(q^2)}{\partial q_2} - \beta q_1^2 p(2) \frac{\partial \eta_2(q^2)}{\partial q_2} \frac{dp_1(q^1_1)}{dq_1} \right) \right)\]

where $A = \beta \frac{p(1)}{q_1^t} \left( \eta_2(q^2_{t+1}) \frac{\partial \nu_2(q^2_{t+1})}{\partial q_1, t} \bar{y}_{t+1}(2) + q_2^2 \frac{\partial \nu_2(q^2)}{\partial q_2} \frac{dp_1(q^1_1)}{dq_1} p(2) \right) = -\frac{dp_1(q^1_1)}{dq_1} \beta q_1 \mu (1 - \mu)(1 - \lambda) \left( q_2 \nu_2(q^2) \left( \frac{\lambda_{y_1}}{u_{1, 1} + \lambda_{y_2}} \right) + q_1 \bar{y}(2) \frac{\partial \nu_2(q^2)}{\partial q_2} \frac{1 - \lambda}{p(1)(u_{1, 1} + u_{2, 1})} \right) > 0$

Notice that conditions (3.6) and (3.6) only differ in term $A$, which is positive. First, the extra term $A$ vanishes, and thus the equilibrium is constrained efficient, if and only if the test is not informative, i.e. $\lambda = 1$. Second, provided that the test is informative, a tax on the entry cost, or equivalently on firms’ profits, would lead the decentralized economy to the social planner’s allocation.

However, this requires some reasoning.

Let $Y_{t+1} = v_{t+1} q_{t+1} \bar{y}_{t+1}(1) + v_{t+1} \eta_{2, t+1}(q^2_{t+1}) \bar{y}_{t+1}(2)$ be the output produced at period $t+1$. Then, since

\[\frac{\partial Y_{t+1}}{\partial q_{1, t}} = \beta \frac{p(1)}{q_1^t} \left( \eta_2(q^2_{t+1}) \frac{\partial \nu_2(q^2_{t+1})}{\partial q_1, t} \bar{y}_{t+1}(2) + q_2^2 \frac{\partial \nu_2(q^2)}{\partial q_2} \frac{dp_1(q^1_1)}{dq_1} p(2) \right)\]

we can rewrite (3.6) as

\[A = \beta \left( \frac{\partial Y_{t+1}}{\partial q_{1, t}} + \frac{\partial (p(1)(1 - \nu_2(q^1_t)) \beta U_2')}{\partial q_{1, t}} \right) = \beta \frac{p(1)}{q_1, t} \left( \eta_2(q^2_{t+1}) \frac{\partial \nu_2(q^2_{t+1})}{\partial q_1, t} \bar{y}_{t+1}(2) + q_2^2 \frac{\partial \nu_2(q^2)}{\partial q_2} \frac{dp_1(q^1_1)}{dq_1} p(2) \right)\]

Now, let us explain the intuition for the inefficiency result. The term $A$ has two components: The first component is the discounted forgone period $t+1$’s output due to the marginal increase in $q_{1, t}$. This term has in turn two other: the effect on the expected productivity $\bar{y}_{t+1}(2)$ conditional
on a certain \( q_{2,t+1} \), i.e. the intensive margin, and the effect on the vacancies’ matching probabilities with older candidates conditional on their expected productivity, i.e. the extensive margin. And the second component of \( A \) is the standard intertemporal effect: it stands for the effect of the marginal change in \( q_{1,t} \) on the discounted utility of the set of unmatched workers at the end of period \( t \).

Two sources of inefficiency are present. First, there is a negative effect of a marginal increase in \( q_{1,t} \) on period \( t+1 \)’s expected productivity, conditional on a certain \( q_{2,t+1} \), which is not captured in equilibrium as period \( t \)’s decisions do not affect the period \( t+1 \)’s equilibrium market values. In the extreme \( \lambda = 0 \) case, this inefficiency vanishes as firms can perfectly discriminate among candidates.

Second, the effect of the marginal increase \( q_{1,t} \) on period \( t+1 \)’s expected output through its partial effect on \( q_{2,t+1} \) is not entirely captured in equilibrium. The fact that the last term in \( A \) is negative indicates that part of this marginal effect on period \( t+1 \)’s output is indeed captured in equilibrium. That is, the effect of the marginal \( q_{1,t} \) on period \( t+1 \) outweighs the standard intertemporal effect on the level of the labor force as it also affects the skill distribution.\(^{24}\) It may be worth looking at the \( \lambda = 0 \) case. This extreme case corresponds indeed to an asymmetric information scenario as firms have perfect information on applicants’ skills, whereas workers do not. The planner is concerned on the asymmetric information problem to the extent of mismatch. Firms in turn have the same concern, but face an extra friction: they have to guarantee the market value to all workers, even to those who do not pass. Notice that the constrained efficient allocation would be attained if and only if \( p(2) = 1 \).

4 Aggregate Shocks and Exogenous Job Destruction

Now, we turn to analyze an economy with shocks on aggregate productivity and the exogenous job-separations idiosyncratic probabilities. The comparative statics analysis in the previous sections with respect to skilled workers’ productivity suggests, consistently with Blanchard and Diamond (1994), that the negative effect of unemployment duration on the hazard rate becomes stronger during recessions. The intuition is clear: during good times, the labor market is very tight, and applicants are likely to face no competition of workers with shorter durations; whereas this rarely happen during recessions in which the ratio of vacancies to applicants is much smaller, and firms may discriminate against those of longer duration. The insight on wage trend differences is based on the

\(^{24}\)The difference between these last two terms from \( A \) can be simplified into the very last term in expression (3.6)
amount of information obtained, and hence on how firms adjust their expectations. During booms, firms rationally anticipate a drastic drop in workers' productivity with duration, and proceed to adjust wages accordingly. In contrast, in depressed labor markets no much information is conveyed leading to relatively flatter productivity losses over duration.

Mortensen and Pissarides (1994) allow for firm-initiated separations, and Menzio and Shi (2009) for endogenous quits along the business cycle. Instead, we are to impose exogenous breaks on productive pairs. This greatly simplifies the model as tenure at the past job would be informative otherwise, and vacant firms would optimally use a two-dimensional ranking recruiting strategy. To keep things simple, we follow the basic shock structure dealt with in Cole and Rogerson (1999).

4.1 Changes in Environment

The economy is now stochastic. The current state of the economy is denoted by $s \in S \equiv \{s_1, s_2\}$, where $s_1$ stand for bad times, and $s_2$ for good ones. To obtain a recursive formulation, the stochastic variable is to be governed by a Markov process. Let $\Phi$ denote the symmetric transition matrix, with $\phi$ denoting the probability of remaining in the same state one period later. Shock persistence requires $\phi > 0.5$. We model the effects of the state of the economy along two dimensions. First, we normalize unskilled workers' productivity to 0 so that they are never employable, and their productivity does not evolve with the aggregate shocks. In contrast, skilled workers' productivity $y_h = y(s)$ evolve according to the aggregate productivity level. For notational simplificity, let $y_i \equiv y(s_i)$, with $y_1 < y_2$. Second, job-worker pairs are exogenously separated at an idiosyncratic probability $\psi(s)$. Again, let $\psi_i \equiv \psi(s_i)$, and $\psi_1 > \psi_2$. That is, aggregate productivity reduces and job-separations are more frequent at bad states, and conversely for good states. Furthermore, the economy is hit by shocks that follow a Markov process.

Every period a cohort of unemployed workers are born, and live for $T$ periods. Contrary to the benchmark setup, the measure of newly unemployed workers is endogenously determined. A new cohort is of measure larger than one as now all dismissed workers are replaced by newborn ones. Upon the realization of new state $s$, job-separations occur. Let $d(s, e)$ denote the dismissed workers at state $s'$ for a given measure $e$ of employed workers at the end of the past period. Therefore, there is a mass $T$ of workers each period.

---

$^{25}$Alternatively, we can interpret the productivity fluctuations in relative terms.

$^{26}$This assumption is to keep things simple. Otherwise, firms might prefer to rank candidates by duration and lifetime span.
Notice that although job destruction is exogenous, and hence tenure at past jobs is not informative, period $t$ vacant firms may obtain information from candidates’ unemployment history prior to last job. We rule out this history-dependent strategy by assuming that unemployment records are kept only since last dismissal.

**Timing.** There are four stages within each period: separation and birth, search, testing and matching, and production. At the separation stage, the state of the economy is realized. Then, job-separations take place, and a positive measure of workers $d(s, e)$ are laid off. A mass, $1 + d(s, e)$, of newly unemployed workers are born. At the second stage, contractual offers are post, and applications are sent out. Next, candidates are screened, and the best available are selected out closing the matching stage. Finally, matched workers produce, and wages are paid.

**Contracts and Matching** The state variable now becomes $(s, u)$, where $u$ stands for the unemployment distribution as above. There are no substantial differences with respect to the benchmark economy. It is worth emphasizing two features, though. First, contractual offers must stipulate a wage at any future date as a function of the realization of the aggregate productivity, and may be contingent on the candidate’s observables at the hiring date (i.e. unemployment duration). Second, we will refer to $q(w, (s, u))$ as the expected queue length associated to each contract $w$ when the market conditions are $(s, u)$. Thus, it may vary with the stochastic state, being potentially relatively lower at times of high aggregate productivity.

**Value Functions** Now, we turn to describe the Bellman equations. First, the employed worker is paid according to the contractual agreement for any given aggregate productivity. As now employment is no longer an absorbing state, an employed worker will lose her job with positive probability.

$$E_r(w; (s, u)) = w_r(s, u) + \sum_{j=1}^{T-r} \beta^j \sum_{s_j \in S} \Phi_j(s_j|s_0 = s)(1 - \psi(s_j))w_r(s_j, u)$$

(18)

where the summation term stands for the discounted expected value over state paths. Notice that the contract does not change over time, therefore wages only change if the new realization of the state differs.

An unemployed worker of type $\tau$ will apply to any posted job offer $w$ that maximizes her utility, taking into account the associated expected queue length $q^\tau(w, (s, u))$, which also depends on the aggregate state of the economy. The applicant gets employed with probability $\nu_r(q(w, (s, u)))$
conditional on succeeding in the test, remaining unemployed one more period otherwise. Thus, her value function is:

\[ U_\tau(s, u) = b + \max_w \left\{ p_\tau(u) \nu_\tau(q^\tau(w, (s, u))) \left( E_\tau(w; (s, u)) - b - \beta E_{\tau+1}(s', u|s) \right) \right\} + \beta E_{\tau+1}(s', u|s) \]

(19)

where \( p_\tau(u) = \lambda + \frac{u_h(\tau)}{u_h(\tau) + u_l(\tau)} (1 - \lambda) \) is the probability of passing the test conditional on age \( \tau \). Notice that this last expression does not depend on the value of the current state.

Now, a firm that has offered contract \( w \) and has filled its vacancy with a type \( \tau \) worker receives worker’s productivity net of wages till worker retires:

\[ J_\tau(w; (s, u)) = \overline{y}_\tau(s, u) - w_\tau(s, u) + \sum_{j=1}^{T-\tau} \sum_{s_j \in S} \beta^j \Phi^j(s_j|s_0 = s)(1 - \psi(s_j))(\overline{y}_\tau(s_j, u) - w_\tau(s_j, u)) \]

(20)

where the expected productivity of the cohort is pinned down by the straightforward update of expression (1). Again, the continuation value of the filled vacancy depends on the unemployment distribution from and age at which the worker was hired. The corresponding value function of a vacant can be written as:

\[ V(w; (s, u)) = -k + \sum_{\tau=1}^{T} \eta_\tau(q^\tau(w, (s, u))) J_\tau(w; (s, u)) \]

(21)

A firm posting a vacancy for one period incurs a recruitment cost \( k \). Recall that the hiring probabilities, \( \eta_\tau \), defined above are based on the (later to be confirmed) assertion that firms rank candidates by duration. That is, they start testing the candidates with the shortest unemployment spell and continue testing until they find a successful applicant.

Given the state at period \( t \), \( s \), the measure of workers who did not succeed in obtaining a job this period (and are of age under \( T - 1 \)) is defined as

\[ \Sigma_u(s) = \sum_{\tau=1}^{T-1} \sum_{i=1}^2 u_i(\tau) (1 - \lambda_i \nu_\tau(\omega, (s, u))) \]

(22)

Furthermore, the measure of employed workers (of age under \( T - 1 \)) at the end of the period is

\[ e(s) = T - \Sigma_u(s) - \sum_{i=1}^2 u_i(T) (1 - \lambda_i \nu_\tau(\omega, (s, u))) \]

(23)

Finally, upon the realization of the shock \( s' \), the measure of displaced workers at the beginning of period \( t \) is

\[ d(s', e(s)) = \psi(s') e(s) \]

(24)
The law of motion of the unemployment distribution is now slightly different. Let \((s, u)\) be the state variable at period \(t\). Then, period \(t+1\)'s unemployment distribution is defined by the following Recursivity condition.

\[
\begin{aligned}
 u'_i(\tau+1; s') &= \begin{cases} 
 u_i(\tau) (1 - \lambda_i \nu_i(\omega, (s, u))) \quad & q_\tau > 0 \\
 u_i(\tau) \quad & \text{o.w.}
\end{cases} \\
 u'_i(1; s') &= \mu_i(1 + d(s', e)) 
\end{aligned}
\]

4.2 Equilibrium

We now turn to define the stationary equilibrium.

**Definition 2** A stationary symmetric recursive equilibrium consists of, for any given pair \((s, u)\), value functions \(V(\cdot; (s, u))\), \(J_\tau(\cdot; (s, u))\), \(E_{\tau}(\cdot, (s, u))\): \([0, y_h]^T \to \mathcal{R}_+\), and \(U(s, u) \in \mathcal{R}_+^T\), a menu of contracts \(\omega(u): S \to [0, y_h]^T\), expected queue lengths \(q(\cdot, (s, u)) : [0, y_h]^T \to \mathcal{R}_+^T\), a measure of displaced workers \(d : S \times [0, T] \to [0, T]\), and the stationary distribution \(M\) over pairs \((s, u)\) such that:

i) Given the other equilibrium objects, the value functions satisfy the corresponding Bellman equations.

ii) Profit maximization and zero-profit condition: Given the state variable \((s, u)\), \(U\), and \(q\), each firm maximizes expected profits, which become null.

\[
V(w; (s, u)) \leq 0, \text{ with equality for } w = \omega
\]

iii) Workers direct their search:

\[
U_\tau(s, u) \geq b + p_\tau(u) \nu_\tau(w, (s, u)) \left( E_\tau(w; (s, u)) - b - \beta E_{s'} \left( U_{\tau+1}(s', u)|s) \right) + \beta E_{s'} \left( U_{\tau+1}(s', u)|s) \right) \right)
\]

with complementary slackness, where the value function \(E\) is defined by (18)

iv) Vacancies condition: \(q_\tau(\omega, (s, u)) = q_1(\omega, (s, u)) \frac{u_h(\tau) + u_l(\tau)\lambda}{u(1; s) \mu + (1 - \mu)\lambda}\)

v) Law of motion of the state variable: \(M(s, u) \equiv (s', u')\), where \(M\) is pinned down by the transition matrix of the Markov chain \(\Phi\), and by the Recursivity condition (25).
The definition of the stationary equilibrium does not substantially differ from the one of the steady state equilibrium in Section 3.2. Items ii) and iii) enforce agents’ optimizing behavior as well as impose firms breaking even in every period. Item iv) is the natural counterpart of the vacancies condition in Section 3.2. Finally, item v) imposes rational expectations when determining the transition matrix for the state vector \((s, u)\), induced by the transition matrix of the state Markov process along with the Recursivity condition.

5 Policy Analysis

Despite the wide set of public policies implemented in most OECD countries targeting long term unemployed workers (ALMP account for up to 2.2\% of Swedish GDP), little normative work has been carried out. Thus, it is of major interest to analyze the effects of public policies on equilibrium wages, job-finding rates, unemployment rate and expected unemployment duration. In this subsection we claim that most public policies targeting long term unemployment have very little returns if firms rank candidates by duration. We will focus here on three policies widely used to help long term unemployed workers: namely, unemployment benefits (the minimum wage legislation), wage and employment subsidies, and job-search assistance programs.

In particular the third policy exhibits drastic differences depending on the mechanism. The focus for now is on the benchmark (informational stigma) mechanism.

Unemployment Benefits  We undertake a comparative statics analysis with respect to the home production parameter, \(b\). Not surprisingly, the higher \(b\), the fewer vacancies posted.\(^{27}\) As firms compete for workers by posting wage contracts, a higher home production obliges firms to raise their wage offers reducing the returns to posting. Thus, job prospects deteriorates. Notwithstanding, job-finding rates for long term unemployed reduce not only because there are fewer vacancies to apply to, but also because the event of those fewer vacancies not being filled in by short term unemployed is of lower probability. In other words, firms’ ranking strategies makes the unemployment benefits effects on job prospects be increasingly stronger with duration. This is complementary to Ljungqvist and Sargent (1998) story for the long term unemployment differences between the US and Europe. In their case, the European higher rates are explained from the supply side:\(^{28}\) the larger the benefits,

\(^{27}\)The proof is for \(T=2\) for the time being. Needs to be extended to the general case.

\(^{28}\)The demand side is not modeled in their paper.
the less intensely job-seekers search. In their case, human capital depreciation amplifies this effect. In contrast, as just explained the mechanism relies on the demand side in our model.

**Minimum Wage** A binding minimum wage legislation may have a well-known effect: it makes unemployed become unemployable after a certain duration as unemployment duration is associated here with low expected productivity. There is a secondary effect. By conditioning hiring of low productive workers, the minimum wage policy restrains firms’ entry as it reduces the expected profits of a given vacancy. Thus, skilled workers turn out to be affected as well. The empirical relevance of this effect remains to be estimated.

**Job-search Assistance Programs** Long term unemployed are usually helped to write their applications and resumes, and prepare their interviews (see Heckman et al.). In our model, these programs can be modeled by raising the parameter \( \lambda \) for long-term unemployed as this parameter stands for the probability of passing the screening. Notice that this policy is inefficient by definition: it reduces the level of information as the testing device becomes less informative. Thus, firms update their beliefs on the imperfectness of the screening technology, and the expected productivity of long term unemployed would fall down reducing firms’ entry. In contrast, any policy ensuring active search are welfare improving (e.g. the incentives from unemployment insurance matter).

**Hiring Subsidies** Subsidizing job creation is a policy widely used across countries. Its usefulness mainly relies on the assumption of human capital deterioration while unemployed, and skill acquisition on the job. However, if the informational stigma mechanism is of some empirical importance, this policy introduces important distortions in the economy. It modifies firms’ hiring incentives: the subsidy makes job-seekers with longer unemployment spells more attractive to firms even though they are less skilled than others in expected terms. In Section ? we estimate the welfare/output losses of this policy.

**Centralized Screening Systems** There is an obvious information-wasting problem in this economy as a negative result at testing is only observable to firms, untradable and imperfectly captured by job-seekers’ unemployment duration. Efficiency gains would be obtained by centralizing testing. Two empirical counterparts of centralized screening are worthwhile mentioning: namely, the US Temporary Help Supply industry and the Japanese school-based hiring system. Regarding the
former, Autor (2001) reports that THS firms grew systematically in the 1990s accounting for 10 percent of employment growth in that decade. Autor (2003) estimates the effects of the changes to the employment-at-will doctrine on the growth of the THS industry at 20% from 1973 to 1995. In addition to training and labor supply, the THS industry offers screening services. In particular, screening is relevant for one fourth of THS services clients, who aim at filling in permanent positions. Japanese high-schools in turn play an active role in the job-assignment of the newly-graduated students, and firms benefit from the bulk of information accumulated by teachers over the education years.

6 Conclusions

This paper builds a theory of ranking by unemployment duration based on informational stigma. Firms form rational expectations on the expected productivity of job applicants according to the observables: their unemployment duration conveys information on past rejections. We show that if this economic mechanism is empirically relevant, the policy implications are important. Many OECD countries have implemented a number of active labor market policies targeting long-term unemployment, such as employment subsidies and job-search assistance programs. We claim here that such policies may introduce distortions in the economy by either making firms hire less productive workers (as in the former case) or making the screening technology less informative. Finally, we quantitatively assess the welfare gains of such policies.

References


7 Appendix

7.1 Model with Skill Attrition

As van der Berg and van Ours (1996) state, unobserved heterogeneity and true state dependence appear to yield very similar hazard rate distributions, what leads to an identification problem of difficult solution. We show here that indeed, two mechanisms that may be responsible for each of those two components of negative duration dependence are observationally equivalent. In this section we consider a more general model in which human capital depreciates while unemployed. We model it as a expected productivity drop, which may vary over unemployment, \( \delta(\tau) \). Therefore, the expected productivity distribution becomes \( y(\tau) = \bar{y}(\tau) - \delta(\tau) \). Clearly, this is an additional reason for recruiting firms to rank candidates by unemployment duration.

First, we show that the stigma mechanism presented above and a mechanism based mainly on human capital depreciation may be observationally equivalent. That is, for a given skill attrition distribution over duration, the two mechanisms support the same equilibrium outcome. The identification problem is thus of major interest, first, to understand the relevance of each channel on the unemployment distribution, and, second, to recommend adequate public policies.

Second, we calibrate the extended model to the US economy, and undertake some counterfactual policy experiments.

7.1.1 Identification Problem

Let us consider a slightly different physical environment, where the differential features are:

1. Workers are ex-ante identical. Thus, testing is useless. 29

2. Unemployed workers’ productivity depreciates every period according to the exogenous distribution \( y(\tau) \).

3. Search discouragement: workers apply with an exogenous intensity, \( s(\tau) \), to jobs as unemployment progresses.

The following proposition claims that the equilibrium allocation characterized by the fixed point condition of function \( f \) may be also obtained in this environment. Two requirements must hold:

29 Alternatively, we might consider that firms disregard screening candidates because of too a high testing cost.
First, workers’ productivity depreciates as unemployment elapses according to the stigma equilibrium expected productivity distribution $\bar{y}_\tau$. Second, the exogenous search intensity declines with duration according to $p(\tau; u)$. These two conditions make the equilibrium characterization problem formally equivalent to the above one.

**Proposition 7.1** Any equilibrium allocation $(u, \omega, U, q)$ derived from the stigma scenario is also an equilibrium outcome in this environment provided that $y(\tau) = \bar{y}_\tau$, and $s(\tau) = p(\tau; u)$.

Two issues are worth highlighting in the comparison. First, despite the equivalence in equilibrium outcomes, the efficiency properties differ. This is not surprising as the physical world must also differ in the centralized economies. Recall that the stigma equilibrium was shown to be constrained inefficient because agents could not internalize the effects of testing on future newborn agents. When making productivity and search intensity exogenously decline with duration, the planner cannot improve upon the decentralized outcome under certain assumptions: namely, the productivity distribution is concave. It is easy to come up with examples in which ranking workers by duration is individually optimal, but not socially. For instance, for $T=2$, and with an $\epsilon$ skill deterioration, the planner may prefer to hire the “long term” unemployed workers if the discounting parameter is low enough.

Second, despite the steady state equivalence, the two propagation mechanisms behave differently when the economy faces a sudden temporary shock. Let us consider an aggregate negative and temporary productivity shock. The flow of vacancies instantaneously reduces due to lower expected returns to posting. With exogenous decline in productivity and search intensity, a temporary unemployment rise would have long effects over time. In contrast, in the benchmark environment the propagation mechanism is affected by the demand side’s reaction to the negative shock. The more loose the labor market is, the less information on the expected productivity of the pool of remaining applicants is conveyed to the future because of less testing. As a result, for any given duration $\tau$, the average productivity of the unemployed pool declines at a lower rate than before the shock. Therefore the transition back to the steady state must be shorter and showing lower unemployment rates in the stigma scenario. (simulation). This is consistent with Lazear’s (1986) results. In his model, stigma is also more of an issue during booms than during recessions.

Despite the obvious interest in disentangling the two channels, no much can be said as even if the human capital depreciation rates are disproportionally high, that may be caused by a not well-pinned
down search discouragement effect. There is little work devoted on isolating the causes of negative
duration dependence whereas a number of public policies have been implemented worldwide to help
the long term unemployed.

The two mechanisms of duration dependence in this paper, however, have been shown to have
quite different implications.

7.2 Proofs

Proof of Lemma 3.3
Let $P(i|\tau, p)$ denote the probability that a worker of age $\tau$ is of type $i$ conditional on having passed
the test (event $p$). We need to show that $P(h|\tau, p)$ is decreasing in $\tau$. Then, by using Bayes’ rule,

$$P(h|\tau, p) > P(h|\tau + 1, p) \iff \frac{P(h)P(\tau, p|h)}{P(h)P(\tau, p|\tau) + (1 - P(h))P(\tau, p|\bar{l})} > \frac{P(h)P(\tau + 1, p|h)}{P(h)P(\tau + 1, p|\tau) + (1 - P(h))P(\tau + 1, p|\bar{l})}$$

Now, after some manipulations, this inequality is true if and only if

$$P(\tau|\bar{l}) < P(\tau + 1|\bar{l}) \iff \lambda_h > \lambda_l$$

where $P(\tau + 1|\tau)$ stands for the probability of staying unemployed one more period conditional on
passing the test. Thus, the expected productivity $\bar{y}_\tau$ falls with $\tau$ provided that type $l$ workers are
less successful in passing the test than type $h$ job-seekers are.

Proof of Proposition 3.2 Let $f : K \to K$ defined as the composition $\phi \circ \psi$. We just need to
show that $f$ is a continuous function, and $K \subset \mathbb{R}^{3T}$ is a compact set. Then, Brower’s Fixed Point
Theorem applies, and ensures the existence of a fixed point of function $f$.

Let us first define $K$. $K \equiv \{ z = (u, U') \in [0, 1]^{2T} \times [0, y_h\beta(1)]^T \mid \frac{u_h(\tau + 1)}{u_l(\tau + 1)} \leq \frac{u_h(\tau)}{u_l(\tau)}, U'_{\tau + 2} \geq \frac{\beta(\tau)\bar{y}(\tau; u) - \beta(\tau + 1)\bar{y}(\tau + 1; u)}{\beta}, \} \}$, where $\beta(\tau) = \frac{1 - \beta^{T-\tau+1}}{1 - \beta}$. In words, $K$ is the nonempty set of
pairs formed by worker distributions in the two dimensional age-productivity space and today’s
expectations on unemployment value functions. The reasons for the defining constraints will be
seen in short. Obviously, the set in question is compact.

We turn now to show that $\psi(z)$ is a continuous function. After substituting out the wages from
the complementary slackness conditions, the firm’s problem can be rewritten as

$$\max_q F(q) \equiv \sum_{\tau=1}^{T} \left( \eta_r(q^\tau)\Delta(\tau) - q_r U_\tau - b - \beta U'_{\tau + 1} \right)$$
where $\Delta(\tau) \equiv \bar{y}_\lambda(\tau)\beta(\tau) - b - \beta U'_{\tau+1}$. Wages were replaced out because unemployed workers strictly prefer employment to unemployment. Then, the complementary slackness condition implies that low wages are penalized with zero queue lengths. In other words, $U_\tau > b + \beta U'_{\tau+1}$. First, we will show that the firm’s problem has a unique solution, which is characterized by the first order conditions. The Hessian of function $F$ is $D^2 F = (h_{ij})_{i,j}$, where for any given pair $(i, j)$, with $i \leq j$,

$$h_{ij} = \sum_{\tau \geq j} \frac{\partial^2 \eta(r(q^\tau))}{\partial q_i \partial q_j} \Delta(\tau) = -\frac{U_j - b - \beta U'_{j+1}}{p\lambda(j)}$$

The last expression for $h_{i,j}$ is only true at the critical values of $F$. Therefore, $h_{i,j} < 0$ at the critical values for all $i, j$. Now, we will show that the Hessian is negative definite at all critical values. That is the case if and only if $v'D^2 F v < 0$, where $v$ is a nonzero vector of dimension $T$. Notice that $v'D^2 F v = \sum_{i=1}^{T} h_{ij} \left( \sum_{l=1}^{T} v_l \right)^2$, which is strictly negative since: first, all hessian components are so, and are multiplied by nonnegative numbers; and second, $\sum_{l=1}^{T} v_l \neq 0$, where $i$ is the subindex of the last nonzero component of vector $v$. As a result, all critical values must be local maximums, i.e. there is a unique global maximum, and the FOC are also sufficient.

Let $z \in K$. Then, $\psi(z)$ is defined by the solution set of the following system:

$$q_\tau = q_1 \alpha(\tau) \text{ for all } \tau < T$$
$$q_T = q_1 \alpha(T)$$
$$\sum_{\tau=1}^{T} (\beta(\tau)g(\tau; u) - b - \beta U'_{\tau+1}) \sum_{j=1}^{\tau} q_j q_\tau \frac{\partial \nu(\tau)}{\partial q_j} = -k$$

(29)

where $\alpha(\tau) = \frac{u_h(\tau) + u_l(\tau)\lambda}{\mu + (1-\mu)\lambda}$. The third condition comes from rearranging terms in the zero-profit condition by using the FOC of the firms’ maximization problem, whereas the remaining equations are the Vacancies condition (13).

It must be shown that the solution set is singleton. For that, the variables $q_\tau$ are replaced out from the last condition imposing the first equalities. Thus, it remains a two equation system with two unknowns $q_1$ and $q_T$. The equation (??) establishes a positive relationship between the two
variables. Finally, after taking derivatives on the equation (29) with respect to \( q_1 \), we obtain

\[
\bar{y}(T; u) e^{-(q_1 \sum_{i}^{T-1} \alpha(i) + q_T)} \left( \sum_{i}^{T-1} \alpha(i) + q_T \right) \frac{dq_T}{dq_1} =
\]

\[
-\sum_{\tau}^{T-1} (\beta(\tau) - \beta U_{\tau+1} - \beta(\tau + 1) U_{\tau+2}) q_1 e^{-q_1 \sum_{i}^{T-1} \alpha(i)} \left( \sum_{i}^{\tau} \alpha(i) \right)^2
\]

\[
-\bar{y}(T) \sum_{i}^{T-1} \alpha(i) e^{-q_1 \sum_{i}^{T-1} \alpha(i) - q_T} \left( q_T \sum_{i}^{T-1} \alpha(i) + q_T \right)
\]

From what it follows \( \frac{dq_T}{dq_1} < 0 \) in \( K \) as the second defining restriction implies that the first term of the right hand side is negative. Hence \( \psi \) is a function on \( K \). Indeed, it is a continuous one. Since \( \phi \) is obviously a continuous function, so is the composed \( f \). Finally, the first defining constraint of the set \( K \) guarantees that the equilibrium is characterized by a declining expected productivity over the lifetime. 

**Proof of Lemma 3.4**

The first part of the statement says that \( J_\tau \) declines with \( \tau \). Its proof is omitted as follows closely Shimer (2005). Regarding the second part, to show that the job-finding rate conditional on passing the test it must be shown that for any \( \tau \),

\[
\frac{1 - e^{-q_\tau}}{q_\tau} > \frac{1 - e^{-q_{\tau+1}}}{q_{\tau+1}}
\]

(30)

or, rearranging terms

\[
e^{q_\tau} \frac{1 - e^{-q_\tau}}{q_\tau} > \frac{1 - e^{-q_{\tau+1}}}{q_{\tau+1}}
\]

(31)

Notice that the left hand side is an increasing function of \( q_\tau \). Thus, the inequality holds provided that \( q_\tau > q_{\tau+1} \). In equilibrium this condition is satisfied due to the *Vacancies condition*. Now, to show that unconditional job-finding probabilities also decline with \( \tau \), we need to prove that

\[
(p(h|\tau + 1) + \lambda(1 - p(h|\tau + 1))) \nu_{\tau+1}(q_{\tau+1}) \leq (p(h|\tau) + \lambda(1 - p(h|\tau))) \nu_{\tau+1}(q_\tau)
\]

(32)

We just proved that \( \nu_{\tau+1}(q_{\tau+1}) \leq \nu_{\tau}(q_\tau) \). As the passing the test probability also decreases with \( \tau \) (32) holds. 

**Proof of Lemma ??**
Let us consider the equilibrium allocation of Section 3. Now, let us consider the Inada conditions on the cost function $c$. Consider an $\epsilon$ measure of rms deviating to pay for $\lambda - \delta$-screening services. That is, unskilled applicants will pass the test with probability $\lambda - \delta$ if applying to this offer, and these firms will pay $c(\delta)$ to some unmodelled intermediaries with screening expertise. To see whether such a deviation may be profitable, let us consider the simple case in which, despite the more stringent screening technology, firms sufficiently compensate workers to make them indifferent to the equilibrium allocation of Section 3. This requires:

$$p(\tau; u)(E_{\tau}(w) - b - \beta U_{\tau+1}(u)) = p_{\delta}(\tau; u)(E_{\tau}(w_{\delta}) - b - \beta U_{\tau+1}(u))$$

Now, the asset value of the vacancy of such deviating firms becomes

$$V_{\delta}(w_{\delta}; u) = -k - c(\delta) + \sum_{\tau} \eta_{\tau}(q^{\tau}(w_{\delta}))(\bar{y}_{\delta}(\tau; u) - w_{\delta})\beta(\tau)$$

Obviously, for $\delta = 0$, that value is 0. We have to show that for positive and sufficiently small values of $\delta$, it is indeed strictly positive. Since the deviation is set for $q^{\tau}(w_{\delta})$ to coincide with the equilibrium queue length, the derivative of the asset value with respect to $\delta$ is

$$\frac{dV_{\delta}}{d\delta} = -c'(\delta) + \sum_{\tau} \eta_{\tau}(q^{\tau}(w_{\delta})) \frac{d(y_{\delta}(\tau; u) - w_{\delta})}{d\delta} \beta(\tau).$$

Given that $c'(0) = 0$, and $\frac{d(y_{\delta}(\tau; u) - w_{\delta})}{d\delta} \beta(\tau) = \frac{u_{l}(\tau)}{u_{h}(\tau) + (\lambda - \delta)u_{l}(\tau)}(\bar{y}_{\delta}(\tau; u)\beta(\tau) - w_{\delta}\beta(\tau) + \beta U_{\tau+1}(u))$, the derivative evaluated at $\delta = 0$ is strictly positive.

**Proof of Proposition ??**

The equilibrium does not need a fixed point problem to be characterized in the environment with skill attrition as the productivity and the search discouragement distributions are exogenous. Notwithstanding, given any pair of such distributions, the equilibrium allocation must be a fixed point of function $f$, defined in Section (3.3). We will show that the equilibrium allocation derived from the informational stigma scenario is also an equilibrium with skill attrition and exogenous search discouragement. Let $(u, U, U', q)$ be an equilibrium allocation in the benchmark scenario.

Let $u(\tau) \equiv u_{h}(\tau) + u_{l}(\tau)$ denote the unemployment measure conditional on duration. Given $(u, U')$, and the exogenous productivity and search effort distributions, the equilibrium allocation must also solve the (dual of the) firm’s problem, and the vacancies condition. The firm’s problem
does not present any difficulty. The vacancies condition must be written in terms of efficient units of search as it is defined from the expected queue lengths. Thus,

\[ q_\tau = q_1 \frac{s(\tau)u(\tau)}{s(1)u(1)} \]

It is easy to see that this expression coincides exactly with condition 13 provided that the search intensities are defined by \( p(\tau; u) \). Then, it only remains to show that the unemployment distribution by duration is equally determined as in function \( f \). Given the definition of \( u(\tau) \), the condition \( s(\tau) = p(\tau; u) \) suffices to show that the dynamics (by duration) of unemployment are obtained by adding the two Recursivity conditions 12 \( u(\tau + 1) = u(\tau)(1 - s(\tau)\nu_\tau(q^\tau)) \).

### 7.2.1 Appendix B. Social Planner Problem

The centralized economy is ruled by a social planner, who maximizes the discounted expected utility of the representative worker subject to a number of constraints. As usual in the literature, that is equivalent to maximizing output net of recruitment costs. Those restrictions are: the benevolent dictator faces the same incomplete information problem as agents do in the decentralized environment. Likewise, it has access to the same imperfect screening technology. However, the results from testing cannot be centralized. That is, the planner gathers information only from unemployment duration. Finally, the planner must assign vacancies to observables, i.e. \( \tau \), but not to individuals.

The benevolent dictator has four decisions to make. First, how many labor submarkets to create. Second, how many vacancies in each submarket. Third, it must tell workers to which labor market to apply. And, finally, the hiring strategy must be determined as a function of observables.

It is straightforward to give an answer to some of these questions. Within any submarket, if randomizing among candidates to fill a given vacancy, the matching technology is CRS, and the job-finding probability is concave in \( \frac{1}{q} \), where \( q \) stands for the ratio of job-seekers to vacancies. Thus, it is optimal to create a single labor market, and indicate all unemployed to send an application to that market although some workers may be discriminated against. That is an optimal decision provided that there are no crowding-out effects. Now, notice that low-skilled workers are as productive at home as at work. As a result, the planner will make passing the test a necessary qualifying condition. Finally, the planner does not have superior information than agents have in the decentralized economy with respect to candidates’ productivity. Unemployment duration is
informative to discriminate against those workers who are equally productive at home than in the market. Therefore, the planner will use unemployment duration to