Earnings Management, Leading Indicators, and Repeated Renegotiation in Dynamic Agency

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Abstract

Using a dynamic LEN model we show how renegotiation based on a non-contractible leading indicator of a forthcoming performance measure introduces demand for discretionary accruals and equilibrium earnings management. In a setting with capital investments, we show how non-discretionary and discretionary accruals combine to alleviate both commitment and induced moral hazard problems. In particular, we show that depreciation policies that match both current cash receipts and the recognition of future cash receipts solve the induced moral hazard problem. In contrast to existing literature, there are several rounds of active renegotiation both due to lack of commitment and non-contractibility.

1 Introduction

Accounting researchers usually view reported earnings as being composed of cash flows and accruals, where accruals are classified into non-discretionary and discretionary. Discussions of earnings quality typically link the accrual component of earnings to the usefulness of earnings in forecasting and valuing firms—the valuation role of earnings. Earnings and their accrual component are relevant for contracting as well, and here the valuation-driven ideas of earnings quality may be at odds with what would make earnings useful as a performance measure for contracting, see Christensen, Feltham, and Şabac (2005)(CFS). While CFS only examine the usefulness of non-discretionary accruals in contracting in a dynamic agency, we also examine the discretionary component of accruals—earnings management.

We consider a “true earnings” measure, composed of cash flows and non-discretionary accruals, which can be manipulated by the manager when he reports earnings. Thus, truthful reporting—no earnings management—consists of reporting the “true earnings” measure. Earnings management is defined as the extent to which the manager’s earnings report differs from true earnings, and it represents shifting of reported income between periods—any under- or over-reporting by the manager in the first period is reversed in the second period. In general, earnings management is not possible unless a full revelation mechanism (“the nemesis of earnings management research”) is blocked by restricting either the contract space, the message space, or possible commitments. Restricting the contract form is considered by Healy (1985) and Chaney and Lewis (1995); restrictions on the message space by Dye (1988), and Demski (1998); and restrictions on commitment by Arya, Glover, and Sunder (1998), Demski and Frimor (1999), and Christensen, Demski, and Frimor (2002).
We restrict contracts to be linear, we restrict communication to reported earnings, and we assume that long-term contracts can be renegotiated. Thus, all assumptions of the revelation principle are violated, and we view discretionary accruals as “window dressing” akin to Feltham and Xie (1994) and Dutta and Gigler (2002), or income shifting as in Liang (2004). The principal controls the cost of earnings management to the agent by controlling the tightness of auditing (compare Demski, Frimor, and Sappington 2004). Auditing tightness ranges from allowing unrestricted earnings management on the agent’s part to restricting it completely. The principal can costlessly preclude earnings management, but he may not find it optimal to do so.

Distinguishing features of our paper are that we allow multiple rounds of renegotiation—before and after the agent’s earnings report—and that we examine the impact of the precision of information at the first renegotiation encounter on earnings management. A leading indicator (such as an analyst earnings forecast) is jointly observable before the first renegotiation encounter, yet it is non-contractible and not directly under the agent’s control (by contrast, Dutta and Gigler 2002 introduce an earnings forecast by the agent). Furthermore, the agent’s earnings management, as well as the precision of the leading indicator are continuous variables that allow a more refined analysis of earnings management than the typically discrete models employed in the earnings management literature (for example, see the comments of Lambert 1999).1

Allowing for multiple renegotiation encounters enables us to consider both renegotiation prior to a contractible performance report as in Fudenberg and Tirole (1990) or Christensen, Demski, and Frimor (2002), and renegotiation following a contractible performance report, as in Fudenberg, Holmström, and Milgrom (1990), and CFS. Furthermore, in our analysis, renegotiation is triggered by the release of information: the non-contractible leading indicator triggers the first renegotiation, while the (manipulated) earnings report triggers the second renegotiation. The leading indicator of true earnings is jointly observable and, thus, the first renegotiation facilitates implicit contracting on this information as in Hermalin and Katz (1991).

When the leading indicator perfectly reveals the true performance measure, the second round of renegotiation becomes irrelevant, and the model becomes similar to CFS (even though the leading indicator is non-contractible). When the leading indicator is uninformative, the first round of renegotiation is similar to the Fudenberg and Tirole (1990) type of renegotiation. Thus, varying the informativeness of the leading indicator relative to the true performance measure allows us to bridge the gap between the two renegotiation settings typically considered in the accounting literature.

1Liang (2004) also has continuous income shifting driven by differences in incentive rates across periods, but in a setting with full commitment, where the differences in incentive rates are due to exogenously specified differences between the periods.
We describe conditions under which non-trivial earnings management is induced by the principal. A first necessary condition is that the information revealed by the leading indicator is imperfect, so that the subsequently reported performance measure is used for insurance purposes by the principal at the first renegotiation stage. Without earnings management, anticipating this insurance motive for the principal at the first renegotiation stage can create negative effort incentives in the first period if the performance reports in the two periods are positively correlated. The second necessary condition is that the performance measures in the two periods have sufficiently high correlation, so that the negative effort externality created by the principal’s desire to provide insurance at the first renegotiation stage is sufficiently severe. The third condition is that the signal-to-noise ratio of the leading indicator is small relative to the agent’s risk aversion. All these conditions ensure that the insurance motive creates high renegotiation costs. Allowing earnings management subsequent to the first renegotiation helps the principal commit not to be too aggressive in how he provides insurance—big differences in incentives rates create strong incentives to shift income between the two periods and, thus, high earnings management costs for the agent for which he must be compensated ex ante.

In cases where the agent manipulates the earnings report, he always underreports, and the amount of underreporting decreases in the precision of the leading indicator. At the same time, the principal’s expected utility is increasing in the precision of the leading indicator, reflecting the lower costs of earnings management and a lower risk premium for which the agent must be compensated. The precision of the leading indicator impacts the optimal tightness of auditing in a more subtle way: there exists a cutoff value—within the interval on which the signal-to-noise ratio of the leading indicator gives rise to non-trivial earnings management—above which the optimal tightness of auditing decreases in the precision of the leading indicator.

A firm’s reported earnings are not only affected by the agent’s discretionary earnings management; they are also affected by the accounting principles used, such as depreciation and revenue recognition policies, which lead to non-discretionary accruals. We extend the model to include non-discretionary accruals by considering a setting with a long-term investment decision in the first period. In addition to facilitating the analysis of the impact of depreciation and recognition policies, this setting gives rise to an induced moral hazard problem similar to that in Dutta and Reichelstein (2003). In this investment setting, we examine the choice of depreciation and revenue recognition policies simultaneously with earnings management by the agent. We show that allowing the principal to optimally choose the depreciation policy solves the investment problem for a broad range of parameters, with the result that the investment problem can be separated from the underlying incentive problem. The key is that the optimal depreciation policy does not only match cash
flows but cash flows plus recognized future benefits from the investment. Recognition of future benefits from the investment in the earnings number (based on a noisy measure of these benefits) introduces accrual estimation errors that are reversed in the subsequent period. In this setting, we find that costless earnings management is always optimal: with aggressive revenue recognition of the investment benefits for positive correlation and ultra-conservative revenue recognition for negative correlation between the performance measures. This result is analogous to the optimal amount of reversible accrual estimation errors in CFS, which brings the correlation of the performance measures to zero.

The paper is organized as follows: in Section 2, we present the basic model; in Section 3, we present the solution of the basic agency problem; in Section 4, we examine optimal audit policies, optimal earnings management, and the impact of the precision of the leading indicator on both; in Section 5, we introduce the induced moral hazard problem on investment; and in Sections 6 and 7, we examine revenue recognition policies. Section 8 concludes the paper, and the Appendix summarizes the details of proofs for the main results.

2 The Model

A risk-neutral principal owns a production technology that requires productive effort from an agent in two periods $t = 1, 2$. The agent is risk- and effort-averse with exponential utility and quadratic effort cost of the form

$$u(w, a, e) = -\exp[-r(w - \frac{1}{2}(a_1^2 + a_2^2 + \frac{1}{\alpha}e^2))],$$

where $w$ is the agent’s terminal wealth, $a = (a_1, a_2)$ is the agent’s productive effort at the start of periods 1 and 2, respectively, and $e$ is the agent’s discretionary earnings management activity. The agent’s earnings management has no direct benefit (or cost) to the principal, but it allows the agent to shift reported performance between the two periods, i.e., $e$ can be viewed as “window dressing” in the first period that must be reversed in the second (see Feltham and Xie 1994). Earnings management is achieved through manipulation of the accounting system, and its cost to the agent is determined by the parameter $\alpha \geq 0$. We assume this parameter is set by the principal and can be interpreted as the inverse intensity of the chosen audit. We allow for $\alpha = 0$ (as a limiting case), i.e., infinitely costly earnings management, which means that the agent is effectively precluded from any income shifting, i.e., $e = 0$. The other extreme, $\alpha = \infty$, corresponds to costless earnings management. We assume the principal does not incur any costs in setting the auditing tightness parameter $\alpha$; thus, we eliminate from consideration auditing costs or expected litigation costs when considering the principal’s decision to allow earnings management.
Assuming \( w \) is normally distributed, the agent’s certainty equivalent, CE, of terminal wealth \( w \) and effort \( a, e \) is

\[
CE(w, a_1, a_2, e) = E[w] - \frac{1}{2} \text{var}(w) - \frac{1}{2} (a_1^2 + a_2^2 + \frac{1}{\alpha} e^2) .
\]

(1)

The agent provides productive effort \( a_t \) in period \( t \) and the principal’s benefit is\(^2\)

\[
x_t = b_t a_t + \psi_t .
\]

(2)

where \( \psi_t \) is an arbitrary mean zero noise term which does not depend on \( a_t \). Both outcomes, \( x_1 \) and \( x_2 \), are not observed until after the termination of the contract at the end of period 2. Hence, the output \( x_t \) only determines the principal’s expected surplus, and since the principal is risk-neutral, no further distributional assumptions are needed regarding \( \psi_t \). The agent’s actions are unobservable. Hence, neither the output nor the agent’s actions are contractible.

The contractible information is given by two performance measures \( \hat{y}_t, t = 1, 2 \), reported by the agent. The agent reports \( \hat{y}_t \) after having observed the “true” performance measure \( y_t \):

\[
y_t = m_t a_t + \varepsilon_t .
\]

(3)

We assume that the performance measures (or equivalently, the noise terms) are joint normally distributed, i.e., \((\varepsilon_1, \varepsilon_2) \sim N([0, 0], \Sigma)\) with variance-covariance matrix

\[
\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} .
\]

The agent’s productive actions affect only the means of the performance measures, the noise terms have mean zero, and their variance-covariance matrix is not affected by the agent’s actions. We assume, without loss of generality, that the variances of the noise terms in the performance measures are scaled to one.

After the agent has provided effort \( a_1 \), a non-contractible “leading indicator” \( z \) is privately observed by both the principal and the agent,

\[
z = m a_1 + \delta .
\]

(4)

where \( \delta \sim N(0, \sigma^2) \) is potentially correlated with \( \varepsilon_t \) and \( \text{cov}(\delta, \varepsilon_t) = \rho \sigma \). The conditional expectations

\(^2\)We assume the principal has no incentive to terminate the firm for any signals he may receive. This can be insured by including a sufficiently large constant in the principal’s benefits, which will only be obtained whenever the firm operates.
and posterior variances conditional on $z$ and the agent’s first-period action $a_1$ are determined as follows:

$$E[y_t|z.] = E[y_t|.] + \frac{\text{cov}(y_t, z.)}{\text{var}(z)} (z - E[z.]) = E[y_t|.] + \frac{\rho_t}{\sigma} (z - ma_1) ,$$

(5)

$$\text{var}(y_t|z.) = \text{var}(y_t - \frac{\rho_t}{\sigma} z) = 1 - \rho_t^2 ,$$

(6)

$$\text{cov}(y_1, y_2|z.,.) = \text{cov}(y_1 - \frac{\rho_1}{\sigma} z, y_2 - \frac{\rho_2}{\sigma} z) = \rho - \rho_1 \rho_2 .$$

(7)

Note that, while the conditional expectations depend on the agent’s (first-period) actions (conjectured by the principal or known to the agent), the conditional variances do not depend on the agent’s actions.

Over the lifetime of a company, accounting income equals cash flow. Hence, the agent is limited to “shifting income” between periods—in the prevailing case between the two reported performance measures. As $e$ represents the agent’s income shifting, our assumptions imply that

$$\hat{y}_1 = y_1 - e$$

$$\hat{y}_1 + \hat{y}_2 = y_1 + y_2 .$$

(8)

That is, any under(over)-reporting by the agent in the first period is reversed in the second period, i.e., $\hat{y}_2 = y_2 + e$, and the agent must report the true aggregate of the two performance measures. Note also that the agent’s earnings management does not affect either the informativeness or the covariance of the performance measures; in our model, earnings management does not result in information garbling in the usual sense.

The principal owns the production technology for both periods and needs an agent to supply productive effort. There is more than one agent that the principal can employ in each period. All agents are identical (the agents have the same ability and the same utility functions) and have equal alternative employment opportunities. Each agent’s reservation certainty equivalent is normalized to zero. Both the principal and the agents are assumed to have discount rates of zero. Utility functions, discount rates, reservation wages, the nature of the production technology, and the information structure are common knowledge. We assume that once the initial contract is accepted by an agent, both the principal and the agent are committed to the full duration of the contract, i.e., the principal cannot replace the agent and the agent cannot quit.

At the start of the first period, the principal and the agent sign an initial contract $c_I$, and the agent provides effort $a_1$. After the agent has taken the first-period action and the leading indicator $z$ is observed,

$3$Neither $m_t = b_t$ nor $s_t = \psi_t$ is precluded. However, the adding up property could also be a consequence of the fact that most accounting manipulation reverses within a limited horizon.
but before the agent observes the first-period performance $y_1$, renegotiation may take place, resulting in a renegotiated contract $c_{R1}$. After re contracting, the agent observes $y_1$ and issues the report $\hat{y}_1$. At the start of the second period, after $\hat{y}_1$ is reported, a second round of renegotiation may take place, resulting in a renegotiated contract $c_{R2}$. In the second period, the agent provides effort $a_2$, observes the second performance measure $y_2$, and issues the report $\hat{y}_2$. At the end of the second period the contract is settled. After the end of the second period (i.e., after the contract is settled), the outcomes $x_1, x_2$ are revealed to the principal. The time line of events is summarized in Figure 1.

Figure 1: Time line of events: $I$ is the initial contracting date, $R_1, R_2$ are the two renegotiation dates.

Renegotiation consists of the principal making a “take-it-or-leave-it” offer to the agent, $c_j, j = R_1, R_2$, that replaces the existing contract, $c_i, i = I, R_1$, if accepted by the agent. The agent can reject the renegotiation offer, which means that the existing contract remains in effect.

We restrict the initial contract and subsequent renegotiation offers to be linear in the contractible performance measures:

$$c_j = f_j + v_{1j}\hat{y}_1 + v_{2j}\hat{y}_2 \quad j = I, R_1, R_2.$$  

Note, however, that the contract parameters may depend on jointly observable information at each renegotiation stage. The resulting contracts are thus the optimal linear contracts in each case (although linear contracts are not optimal).

Given linear contracts and normally distributed performance measures, the agent’s wealth is (conditional) normally distributed as well, which implies that the agent’s certainty equivalent of wealth and effort is given by equation (1). The agent’s wealth $w$ is the total compensation to be received by the agent, and the agent is indifferent as to the timing of consumption. Thus, the agent’s domain additive utility ensures that there are no inter-temporal consumption smoothing issues. In addition, the agent’s exponential utility
eliminates wealth effects, in that compensation paid (earned) does not impact the agent’s risk preferences.\footnote{The results are qualitatively no different if we assume time-additive agent utility with unrestricted borrowing and lending, and an infinite consumption horizon, see Dutta and Reichelstein (1999), Christensen et al. (2004), Šabac (2005). Since consumption smoothing by the agent is not an issue, we use the simpler model.}

## 3 Optimal Contracts with Renegotiation

In what follows, an equilibrium is characterized by a sequence of contracts \((c_I, c_{R1}, c_{R2})\), such that renegotiation offers are rationally anticipated by the agent and accepted. We solve for the optimal sequence of contracts backwards in time starting with the second round of renegotiation.

At the second renegotiation stage, \(R_2\), the only remaining uncertainty pertains to the second-period performance, \(y_2\) (and the non-contractible variables \(x_1\) and \(x_2\)). Admittedly, the principal can observe neither the first-period outcome \(x_1\), first-period effort \(a_1\), nor the accounting manipulation \(e\), which then—in principle—are random. However, as the principal foresees the agent’s equilibrium behavior, and as we only consider pure strategies, the principal acts as if the agent’s private information is observable at the second renegotiation stage. This implies that the principal knows the agent’s continuation certainty equivalent under the existing contract, \(c_{R1}\). Hence, the principal offers a contract, \(c_{R2}\), which will maximize the principal’s continuation expected utility subject to an interim participation and an interim incentive compatibility constraint. Since there are no wealth effects, it follows that the second-period incentive rate offered at \(R_2\) is (see Feltham and Xie 1994 and Christensen, Feltham, and Šabac 2003, 2005)

\[
v_{2R2} = \frac{m_2 b_2}{m_2^2 + r \text{var}(\hat{y}_2 | z, \hat{y}_1)}.
\]

That is, \(c_{R2} = f_{R2}(\hat{y}_1, z, \hat{a}_1, \hat{e}) + v_{2R2} \hat{y}_2\), where \(v_{2R2}\) is given by (10) and where \(f_{R2}(\hat{y}_1, z, \hat{a}_1, \hat{e})\) is set so as to provide the agent with the same continuation expected utility given acceptance of the new offer, \(c_{R2}\), as could be achieved under the existing contract, \(c_{R1}\), given the principal’s conjecture of the agent’s first-period actions, \(\hat{a}_1\) and \(\hat{e}\).

At the first renegotiation stage, we assume without loss of generality that the principal offers renegotiation-proof contracts. As Christensen, Feltham, and Šabac (2003, 2005) show, any contract offered at \(R_1\) that has \(v_{2R1} = v_{2R2}\), where \(v_{2R2}\) is the second-period incentive rate optimally chosen at \(R_2\), is renegotiation-proof at \(R_2\). Thus, in what follows, we assume \(c_R = c_{R1} = c_{R2}\) is a renegotiation-proof contract offered by the
principal at $R_1$ satisfying (10). Accordingly, we use the simplified notation

$$c_R = f_R + v_{1R} \tilde{y}_1 + v_{2R} \tilde{y}_2.$$ 

While we can assume without loss of generality in equilibrium that $c_R$ is renegotiation-proof at $R_2$, and while the principal could offer an ex-ante renegotiation-proof contract $c_I$, this may not be optimal for the principal because $z$ is not contractible. As we will show later, the first round of renegotiation allows implicitly incorporating $z$ in the agent’s incentive compensation through $c_R$, and renegotiation will take place at $R_1$ in equilibrium (compare Hermelin and Katz 1991).

3.1 The Optimal Renegotiation-proof Contract at R1

At the first renegotiation stage, the principal and the agent both know the non-contractible information $z$. The agent knows the first-period action $a_1$, whereas the principal holds a conjecture $\hat{a}_1$ regarding the first-period effort chosen by the agent. As previously mentioned, the principal foresees the agent’s equilibrium behavior and, in equilibrium, $\hat{a}_1 = a_1$. At $R_1$, the agent’s choice of first-period effort is sunk and, hence, the principal’s only concern is to maximize the continuation expected value. When considering the continuation expected value, given the agent accepts the renegotiation offer $c_R = f_R + v_{1R} \tilde{y}_1 + v_{2R} \tilde{y}_2$, there are two choice variables but only one tradeoff to be considered. As argued above, the principal is restricted in setting the second-period incentive rate $v_{2R}$, and only two choice variables remain, $f_R$ and $v_{1R}$.

Furthermore, as the first-period effort is sunk and as the second-period incentive rate (and, thus, the second-period effort) is constrained by renegotiation, the only role of the first-period incentive rate $v_{1R}$ is to reduce the riskiness of the continuation compensation $c_R$. That is, the remaining risk in the first-period performance measure given $z$ is used to insure the risk in the second-period measure. However, letting the first- and second-period incentive rates differ, $v_{1R} \neq v_{2R}$, implies the agent has an incentive to manipulate reported outcomes, i.e., to set $e \neq 0$, see also Demski and Frimor (1999). Thus, the principal faces a trade-off between risk reduction and manipulation. From a risk-sharing perspective at $R_1$, different incentive rates could well be beneficial. However, this benefit must be traded off against the added cost of compensating

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5If $v_{2R1} = v_{2R2}$ as determined by (10), then the incentive rate under $c_{R1}$ is identical to the incentive rate which will be offered at $R_2$. Hence, as the only remaining uncertainty pertains to $y_2$, the principal gains only if the agent accepts a lower continuation expected utility under $c_{R2}$ at $R_2$. Obviously, the agent rejects such offers and, hence, any contract $c_R$ accepted at $R_1$, for which $v_{2R1} = v_{2R2}$, is renegotiation-proof at $R_2$.

6Following Fudenberg and Tirole (1990), and disregarding manipulation concerns, it is clear that $v_{1R} = 0$, provided the first- and second-period outcomes are uncorrelated given $z$, i.e., $\text{cov}(y_1, y_2|z) = 0$. 

9
the agent for the induced manipulation.

The principal correctly conjectures the agent’s continuation certainty equivalent under the existing contract $c_I$ and offers a contract $c_R$ which will maximize the principal’s continuation expected utility subject to an interim participation constraint, an interim incentive compatibility constraint, and the second-period renegotiation-proofness constraint (10).

After the agent has taken action $a_1$ and the non-contractible information $z$ has been observed, the agent’s certainty equivalent at the first renegotiation date is, conditional on acceptance of $c_R$, and conditional on the rationally anticipated actions $e$ and $a_2$,

\[
\text{CE}[c_R|z, e, \tilde{a}_1, a_2] = f_R + v_1R \mathbb{E}[\hat{y}_1|z, e, \tilde{a}_1] + v_2R \mathbb{E}[\hat{y}_2|z, e, \tilde{a}_1, a_2] \\
- \frac{1}{2} r \text{var}(c_R|z) - \frac{1}{2} (a_2^2 + \frac{1}{\alpha} e^2)
\]

(11)

where $\tilde{a}_1 \in \{a_1, \hat{a}_1\}$ denotes the agent’s chosen or conjectured (by the principal) first-period effort. Let $a_2, e_R$ be the (optimal) actions induced by $c_R$. It then follows that the agent’s optimal choice of income shifting $e_R$ is given by setting the derivative of (11) with respect to $e_R$ equal to zero and solving for $e_R$,

\[
e_R = \alpha (v_2R - v_1R).
\]

Similarly, the second-period optimal action is determined by

\[
a_{2R} = m_2 v_{2R},
\]

(13)

where the second-period incentive rate $v_{2R}$ is set by the constraint (10) that the contract is renegotiation-proof.

As noted, the interim participation constraint ensures the agent is willing to accept the renegotiation (continuation) offer $c_R$ at $R1$. Hence, the principal must offer the agent at least the same expected utility under $c_R$ as under the existing contract $c_I$. Recognizing the choice of $v_1R$ and $v_{2R}$ affects the continuation certainty equivalent, the principal chooses $f_R$ to ensure acceptance of the renegotiation offer by the agent,

\[
\text{CE}[c_I|z, e_I, \tilde{a}_1, a_{2I}] \leq \text{CE}[c_R|z, e_R, \tilde{a}_1, a_{2R}].
\]

(14)
where $\hat{a}_1$ represents the principal’s conjecture regarding the agent’s unobservable first-period action, and $a_{21}, e_I$ represent the actions induced if the initial contract were to remain in effect. In equilibrium, the principal will set $f_R$ so that (14) is binding:

$$f_R = -v_1R E[\tilde{y}_1|z,e_R,\hat{a}_1] - v_2R E[\tilde{y}_2|z,e_R,\hat{a}_1,a_{21}] + \frac{1}{2} r \text{var}(c_R|z) + \frac{1}{2} a_{21}^2 R + \frac{1}{2 \alpha} e_R^2 + CE[e_I|z,e_I,\hat{a}_1,a_{21}].$$

Substituting in $c_R$ yields

$$c_R = v_1R (\tilde{y}_1 - E[\tilde{y}_1|z,e_R,\hat{a}_1]) + v_2R (\tilde{y}_2 - E[\tilde{y}_2|z,e_R,\hat{a}_1,a_{21}]) + \frac{1}{2} r \text{var}(c_R|z) + \frac{1}{2} a_{21}^2 R + \frac{1}{2 \alpha} e_R^2 + CE[e_I|z,e_I,\hat{a}_1,a_{21}].$$

As can be seen from (16),

$$E[c_R|z,e_R,\hat{a}_1,a_{21}] = \frac{1}{2} r \text{var}(c_R|z) + \frac{1}{2} a_{21}^2 R + \frac{1}{2 \alpha} e_R^2 + CE(e_I|z,e_I,\hat{a}_1,a_{21}).$$

Hence, when renegotiating at $R_1$, the principal faces a tradeoff between reducing the continuation risk premium $1/2 r \text{var}(c_R|z)$ and increasing the (payment for the) agent’s manipulation, $(1/2 \alpha)e_R^2$. More formally, the principal’s problem at renegotiation date $R_1$ reduces to choosing $v_1R$ to maximize

$$b_1 \hat{a}_1 + b_2 a_{21} - E[c_R|z,e_R,\hat{a}_1,a_{21}],$$

subject to the renegotiation-proofness constraint (10) that determines $v_2R$, the incentive compatibility constraints (13) and (12) that determine $a_{21}$ and $e_R$, and the interim individual rationality constraint (14) which we replace by (15). Taking expectations in (16) and substituting (12) reduces the principal’s problem to minimizing the following expression with respect to $v_1R$:

$$\frac{1}{2} r \text{var}(c_R|z) + \frac{1}{2 \alpha} e_R^2 = \frac{1}{2} r \left[ v_1^2 \text{var}(y_1|z) + 2 v_1R v_2R \text{cov}(y_1, y_2|z) + v_2^2 \text{var}(y_2|z) \right] + \frac{1}{2 \alpha} a_{21}^2 (v_2R - v_1R)^2.$$  

Differentiating (18) with respect to $v_1R$ gives the first-order condition for $v_1R$:

$$rv_1R \text{var}(y_1|z) + rv_2R \text{cov}(y_1, y_2|z) - \alpha (v_2R - v_1R) = 0.$$
Solving for \( v_1 \) gives \( v_1 = \frac{\alpha - r \text{cov}(y_1, y_2 | z)}{\alpha + r \text{var}(y_1 | z)} \). When \( \alpha = 0 \), accounting manipulation is infinitely costly to the agent and, hence, \( v_1 \) is determined purely by risk insurance considerations, i.e., 
\[ v_1 = -\frac{\text{cov}(y_1, y_2 | z)}{\text{var}(y_1 | z)} v_2. \]
At the other extreme, if \( \alpha = \infty \), then accounting manipulation is costless to the agent, and risk insurance is completely ignored by setting \( v_1 = v_2 \).

### 3.2 First-period Action Choice and Initial Contract

Note that the fixed wage of the contract negotiated at \( R_1, c_R \), as given by (15) has three terms which depend linearly on the non-contractible leading indicator \( z \):

\[
-v_1 R E[\hat{y}_1 | z, e_R, \hat{a}_1] = -v_1 R \left[ m_1 \hat{a}_1 + \frac{\rho_1}{\sigma} (z - \hat{m}_1) - e_R \right],
\]
\[
-v_2 R E[\hat{y}_2 | z, e_I, \hat{a}_1, a_2 R] = -v_2 R \left[ m_2 a_2 R + \frac{\rho_2}{\sigma} (z - \hat{m}_1) + e_R \right],
\]
\[
\text{CE}(c_I | z, e_I, \hat{a}_1, a_2 I) = f_I + v_1 I \left[ m_1 \hat{a}_1 + \frac{\rho_1}{\sigma} (z - \hat{m}_1) \right] + v_2 I \left[ m_2 a_2 I + \frac{\rho_2}{\sigma} (z - \hat{m}_1) \right] - \frac{1}{2} r \text{var}(c_I | z) - \frac{1}{2} \frac{\sigma^2}{\sigma^2} + (v_2 I - v_1 I) e_I - \frac{1}{2\alpha} e_I^2.
\]

The first two reflect the impact of \( z \) on the expected compensation with the continuation contract, while the third reflects the impact of \( z \) on the expected compensation for the initial contract, i.e., the three terms adjust for the change in expected compensation conditional on \( z \) when replacing the initial contract with a renegotiated contract. Hence, from the perspective of \( t = 0 \), and rationally anticipating the renegotiation-proof contract at \( R_1 \), the effective contract can be viewed as a linear contract of the form

\[
c = \text{Const.} + v_2^e z + v_1 R \hat{y}_1 + v_2 R \hat{y}_2,
\]
where the constant and the “effective incentive rate”, \( v_2^e \), for the leading indicator depend on the initial and renegotiation-proof contract parameters, as well as the principal’s conjecture regarding the agent’s first-period effort \( \hat{a}_1 \), but not on the agent’s actual first-period action \( a_1 \). Collecting terms, the effective incentive
rate for \( z \) is

\[
v_z^e = (v_1 I - v_1 R) \frac{\rho_1}{\sigma} + (v_2 I - v_2 R) \frac{\rho_2}{\sigma}.
\]  

(21)

Applying similar derivations as at \( t = 1 \), the induced first-period action is

\[
a_1 = v_1 R m_1 + v_z^e m.
\]

That is, the agent’s first-period action is influenced by both the explicit first-period incentive, \( v_1 R m_1 \), and the implicit incentive, \( v_z^e m \), created by the renegotiation based on the leading indicator. The latter is determined by the difference between the initial incentive rates and the renegotiated incentive rates. Note that, if the principal sets the initial incentive rates equal to the rationally anticipated renegotiation-proof incentive rates at \( R_1 \), then there is no implicit incentive (and no renegotiation based on \( z \)). But is that optimal? The answer is NO! The key is that renegotiation allows contracting on \( z \) in this case.

To see why, consider as a benchmark case a setting in which the leading indicator is directly contractible with incentive rate \( v_z \). Note that the renegotiation-proof incentive rates \( v_1 R \) and \( v_2 R \) as given by (20) and (10), respectively, remain unchanged. Hence, the principal’s decision problem at the initial contracting stage reduces to maximizing the following expression with respect to \( v_z \):

\[
U^p \sim b_1 a_1 - \frac{1}{2} a_1^2 - \frac{1}{2} r \text{var}(v_z z + v_1 R y_1 + v_2 R y_2)
= b_1 [v_z m + v_1 R m_1] - \frac{1}{2} [v_z m + v_1 R m_1]^2 - \frac{1}{2} r \text{var}(v_z z + v_1 R y_1 + v_2 R y_2),
\]

(22)

where the variance of the contract is

\[
\text{var}(v_z z + v_1 R y_1 + v_2 R y_2) = v_z^2 \sigma^2 + 2v_z [v_1 R \rho_1 + v_2 R \rho_2] \sigma + v_1^2 R + v_2^2 R + 2v_1 R v_2 R \rho.
\]

(23)

The first-order condition for (22) with respect to \( v_z \) is

\[
b_1 m - v_z m^2 - v_1 R m_1 m - r \left[ v_z \sigma^2 + (v_1 R \rho_1 + v_2 R \rho_2) \sigma \right] = 0.
\]

Solving for \( v_z \) yields

\[
v_z = \frac{b_1 m}{m^2 + r \sigma^2} - \frac{v_1 R m_1 m + [v_1 R \rho_1 + v_2 R \rho_2] r \sigma}{m^2 + r \sigma^2}.
\]

(24)

Hence, in general, it is optimal to use a non-zero incentive rate on the leading indicator due to \( z \) being informative about both the agent’s first-period action, i.e., \( m \neq 0 \), and the forthcoming performance measures,
i.e., $\rho_t \neq 0$. In the setting with a non-contractible leading indicator, this can only be achieved (as noted above) if the initial incentive rates are chosen to be different from the renegotiation-proof incentive rates at $R_1$. The same induced first-period action and risk premium are achieved if the principal chooses the initial incentive rates such that the effective incentive rate for the leading indicator $v_z^e$ is equal to $v_z$, i.e., equating (21) and (24) yields

$$(v_1 - v_1^R) \frac{\rho_1}{\sigma} + (v_2 - v_2^R) \frac{\rho_2}{\sigma} = \frac{b_1 m}{m^2 + r \sigma^2} - \frac{v_1^R m_1 m + [v_1^R \rho_1 + v_2^R \rho_2] r \sigma}{m^2 + r \sigma^2}.$$

Solving for the initial incentive rates yields

$$\frac{\rho_1 v_1 + \rho_2 v_2}{\sigma} = \frac{b_1 m}{m^2 + r \sigma^2} - \frac{m [v_1^R m_1 - m \sigma^{-1} (\rho_1 v_1^R + \rho_2 v_2^R)]}{m^2 + r \sigma^2}.$$ (25)

Note that there is an indeterminacy of the initial incentive rates. This reflects the fact that the initial incentive rates must determine a unique sensitivity of the conditional expected compensation for the initial contract with respect to $z$, but that sensitivity can be achieved with a linear combination of the two initial incentive rates. In order to reflect this indeterminacy and to simplify notation, define $w_I = (\rho_1 v_1 + \rho_2 v_2) / \sigma$ and $w_R = (\rho_1 v_1 + \rho_2 v_2) / \sigma$ such that the effective incentive rate for the leading indicator is $v_z^e = w_I - w_R$. Substituting $w_I$ and $w_R$ into (25) yields

$$w_I = \frac{m b_1}{m^2 + r \sigma^2} - \frac{m (m_1 v_1^R - m w_R)}{m^2 + r \sigma^2}.$$ (26)

To conclude, note that the optimal induced first-period action is

$$a_1 = m_1 v_1^R + m (w_I - w_R),$$ (27)

and that the risk premium is determined by the contract variance which by substituting $v_z = v_z^e = w_I - w_R$ into (23) and simplifying is given by

$$\text{var}(c_R) = w_I^2 \sigma^2 + v_1^2 (1 - \rho_1^2) + 2 v_1 v_2^R (\rho - \rho_1 \rho_2) + v_2^2 (1 - \rho_2^2).$$ (28)

Before proceeding, it is worth noting that $w_I$ is not the only contracting parameter available to the principal. The principal can also determine the auditing technology, $\alpha$, and as can be seen from (20), the auditing technology, i.e., the parameter $\alpha$, influences the equilibrium choice of $v_1^R$. If, for example, $\alpha \to \infty$, it is infinitely costly to set $v_1^R \neq v_2^R$ and, hence, $\alpha \to \infty$ implies $v_1^R = v_2^R$. Provided periods are identical
as well as independent and provided \( \rho_2 = 0 \), it is an optimal policy to set \( \alpha^{-1} = w_I = 0 \). Doing so ensures \( v_{1R} = v_{2R} = v^* \), where \( v^* \) is equal to the optimal incentive rate(s) under full commitment.

# 4 Leading Indicator

In this section, we assume the non-contractible early information \( z \) is a noisy forecast of the first-period performance measure (such as an analyst earnings forecast), i.e.,

\[
z = y_1 + \varepsilon,
\]

where the noise term \( \varepsilon \) is independent of the performance measures and has variance \( \text{var}(\varepsilon) = \lambda^2 \). Consequently, the results in the previous section apply with \( m = m_1, \delta = \varepsilon + \varepsilon_1, \) and

\[
\sigma = \sqrt{1 + \lambda^2}, \quad \rho_1 = \frac{1}{\sqrt{1 + \lambda^2}}, \quad \rho_2 = \frac{\rho}{\sqrt{1 + \lambda^2}}.
\]

We assume the incremental noisiness \( \lambda \) of the forecast variable \( z \) is fixed and that the principal controls the parameter \( \alpha \) through the internal audit system. For \( \alpha = 0 \), income shifting is entirely precluded, being infinitely costly. For \( \alpha = \infty \), costless income shifting is allowed, which makes the principal offer \( v_{1j} = v_{2j}, j = I, R \), in order to prevent the agent from engaging in an “infinite transfer” between periods (if offered an initial contract with different incentive rates, the agent’s certainty equivalent is unbounded and any renegotiation offer is rejected).

Below, we consider two extreme cases for the informativeness of the leading indicator that highlight the basic tradeoff to be made in the principal’s optimal choice of audit policy, \( \alpha \).

## 4.1 Perfect Early Information

Assume the leading indicator perfectly reveals the “true” first-period performance, i.e., \( z = y_1 \). In this case, there is no remaining uncertainty pertaining to \( y_1 \) at the first renegotiation stage, \( R1 \). Hence, the first-period performance cannot be used to insure against risk in second-period performance. This implies the principal’s only interest at \( R1 \) is to avoid paying for the agent’s manipulation, which translates into setting \( v_{1R} = v_{2R} \). Moreover, the incentive rate on first-period performance at \( R1 \) has no impact on the agent’s \textit{ex ante} choice of first-period effort (set \( m = m_1, \sigma = 1, \) and \( \rho_1 = 1 \) in (27) and note that \( w_R = v_{1R} + \rho v_{2R} \), so \( a_1 = m_1(w_I - \rho v_{2R}) \))—an increase in this rate is countered by an equivalent reduction in the fixed wage offered at \( R1 \). Hence, in the principal’s \textit{ex ante} choice of contract only the initial incentive rate on
first-period performance $y_1$ matters, i.e., it is costless for the principal to set $v_1R = v_2R$ and, thus, the same solution with no income shifting is obtained for any audit policy. For example, an optimal policy is $\alpha = 0$, which is equivalent to the model in CFS in which there is no earnings management and there is a single round of renegotiation after $y_1$ is reported. It follows that a necessary condition for earnings management to have value is that $y_1$ is random at $R1$ so that first-period performance is useful in insuring second-period performance.\footnote{Note that in CFS, it is never valuable to allow costly earnings management by the agent, provided the principal can preclude earnings management through a costless audit technology.}

\subsection*{4.2 No Early Information}

When $\text{var}(z) \rightarrow \infty$, $z$ is uninformative regarding $y_t$. Accordingly, we have $\rho_1 = \rho_2 = 0$ and $\sigma = \infty$. Assume for the moment that the agent cannot manipulate income. Then the principal’s only concern at $R1$ is to induce second-period effort at the lowest possible risk premium and he can provide full insurance with respect to $y_1$ implying zero induced first-period effort, which is the argument in Fudenberg and Tirole (1990). This is optimal if the two performance measures are uncorrelated. However, if the two performance measures are correlated, then $y_1$ is insurance-informative about the noise in $y_2$, and the optimal incentive rate on $y_1$ is, see (20),

$$v_1R = -\rho v_2R.$$ 

Note that, if $\rho < 0$, the optimal insurance of the noise in $y_2$ has the additional benefit (relative to the zero correlation case) of a positive first-period incentive rate and, thus, positive first-period effort. However, if $\rho > 0$, the problem encountered in Fudenberg and Tirole (1990) is exacerbated because the negative incentive rate on first-period performance implies a negative first-period action.

In a setting similar to the zero correlation case, Christensen, Demski, and Frimor (2002) show how the choice of discretionary accounting policies can mitigate the problem of inducing zero first-period effort—essentially by allowing the agent to build secret reserves. This forces the principal to choose a positive incentive rate on the first-period performance report.

In our setting, the principal chooses the optimal audit policy between two extremes to adjust the first-period incentive rate. At one extreme, $\alpha = 0$, earnings management is precluded and $v_1R$ is only determined by insurance considerations, as described above. At the other extreme, $\alpha = \infty$, allowing free earnings management commits the principal to using equal incentive rates in the two periods. Between these two extremes, (20) implies that the principal can use the tightness of the auditing system to affect his incentives
to (re)set $v_1^R$, 

$$v_1^R = \frac{\alpha - rp}{\alpha + r} v_2^R.$$  \hspace{1cm} (31)

The incentive rate $v_1^R$ increases from $v_1^R = -\rho v_2^R$ for $\alpha = 0$ to $v_1^R = v_2^R$ for $\alpha = \infty$. Since the first-period productive effort is uniquely determined by $a_1 = m_1 v_1^R$ when $z$ is uninformative, it also increases in $\alpha$. It follows that the principal’s expected benefit net of effort cost increases in $\alpha$ (as first-period effort increases towards first-best).

Any initial contract in which $v_{tI} \neq v_{tR}$ is known to be renegotiated and, hence, we can restrict ourselves to renegotiation-proof contracts in which $v_{tI} = v_t^R$ and in which $v_t^R$, $t = 1, 2$, are given by (31) and (10).\footnote{This is only true when $z$ is uninformative, so that the initial contract cannot contribute to the incentives for $a_1$ through implicit contracting on $z$.} If $\alpha = \infty$ is set so that earnings management is costless to the agent, it follows from (31) that $v_1^R = v_2^R$. If periods are identical, i.e., $b_1 = b_2 = b$, $m_1 = m_2$, and there is positive correlation, we know from CFS that the second-period incentive rate $v_2^R$ is set too high (relative to the full commitment solution) from an ex ante perspective (since it is based exclusively on the posterior variance given the first-period performance report). In CFS, this leads the principal to reduce the first-period incentive rate because of the impact of the positive correlation on the ex ante risk premium. However, with costless earnings management, the first and second-period incentive rates must be equal, which further increases the ex ante risk premium.

Costly earnings management, on the other hand, implies that $v_1^R < v_2^R$. This can be preferred by the principal, since it reduces the ex ante risk premium due to having too strong incentives in both periods. However, it also induces earnings management and thus increases the agent’s earnings management cost for which he must be compensated. In aggregate, the risk premium plus the cost of earnings management is increasing in $\alpha$.

The tradeoff that determines the optimal audit policy $\alpha$ is between the higher net benefit of higher first-period productive effort on one hand, and higher risk premium plus earnings management cost on the other. Costly earnings management is optimal if the impact on the risk premium is sufficiently strong compared to the impact on the earnings management cost, i.e., the correlation is sufficiently high. The characterization of the optimal auditing policy $\alpha^*$ is given in the following proposition.

**Proposition 1** Assume identical periods: $b_1 = b_2 = b$, $m_1 = m_2 = m$, and no early information: $\lambda \to \infty$.

1. If $\rho \leq 1/2$, then $\alpha^* = \infty$ is optimal.
2. If \( \rho > 1/2 \) and the following condition is satisfied,

\[
\rho - \frac{1}{2} < \frac{m^2}{r},
\]

then there is an interior \( \alpha^* \) determined by

\[
\alpha^* = \frac{m^2}{\rho - 1/2 - r}.
\]

3. In all other cases in which \( \rho > 1/2 \), \( \alpha^* = 0 \) is optimal.

First, note that at the extremes, whenever the principal sets \( \alpha = \infty \) or 0, there is no earnings management in equilibrium: in the first case, because incentive rates are equal and the agent has no benefit from earnings management; in the second case, because it is infinitely costly to the agent.

In case 1, if the correlation is low enough (\( \rho \leq 1/2 \)), by committing to equal incentive rates (\( \alpha^* = \infty \)), the principal can induce productive effort in the first period and the risk premium is not too high. In this case, prohibiting earnings management (\( \alpha = 0 \)) and relying on the principal’s insurance motive in renegotiation (\( v_1 R = -\rho v_2 R \)) with the associated effort externality to provide incentives is not efficient: with negative correlation because positive effort is induced, but not enough since the ex ante risk premium is low; with positive and small correlation because negative effort is induced and the risk premium is low. Allowing some earnings management (\( 0 < \alpha < \infty \)) commits the principal to a higher first-period incentive rate relative to relying on the insurance motive at renegotiation, but this is always less than the second-period incentive rate, and is suboptimal since the risk premium is relatively low with equal incentive rates.

In case 3, if the correlation is high enough (\( \rho > 1/2 \)), the ex-ante risk premium is high enough to be the dominant concern. If the sensitivity of the performance measures is small enough, the negative effort externality resulting from the principal’s insurance motive at renegotiation time, \( a_1 = m v_1 R = -m \rho v_2 R \), is not too high, so prohibiting costly earnings management is optimal (\( \alpha^* = 0 \)).

In case 2, both the ex ante risk premium, and the negative effort externality resulting from the principal’s insurance motive at renegotiation time are high enough, so that the principal commits to a higher incentive rate in the first period by allowing the agent to shift income. If the sensitivity of the performance measures is high enough, the effort externality becomes large enough so that the cost of earnings management is set sufficiently low and positive productive effort is induced in the first period.

The optimal audit policy is increasing in \( m \) and decreasing in \( \rho \). As \( m \) increases, the externality of having the net benefit of first-period effort determined only by insurance considerations at \( R1 \) increases;
it is then optimal to increase $v_1R$ by increasing $\alpha$ in order to increase first-period productive effort. As $\rho$
increases, the (covariance) risk premium cost of increasing $v_1R$ increases, so it is optimal to reduce $\alpha$. Note
also that the optimal interior $\alpha$ does not depend on the principal’s marginal benefit of agent effort, $b$. This is
only true for the identical periods case, in particular $b_1 = b_2 = b$ suffices for $\alpha$ to be independent of $b$.

4.3 Imperfect Early Information

The analysis in the preceding subsection demonstrates that the optimal auditing policy is determined by
the tradeoff between the earnings management cost plus the ex ante risk premium and the net benefit of
first-period effort. With no early information, first-period effort is only determined by $v_1R$ and the prin-
cipal’s insurance motive at $R1$, possibly mitigated by earnings management whenever $\alpha \in (0, \infty)$. With
imperfect early information, the induced first-period effort depends on both the initial contract parameters
and on the incentive rates in the final renegotiation-proof contract negotiated at the first renegotiation stage
(see (27)). Effectively, the renegotiated contract $c_R$ allows the principal to use $z$ as an (implicit) contract
variable. Hence, the tradeoffs in determining the optimal auditing policy are more complex although the
basic intuition remains. The leading indicator, being a noisy forecast of $y_1$, would never be used under full
commitment, even if it were contractible. However, without full commitment the leading indicator $z$ may
be a valuable contract variable due to its role in motivating productive first-period effort. The performance
measure $y_1$ is used by the principal purely for its insurance role, while its impact on $a_1$ is an externality.

The following proposition characterizes the optimal auditing policy and comparative statics are provided
in the following subsection.

**Proposition 2** Assume identical periods: $b_1 = b_2 = b, m_1 = m_2 = m$.

1. If $\rho \leq 1/2$, then $\alpha^* = \infty$ is optimal.

2. If $\rho > 1/2$ and the following condition is satisfied,

$$\frac{1 + \lambda^2}{1 + 2\lambda^2} (2\rho - 1) < \frac{m^2}{r} < (1 + \lambda^2)(2\rho - 1),$$

then there is an interior $\alpha^*$ determined by

$$\alpha^* = -\frac{r\lambda^2(m^2(1 + 2\lambda^2) - r(1 + \lambda^2)(2\rho - 1))}{(1 + \lambda^2)(m^2 - r(1 + \lambda^2)(2\rho - 1))}.$$  \hspace{1cm} (35)

3. In all other cases in which $\rho > 1/2$, $\alpha^* = 0$ is optimal.
As in the previous section, if the correlation \( \rho \) is small enough, the risk premium is not too high, even for equal incentive rates, so it is optimal to motivate first-period effort without costly earnings management by committing to equal incentive rates, \( \alpha = \infty \).

Consider the case with high correlation, \( \rho > 1/2 \), where the risk premium is also high. If the sensitivity of the performance measures is small, the externality of having first-period effort driven by insurance considerations is small; hence, it is not efficient to incur earnings management costs, and \( \alpha^* = 0 \). As the sensitivity of the performance measures increases, while the sensitivity of \( z \) to \( a_1 \) remains small, it is better to provide incentives for \( a_1 \) through \( y_1 \); hence, committing to putting incentives on \( y_1 \) by allowing some earnings management is efficient, and \( \alpha^* \) is interior. Finally, when the sensitivity of \( z \) becomes high enough, first-period effort is better induced by controlling the incentive weight on \( z \), and inducing earnings management is again inefficient, and \( \alpha^* = 0 \).

### 4.4 Comparative statics

We have established the optimal auditing policy \( \alpha^* \) chosen by the principal given the precision of the leading indicator concerning the forthcoming first-period performance, i.e., \( 1/\lambda^2 \). We now examine how this precision affects the optimal audit policy and the principal’s utility given the optimal auditing policy.

**Proposition 3** Assume identical periods. The principal’s expected utility, given the optimal audit policy \( \alpha^* \) as determined in Proposition 2, is increasing in the precision \( 1/\lambda^2 \) of the leading indicator. The principal’s expected utility given the optimal audit policy is decreasing in the correlation \( \rho \) of the performance measures.

Thus, the more precise \( z \) is relative to \( y_1 \), the weaker is the negative effect of the first round of renegotiation, the extreme case being \( \lambda = 0 \), which makes the problem equivalent to having a single round of renegotiation (see Subsection 4.1). Increasing the precision of \( z \) reduces the negative effects of the first round of renegotiation by reducing the degree of uncertainty regarding \( y_1 \) at the first round of renegotiation, thereby reducing the usefulness of setting \( v_{1R} \) for insurance purposes at \( R1 \).

At the other extreme, if \( z \) is uninformative about \( y_1 \), then \( v_{1R} \) is determined strictly by insurance considerations in the absence of earnings management possibilities (see Subsection 4.2). Alternatively, we can think in terms of the principal contracting on \( z \) to motivate first-period effort. As \( z \) becomes a more precise indicator of \( y_1 \), its sensitivity relative to \( a_1 \) increases; hence, the principal can use the (implicit) incentive weight on \( z \) as a less costly alternative to earnings management in order to induce first-period effort.
The productive effort induced in the second period does not depend on either $\alpha$ or $\lambda$, since the agent chooses his action after having chosen the income shifting level, and after $\hat{y}_1$ is reported. If the optimal audit policy is $\alpha^* = 0$, the first-period effort increases in the precision $1/\lambda^2$ of the leading indicator because $z$ is more sensitive and the usefulness of $y_1$ for insurance at $R1$ is lower. If the optimal audit policy is $\alpha^* = \infty$, the first-period effort decreases with the precision $1/\lambda^2$ of the leading indicator $z$ for positive correlation $\rho$, and increases for negative correlation $\rho$. In this case, since the principal is committed to $v_1^R = v_2^R$, with positive (negative) correlation, $z$ can be used to reduce the aggregate risk premium, and this reduction results in a relatively higher reduction (increase) in first-period effort the more sensitive $z$ is.

Interestingly, if the optimal auditing policy $\alpha^*$ is interior, the optimal productive effort induced in the first period—given the optimal audit policy—does not depend on $\lambda$, while the earnings management depends on $\lambda$ through the difference in the incentive rates and the cost of earnings management. Thus, the impact of $\lambda$ on the principal’s expected utility is exclusively through the cost of earnings management and the risk premium.

In general, the principal’s expected utility decreases in the correlation of the performance measures. This is a result of a direct increase in the risk premium and the impact of correlation on earnings management, which in general is ambiguous. The risk premium effect dominates, so the principal’s utility decreases in the correlation as in the basic model of CFS.

The following proposition examines the impact of the precision of the leading indicator variable and the correlation of the performance measures on the optimal audit policy and on the induced earnings management.

**Proposition 4** Assume identical periods and the optimal auditing policy determined in Proposition 2.

**A. Optimally induced earnings management:** $e^* = \alpha(v_2^R - v_1^R)$. If the optimal auditing policy is $\alpha^* = 0$ or $\alpha^* = \infty$, the principal induces no earnings management, i.e., $e^* = 0$. If the optimal auditing policy is interior, i.e., $\alpha^* \in (0, \infty)$, the principal induces under-reporting in the first period, i.e., $e^* > 0$, and the amount of under-reporting decreases with the precision of the leading indicator. The optimal amount of under-reporting can either increase or decrease in the correlation $\rho$.

**B. Optimal audit policy:** interior $\alpha^* \in (0, \infty)$. If the optimal audit policy is interior, then the optimal audit policy $\alpha^*$ increases in the precision of the leading indicator if, and only if, the following condition is satisfied (which implies (34)),

$$\left(\frac{1 + \lambda^2}{1 + (2 - \sqrt{2})\lambda^2}\right)(2\rho - 1) < \frac{m^2}{r} < (1 + \lambda^2)(2\rho - 1).$$

(36)
The optimal audit policy $\alpha^*$ is decreasing in the correlation $\rho$.

The first-period incentive increases in $\alpha$ and $v_{1R} < v_{2R}$ (for insurance purposes) for $\alpha < \infty$, so that the principal always induces under-reporting, and the absolute amount of under-reporting increases in $\alpha$.\(^9\) The amount of under-reporting decreases in the precision of the leading indicator because the residual risk in $y_1$ conditional on $z$—which the principal uses to insure the risk in $y_2$—decreases, and at the same time the sensitivity of $z$ to $a_1$ increases; both effects reduce the demand for costly earnings management as a way to motivate first-period productive effort.

The optimal interior audit policy is generally increasing in the precision of the leading indicator, except for a small interval in which the sensitivity $m$ is small enough. This is primarily due to the effect of increasing precision on the posterior risk at $R_1$: as $1/\lambda^2$ increases, both the posterior variances (conditional on $z$) and the posterior correlation of the performance measures decrease. This allows the principal to set a higher $v_{1R}$ by increasing $\alpha$ at a lower cost in terms of risk premium.

The optimal interior audit policy is decreasing in the correlation $\rho$ due to the increase in the risk premium associated with a higher $\alpha$ for high positive correlation (recall that $\rho > 1/2$ is necessary for $\alpha^*$ to be interior).

5 Induced Moral Hazard with Revenue Recognition and Depreciation

The preceding analysis examines the impact of earnings management and the associated optimal audit policy. However, the timing of a firm’s reported earnings are not only affected by the agent’s earnings management through, for example, discretionary accruals. It is also affected by accounting principles (chosen by the principal) such as depreciation and recognition policies. We extend the model to include these characteristics of the accounting system by considering a setting with a long-term investment decision in the first period. In addition to facilitating the analysis of the impact of depreciation and recognition policies, this setting gives rise to an induced moral hazard problem similar to that in Dutta and Reichelstein (2003). We examine the optimal choice of depreciation and revenue recognition policies simultaneously with earnings management by the agent in the presence of multiple rounds of renegotiation.

The basic model from the preceding sections is modified as follows. In addition to the operating decisions in each period, $a_t$, the manager decides at the beginning of the first period on a long-term action (investment) $i$. It has no direct personal cost to the agent, but it has a direct cost to the principal of $i$. The

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\(^9\)The under-reporting converges to a finite limit at $\alpha = \infty$, so that there is a “discontinuity” in earnings management at $\alpha = \infty$. At the same time, the cost of earnings management is well behaved with respect to $\alpha$, having a limit of zero at infinity.
expected benefit to the principal from the investment is reflected in the concave increasing function \( b(i) \) with \( b(0) = 0 \). The expected (cash) benefit is realized over the two periods, with a fraction \( \gamma b(i) \) being realized in the first period and the fraction \((1 - \gamma)b(i)\) in the second period. The randomness in the benefits is included in the noise terms of the performance measures, \( \varepsilon_1 \) and \( \varepsilon_2 \). There exists a noisy measure of the expected benefit (revenue) from investment,

\[
q = b(i) + \eta, \quad (37)
\]

where \( \eta \) is generally correlated with \( \varepsilon_t \) and has variance \( \text{var}(\eta) = \theta^2 \). The noisy investment revenue measure is aggregated by the accounting system in earnings and is neither directly observable nor contractible. The accounting revenue recognition policy is such that the fraction \( kq \) of the benefit measure is recognized in the first period in addition to the realized first-period cash flow. This accrual is reversed in the second period, so that the total cash benefit is recognized over the two periods.\(^\text{10}\)

The initial investment \( i \) is capitalized and depreciated over the two periods, with \( di \) being depreciated in the first period and \((1 - d)i \) in the second period. The two performance measures (i.e., earnings numbers) available for contracting are then

\[
\begin{align*}
y_1 &= m_1a_1 + \gamma b(i) - di + kq + \varepsilon_1 \\
y_2 &= m_2a_2 + (1 - \gamma)b(i) - (1 - d)i - kq + \varepsilon_2.
\end{align*} \quad (38)
\]

The noisy forecast of the first-period performance measure is available as in the preceding sections,

\[
z = y_1 + \delta, \quad (39)
\]

where \( \text{var}(\delta) = \lambda \), \( \delta \) may be correlated with \( \varepsilon_t \), and \( \delta \) is independent of \( \eta \).

Note that the timing of the expected cash benefit from investment over the two periods is \((\gamma, 1 - \gamma)b(i)\), while the timing of accruals of the benefit from investment is \((k, -k)b(i)\). For \( k = 0 \), we have ultra-conservative accounting, or cash accounting. For \( k = 1 - \gamma \), we have fair value accounting. The revenue recognition parameter \( k \) is to be chosen \textit{ex ante} by the principal.

\(^{10}\)Note that benefit measure is a variable correlated with the actual cash benefits, but does not have the form of a "forecast of the cash benefits" such as cash benefits plus noise. This specification facilitates our analysis in the following section of a setting in which we assume that \( kq \) is a pure accrual estimation error (independent of cash flows). Of course, it would also be interesting to examine a setting in which the revenue recognition accrual is a fraction of forecasted future cash flows. However, in that setting, the revenue recognition affects the allocation of noise across periods. We leave this for future research.
This setting is similar to the induced moral hazard setting of Dutta and Reichelstein (2003), with the important difference that we introduce an accrual that aggregates the additional measure \( q \) into the earnings numbers, \( y_1 \) and \( y_2 \), and we allow the principal to choose the depreciation rate \( d \). Allowing the additional degree of freedom of choosing the depreciation parameter resolves the induced moral hazard problem as shown in the following proposition.

**Proposition 5** *For any linear contract in effect at the time the agent selects the investment level*

\[
c = f + v_1 \hat{y}_1 + v_2 \hat{y}_2
\]

*and for any given parameters \( \gamma \) and \( k \) such that \( \gamma + k \leq 1 \), first-best investment is achieved with \( d = \gamma + k \).*

The proof is straightforward, since the impact of the investment on the agent’s total compensation only depends on \( (v_1 + v_2)(\gamma + k)(b(i) - i) \). Thus, first-best investment is achieved regardless of the linear contract offered for any benefit function \( b(i) \). As a result, if \( \gamma + k \leq 1 \), an appropriately chosen depreciation policy gives first-best investment and allows us to eliminate the induced moral hazard problem from consideration. However, note that the depreciation policy is such that depreciation is not only matched with cash receipts, but with cash receipts plus recognized expected future benefits; in other words, the optimal depreciation matches the timing of accrual accounting measures rather than that of cash flow.\(^{11}\)

### 6 Accrual Estimation Errors

In the previous section, we considered \( \alpha \) and \( k \) as exogenously given. We now consider the simultaneous endogenous determination of \( k \) and \( \alpha \). The principal has two dials to turn: the conservatism parameter of the revenue recognition policy \( k \), and the reporting discretion parameter on the cost to the agent of earnings management \( \alpha \). From Subsection 4, we know conditions under which there is demand for earnings management by the agent, and CFS give conditions under which there is demand for stochastic non-discretionary reversible accruals. The latter are introduced in the current model through the recognition of the noisy

\(^{11}\)We have assumed zero interest rates. This is without loss of generality. For positive interest rates, the solution to the congruity problem with respect to investment incentives is similar, provided residual income (instead of income) is used as the performance measure in each period. In this case, the optimal \( d \) is given by \( d + R = (k + \gamma)/(\beta^2(1 - \gamma) + \beta \gamma) \), where \( \beta = (1 + R)^{-1} \) is the discount factor. Note that the restriction on the revenue recognition parameter becomes \( k \leq \beta(1 - \gamma) \). In other words, revenue recognition is restricted to be less than the present value of future cash benefits from the investment. The incentive problem with respect to the agent’s effort \( a_t \) is not affected by the use of either residual income or income as performance measures, since the information available at contracting time and at renegotiation time is the same in both cases.
measure of future benefits from the investment, implying that the correlation between the two performance measures decreases. For tractability, we assume for the remainder of the paper that the noise \( \eta \) in \( q \) given by (37) is uncorrelated with any of the other random variables in the model. This means we restrict our attention to cases in which revenue recognition is only accompanied by pure measurement error.

We begin by assuming no restrictions on the value of \( k \) and \( d \), so that first-best investment can be obtained and the investment problem does not affect the choice of \( k \) and \( \alpha \). We then have the following result.

**Proposition 6** Assume identical periods and a fixed incremental level of noise \( \lambda \) in \( z \). In all cases it is optimal to allow for costless earnings management, \( \alpha^* = \infty \).

If \( \rho \geq 0 \), it is optimal to set \( k^* = \sqrt{\rho} / \theta \), in which case the performance measures become uncorrelated and the full commitment solution is achieved.

If \( \rho < 0 \), it is optimal to set \( k^* = 0 \).

If the correlation is positive and low enough, it is optimal to commit to equal incentive rates by allowing costless earnings management. The additional degree of freedom given by \( k \) allows the principal to get uncorrelated periods if \( \rho \geq 0 \), in which case the full commitment solution can be obtained. For high positive correlation, the principal can use \( k \) to get uncorrelated performance measures, in which case equal incentive rates are optimal. This can be achieved by allowing costless earnings management \( \alpha^* = \infty \), and again the full commitment solution is obtained. Thus, with positive correlation, the optimal revenue recognition increases in the correlation of performance measures and decreases in the measurement noise.

If the correlation is negative, there is no benefit from reversible accrual noise, but it is still optimal for the principal to commit to identical incentive rates, in which case it is optimal to set \( k^* = 0 \). In other words, conservative accounting is optimal, and costless earnings management is allowed.

For \( \lambda = 0 \), the above proposition recovers the result on accrual estimation errors from CFS. Furthermore, note that the results in the proposition are independent of \( \lambda \), that is, the precision of the leading indicator is irrelevant for both earnings management and revenue recognition.

### 7 Exogenous Revenue Recognition

The analysis in the preceding section shows that, when both income shifting and revenue recognition are controlled by the principal, it is optimal to allow for costless income shifting (thereby committing to equal incentive rates), while the revenue recognition parameter is set optimally as a function of the correlation
between performance measures. However, accounting standards specify revenue recognition policies that are the same across firms, or the amount of accrual estimation error may be fixed. Therefore, in this section, we consider the principal’s welfare, the optimal audit policy, and the optimally induced income shifting as functions of exogenously specified non-discretionary accruals as characterized by the parameters $k$ and $\theta^2$.

For simplicity, throughout this section we assume that no information is available at the first round of renegotiation, that is $\lambda = \infty$. The parameter $k$ is exogenously fixed and satisfies $0 \leq k \leq 1$. We also assume away the (induced moral hazard) investment problem, which amounts to assuming $k + \gamma \leq 1$.

The first task is to establish the optimal audit policy chosen by the principal for any given $k$ and $\theta^2$, which we denote by $\alpha(k, \theta)$. The following result analogous to Proposition 1 obtains.

**Proposition 7** Assume identical periods: $b_t = b$, $m_t = m$, and no early information: $\lambda \rightarrow \infty$.

1. If $\rho \leq 1/2 + 3/2 k^2 \theta^2$, then $\alpha(k, \theta) = \infty$ is optimal.

2. If $\rho > 1/2 + 3/2 k^2 \theta^2$ and the following condition is satisfied,

$$\rho - \frac{1}{2} - \frac{3}{2} k^2 \theta^2 < \frac{m^2}{r},$$  \hspace{1cm} (40)

then there is an interior $\alpha(k, \theta)$ determined by

$$\alpha(k, \theta) = \frac{m^2}{\rho - 1/2 - 3/2 k^2 \theta^2} - r.$$  \hspace{1cm} (41)

3. In all other cases in which $\rho > 1/2 + 3/2 k^2 \theta^2$, $\alpha(k, \theta) = 0$ is optimal.

The results in the above proposition can be easily obtained by recognizing that, for given $k$ and $\text{var}(\eta) = \theta^2$, the normalized sensitivities and correlation of the performance measures that include measurement error are $m' = m/\sqrt{1 + k^2 \theta^2}$, and $\rho' = (\rho - k^2 \theta^2)/(1 + k^2 \theta^2)$, respectively. The tradeoffs that determine $\alpha(k, \theta)$ are the same as described for Proposition 1, for the given values of sensitivity and correlation, $m', \rho'$.

We can now state the comparative statics with respect to $k$ for the principal’s welfare, for the optimal audit policy, and for the optimally induced earnings management. Note that the revenue recognition parameter $k$ and the noisiness of the non-discretionary accruals enter the analysis only through the combined term $k^2 \theta^2$. Therefore, all comparative statics with respect to $k$ hold the same way with respect to $\theta$.

**Proposition 8** Assume identical periods: $b_t = b$, $m_t = m$, and no early information: $\lambda \rightarrow \infty$. 
A. The principal’s welfare: Let $\alpha(k, \theta)$ be the optimal audit policy determined in Proposition 7, then

$$\left(\rho - k^2 \theta^2\right) \frac{d}{dk} U^P(\alpha(k, \theta)) \geq 0.$$  \hfill (42)

B. The optimal audit policy: Let $\alpha(k, \theta)$ be the optimal (interior) audit policy determined in Proposition 7, then $\alpha(k, \theta)$ increases in $k$.

C. The optimally induced earnings management: Let $e(k, \theta)$ denote the induced income shifting, given the optimal interior $\alpha(k, \theta)$. There exists a cutoff $g$ that depends on $k^2 \theta^2$ and $\rho$, such that $1/2 - 3/2 k^2 \theta^2 < g$ and

$$1 - \frac{3}{2} k^2 \theta^2 < \frac{m^2}{r} < g \implies \frac{d}{dk} e(k, \theta) > 0 \quad \text{and} \quad g < \frac{m^2}{r} \implies \frac{d}{dk} e(k, \theta) < 0.$$  \hfill (43)

Part A characterizes the same tradeoff present with accrual estimation errors in CFS. The key is that an increase in $k$ reduces both the correlation $\rho'$ and the normalized sensitivities $m'$ of the performance measures. If the correlation is negative, an increase in $k$ further reduces the correlation and the sensitivity, such that the gain from reduced correlation is less than the loss from increased noisiness, so the net effect is a reduction in the principal’s surplus. For positive correlation, a reduction in correlation is only beneficial up to the point at which the performance measures become uncorrelated. Thus, the possibility of earnings management does not alter the basic insights from CFS.

For part B, we know—from the discussion following Proposition 1—that $\alpha$ decreases in $\rho'$ and increases in $m'$. While $\rho'$ and $m'$ both decrease in $k$, the effect on $\alpha$ of reduced correlation is stronger, so the optimal audit policy $\alpha(k)$ increases in $k$.

For part C, the dependence of earnings management on $k$ is ambiguous. While $\alpha(k)$ is increasing in $k$, the changes in correlation and sensitivity also affect the incentive rates directly. While $\alpha$ decreases in $\rho'$, the difference in incentive rates $v_2R - v_1R$ is increasing in $\rho'$ (keeping $\alpha$ constant). Thus, the total impact on earnings management $e = \alpha(v_2R - v_1R)$ is ambiguous.

To summarize, the possibility of income shifting by the agent does not alter the conclusions in CFS on non-discretionary accrual estimation errors. Noisier non-discretionary accrual estimates or more aggressive revenue recognition rules optimally induce a more lax audit policy (higher parameter $\alpha$), while the net impact on the earnings management itself is ambiguous.
8 Conclusions

This paper is most closely related to, and extends the results in Christensen, Demski, and Frimor (2002) (CDF) and Christensen, Feltham, and Şabac (2005) (CFS). We extend CDF by introducing both correlated periods and additional (non-contractible) information at the renegotiation stage, and examining the impact of performance measure correlation and leading information precision on audit policy, earnings management, and the principal’s welfare. We extend CFS by introducing a second round of renegotiation (prior to the first performance report), a leading indicator (forecast of first-period performance), and the possibility of earnings management.

Our model bridges the gap between CFS and CDF: if the leading indicator perfectly reveals the first-period performance measure, the model becomes similar to that in CFS, while if the leading indicator is uninformative, we obtain a more general LEN version of CDF. We characterize conditions in which non-trivial earnings management is optimally induced in equilibrium. By contrast, in CDF, no earnings management is induced in equilibrium, i.e., the possibility of income shifting by the agent is used by the principal to commit to incentive rates. We also show that supplying information at the first round of renegotiation increases the principal’s welfare by reducing the negative effects of renegotiation prior to the first performance report.

If earnings management is induced, it is decreasing in the precision of the leading indicator, while the audit policy is generally relaxed. While the principal’s welfare decreases in the correlation between the performance measures as in CFS, the optimal audit policy becomes tighter and the effect of correlation on earnings management is ambiguous.

We also extend both CFS and CDF by simultaneously considering discretionary accruals (earnings management) and non-discretionary accruals (accrual estimation errors) as a means to alleviate the contracting frictions created by renegotiation. We find that the main conclusions of CFS on the usefulness of accrual estimation errors in contracting do not change, since controlling accrual estimation error is less costly to the principal than allowing earnings management.\(^{12}\)

If the control of accrual estimation error by the principal is exogenously constrained, the principal’s welfare may increase or decrease in the variance of accrual estimation errors depending on whether the resulting performance measure correlation is positive or negative. While firms with higher accrual estimation errors would prefer more lax auditing policies, the impact of accrual estimation errors on the induced earnings management is ambiguous.

\(^{12}\)In both cases, these costs are indirect, through the provision of incentives, since we have assumed away any direct costs to the principal of either information system design or auditing.
If changes in accounting regulation result in more conservative revenue recognition policies, the principal’s welfare will decrease (increase) if the resulting performance measure correlation is positive (negative). While a regulatory change to more conservative revenue recognition also increases the optimal tightness of internal auditing (making earnings management costlier), the impact on the induced earnings management is ambiguous.

To summarize, the main empirical implications that can be derived from our analysis are the following. First, earnings management is only induced if the performance measures used in contracting are highly positively correlated. Second, if earnings management is induced, the amount of under-reporting is lower for firms with better analyst following/forecasts. Third, the tightness of the audit policy regarding income shifting is higher for firms with highly correlated performance measures. Fourth, the tightness of the audit policy is lower for firms with higher accrual estimation errors (low “earnings quality”) and is increased by a regulatory change to more conservative revenue recognition. Finally, the relation between the induced earnings management and either the correlation of performance measures, accrual estimation errors, or the conservatism of revenue recognition is ambiguous.

Appendix: Proofs

Proof of Proposition 1. Set \( b_1 = b_2 = b, m_1 = m_2 = m \) and take the limit of the principal’s surplus for \( \lambda \to \infty \). Differentiating with respect to \( \alpha \) gives

\[
\frac{\partial U^p}{\partial \alpha} = \frac{b^2 m^2 r^2 (1 + \rho)^2 [2m^2 + (r + \alpha)(1 - 2\rho)]}{2(r + \alpha)^3 (m^2 + r - r\rho^2)^2}.
\]

The sign of the derivative with respect to \( \alpha \) is then determined by

\[
2m^2 + (r + \alpha)(1 - 2\rho).
\]

It follows that, for \( \rho \leq 1/2 \), the principal’s surplus is increasing in \( \alpha \geq 0 \), and it is optimal to set \( \alpha^* = \infty \). If \( \rho > 1/2 \), then \( 2m^2 + r(1 - 2\rho) < 0 \) gives the corner solution \( \alpha^* = 0 \), while \( 2m^2 + r(1 - 2\rho) > 0 \) gives the interior solution \( \alpha^* = m^2 / (\rho - 1/2) - r \). \( \square \)

Proof of Proposition 2. The proof is similar to that of Proposition 1. Set \( b_1 = b_2 = b, m_1 = m_2 = m \).
Differentiating with respect to $\alpha$ gives

$$\frac{\partial U^p}{\partial \alpha} = \frac{b^2m^2r^2(1 + \rho)^2\lambda^4}{2(m^2 + r + r\lambda^2)(\alpha + (r + \alpha)\lambda^2)^3(m^2 + r - r\rho^2)^2} \times [m^2(\alpha + (r + \alpha)\lambda^2 + 2r\lambda^4) + r(1 + \lambda^2)(\alpha + (r + \alpha)\lambda^2)(1 - 2\rho)].$$

The sign of the derivative with respect to $\alpha$ is then determined by

$$[m^2(\alpha + (r + \alpha)\lambda^2 + 2r\lambda^4) + r(1 + \lambda^2)(\alpha + (r + \alpha)\lambda^2)(1 - 2\rho)].$$

□

**Proof of Proposition 3.** Substituting $\alpha^*$ given by (35) in the principal’s surplus yields

$$U^p(\alpha^*) = \left( b^2[8m^4(1 + \lambda^2) + r^2(1 + \lambda^2)(1 - \rho - 2\rho^2)^2 + m^2r(1 + \rho)(9 - 15\rho + 8\lambda^2(1 - 2\rho))] \right) / \left( 8(1 + \lambda^2)(m^2 + r - r\rho^2)^2 \right).$$

Differentiating the above with respect to $\lambda$ gives

$$\frac{d}{d\lambda} \left[ U^p(\alpha^*) \right] = - \frac{b^2m^2r\lambda(1 + \rho)^2}{4(1 + \lambda^2)^2(m^2 + r - r\rho^2)^2} < 0.$$

Differentiating with respect to $\rho$ gives

$$\frac{d}{d\rho} \left[ U^p(\alpha^*) \right] = - \left( b^2r[m^4(\rho - 3 - 4\lambda^2) + r^2(1 + \lambda^2)(1 + \rho)^3(2\rho - 1) - m^2r(1 + \rho)(4 + \rho(-4 + 7\rho) + \lambda^2(5 - 2\rho + 8\rho^2))] \right) / \left( 4(1 + \lambda^2)(m^2 + r - r\rho^2)^3 \right).$$

The function that determines the sign of the derivative is a quadratic polynomial in $m^2/r$, and we find that the larger root is always less than the left-hand side in the condition that ensures an interior $\alpha^*$ (by numerical analysis on the explicit expressions involved). It follows that, for all $m^2/r$ in the interval on which $\alpha^*$ is interior, $U^p$ is decreasing in $\rho$. 

30
Substituting $\alpha = 0$ in the principal’s surplus yields

$$U^p(\alpha = 0) = (b^2 m^2[2m^4 + m^2 r(4 + \lambda^2 - 2(1 + \lambda^2)\rho - (4 + \lambda^2)\rho^2)$$

$$+ r^2(1 - \rho^2)(2 - \rho(2 + \rho) = \lambda^2(1 - 2\rho)))/(2(m^2 + r - r\lambda^2)(m^2 + r - r\rho^2)).$$

Differentiating the above with respect to $\lambda$ gives

$$\frac{d}{d\lambda} [U^p(\alpha = 0)] = -\frac{b^2 m^2 r\lambda(1 + \rho)^2(m^2 + r - r\rho^2)^2}{(m^2 + r + r\lambda^2)^2(m^2 + r - r\rho^2)^2} < 0.$$

Differentiating with respect to $\rho$ gives

$$\frac{d}{d\rho} [U^p(\alpha = 0)] = -\left(b^2 m^2 r[m^4(1 + \lambda^2(1 + \rho)) + r^2(1 + \lambda^2)(1 - \rho)(1 + \rho^3)

+ m^2 r(2 - \rho + 2\rho^3 + \lambda^2(2 + \rho^3))] / (m^2 + r + r\lambda^2)(m^2 + r - r\rho^2)^3\right) < 0,$$

since $2 - \rho + 2\rho^3 > 0$ for $\rho \in [-1, 1]$.

Finally, taking limits with respect to $\alpha \to \infty$ in the principal’s surplus yields

$$U^p(\alpha \to \infty) = (b^2 m^2[2m^4 + m^2 r(2 + \lambda^2 - \rho(1 + 2\rho))$$

$$+ r^2(1 + \rho)(2 + 2\lambda^2(1 - 2\rho) - \rho(4 - \rho - \rho^2))]$$

$$/(2[m^2 + r(1 + \lambda^2)]^2[m^2 + r(1 - \rho^2)])^2.$$

Differentiating the above with respect to $\lambda$ gives

$$\frac{d}{d\lambda} [U^p(\alpha \to \infty)] = -\frac{b^2 m^2 r^3\lambda\rho^2(1 + \rho)^2}{(m^2 + r + r\lambda^2)^2(m^2 + r - r\rho^2)^2} < 0.$$

Differentiating with respect to $\rho$ gives

$$\frac{d}{d\rho} [U^p(\alpha \to \infty)] = -\left(b^2 m^2 r[m^4 + m^2 r(2 + \lambda^2 - \rho + 2\rho^3)

+ r^2(1 - \rho + \rho^3 - \rho^4 + \lambda^2(1 + 3\rho^2 + 4\rho^3))] / (m^2 + r + r\lambda^2)(m^2 + r - r\rho^2)^3\right) < 0,$$

since $2 - \rho + 2\rho^3 > 0$ for $\rho \in [-1, 1]$. □
Proof of Proposition 4. A. For any given \( \alpha \), the optimally induced income shifting is
\[
e(\alpha) = \frac{bmr\lambda^2(1 + \rho)}{(\alpha + (r + \alpha)\lambda^2)(m^2 + r(1 - \rho^2))} \geq 0 ,
\]
so the contract always induces under-reporting in the first period.

Assuming an interior optimal audit policy, the optimally induced under-reporting is
\[
e(\alpha^*) = \frac{br(1 + \rho)[m^2(1 + 2\lambda^2) + r(1 + \lambda^2)(1 - 2\rho)]}{2m(1 + \lambda^2)(m^2 + r(1 - \rho^2))} .
\]
Differentiating with respect to \( \lambda \) yields
\[
\frac{d}{d\lambda}[e(\alpha^*)] = \frac{bmr\lambda(1 + \rho)}{(1 + \lambda^2)^2(m^2 + r(1 - \rho^2))} \geq 0 ,
\]
so the absolute amount of under-reporting is decreasing in the precision \( 1/\lambda \) of the leading indicator. The comparative statics with respect to \( \rho \) are ambiguous; there exists a parameter range over which the optimal amount of induced under-reporting decreases with \( \rho \). Differentiating the optimal amount of under-reporting with respect to \( \rho \) yields
\[
\frac{d}{d\rho}[e(\alpha^*)] = br[m^4(1 + 2\lambda^2) + m^2r(\rho(\rho - 2) + \lambda^2(1 + 2\rho^2))
- r^2(1 + \lambda^2)(1 + \rho^2)]/2m(1 + \lambda^2)(m^2 + r(1 - \rho^2)) ,
\]
which is generally positive for a large enough \( m \). In particular, for \( \lambda \to \infty \), the following parameter range makes the above derivative negative:
\[
\rho - \frac{1}{2} < \frac{m^2}{r} < \frac{1}{4} \left\{-1 + 2\rho^2 + \sqrt{4\rho^4 + 12\rho^2 + 16\rho + 9} \right\} .
\]

B. Assuming an interior audit policy and differentiating \( \alpha^* \) from Proposition 2 with respect to \( \lambda \) yields
\[
\frac{\partial \alpha^*}{\partial \lambda} = -2r\lambda[m^4(2\lambda^4 + 4\lambda^2 + 1) + 2m^2r(1 - 2\rho)(2\lambda^4 + 3\lambda^2 + 1)
+ r^2(1 + \lambda^2)^2(1 - 2\rho)^2]/[(1 + \lambda^2)^2(m^2 + r(1 + \lambda^2)(1 - 2\rho))^2] .
\]
Since this derivative is positive if, and only if,
\[
\frac{(1 + \lambda^2)}{1 + (2 + \sqrt{2})\lambda^2} \frac{(2\rho - 1)}{r} < \frac{m^2}{r} < \frac{(1 + \lambda^2)}{1 + (2 - \sqrt{2})\lambda^2} \frac{(2\rho - 1)}{r},
\]
and since the condition for interior \(\alpha\) is
\[
\frac{1 + \lambda^2}{1 + 2\lambda^2} \frac{(2\rho - 1)}{r} < \frac{m^2}{r} < (1 + \lambda^2)(2\rho - 1),
\]

it follows that \(\alpha^*\) is decreasing in \(\lambda\) if, and only if,
\[
\frac{(1 + \lambda^2)}{1 + (2 - \sqrt{2})\lambda^2} \frac{(2\rho - 1)}{r} < \frac{m^2}{r} < (1 + \lambda^2)(2\rho - 1).
\]

For the last inequality, we used
\[
\frac{1}{1 + (2 + \sqrt{2})\lambda^2} < \frac{1}{1 + 2\lambda^2} < \frac{1}{1 + (2 - \sqrt{2})\lambda^2} < 1.
\]

Assuming an interior audit policy and differentiating \(\alpha^*\) from Proposition 2 with respect to \(\rho\) yields
\[
-\frac{4m^2r^2\lambda^4}{(m^2 - r(1 + \lambda^2)(2\rho - 1))^2} < 0.
\]

\(\Box\)

**Proof of Proposition 6.** Using the explicit expression for the principal’s expected utility and differentiating with respect to \(\alpha\) gives
\[
\frac{\partial U^p}{\partial \alpha} = b^2 m^2 r^2 (1 + k^2\theta^2)(1 + \rho)^2 \lambda^4[m^2(\alpha + (r + \alpha)\lambda^2 + 2r\lambda^4)
+ r(1 + \lambda^2)(\alpha + (r + \alpha)\lambda^2)(1 - 2\rho + 3k^2\theta^2)]/[2(\alpha + (r + \alpha)\lambda^2)^3
(m^2 + r(1 + \lambda^2)(1 + k^2\theta^2))(m^2(1 + k^2\theta^2) + r(1 + \rho)(1 - \rho + 2k^2\theta^2))^2].
\]

It follows that the principal’s utility is increasing in \(\alpha\) for any \(\rho \leq 0\) and for any \(k\). Thus, for negative \(\rho\) it is optimal to set \(\alpha^* = \infty\). Taking limits in \(U^p\) with \(\alpha \to \infty\) and differentiating with respect to \(k\), gives a negative function for all \(k\). It follows then that the optimal \(k\) is zero.
To conclude the proof, we note that
\[
\left( \lim_{\alpha \to \infty} U^p \right)_{|k=\sqrt{\rho}/\theta} = \frac{b^2 m^2}{m^2 + r(1 + \rho)},
\]
which is clearly a global optimum, being the principal’s expected utility with full commitment. \(\square\)

**Proof of Proposition 7.** Set \(b_1 = b_2 = b, m_1 = m_2 = m\) and take the limit of the principal’s surplus for \(\lambda \to \infty\). Differentiating with respect to \(\alpha\) gives
\[
\frac{\partial U^p}{\partial \alpha} = \frac{b^2 m^2 r^2 (1 + \rho)^2 [2m^2 + (r + \alpha)(1 + 3k^2 \theta^2 - 2\rho)]}{2(r + \alpha)^3 (m^2 (1 + k^2 \theta^2) + r (1 + 2k^2 \theta^2 - \rho)(1 + \rho))^2}.
\]
The sign of the derivative with respect to \(\alpha\) is then determined by
\[
2m^2 + (r + \alpha)(1 + 3k^2 \theta^2 - 2\rho).
\]
It follows that, for \(\rho \leq 1/2 + 3/2 k^2 \theta^2\), the principal’s surplus is increasing in \(\alpha \geq 0\), and it is optimal to set \(\alpha(k) = \infty\). If \(\rho > 1/2 + 3/2 k^2 \theta^2\), then \(2m^2 + r (1 + 3k^2 \theta^2 - 2\rho) < 0\) gives the corner solution \(\alpha(k) = 0\), while \(2m^2 + r (1 + 3k^2 \theta^2 - 2\rho) > 0\) gives the interior solution \(\alpha(k) = m^2 / (\rho - 1/2 - 3/2 k^2 \theta^2) - r\). \(\square\)

**Proof of Proposition 8.** For part A, we substitute the optimal audit policy in the principal’s surplus and differentiate with respect to \(k\):
\[
\frac{d}{dk} [U^p(\alpha(k) = 0)] = b^2 km^2 \theta^2 (1 + \rho)^2 (2m^4 + 3m^2 r (1 + k^2 \theta^2)
\]
\[
+ r^2 (1 + 2k^2 \theta^2 - \rho)(1 + \rho)) / (m^2 (1 + k^2 \theta^2) + r (1 + 2k^2 \theta^2 - \rho)(1 + \rho))^3).
\]
\[
\frac{d}{dk} [U^p(\alpha(k))] = b^2 kr^2 \theta^2 (1 + \rho)^3 (2m^2 + 2r - k^2 \theta^2)
\]
\[
+ r (1 + 3k^2 \theta^2 - 2\rho)(1 + \rho)) / (m^2 (1 + k^2 \theta^2) + r (1 + 2k^2 \theta^2 - \rho)(1 + \rho))^3).
\]
\[
\frac{d}{dk} [U^p(\alpha(k) = \infty)] = \frac{4b^2 km^2 r^2 \theta^2 (\rho - k^2 \theta^2)(1 + \rho)^3}{(m^2 (1 + k^2 \theta^2) + r (1 + 2k^2 \theta^2 - \rho)(1 + \rho))^3}.
\]
Part B is obvious from the expression for the interior \(\alpha(k)\).

For part C, we substitute the optimal audit policy \(\alpha(k)\) in the expression for the induced income shifting and we obtain
\[
e(k) = \frac{br \sqrt{1 + k^2 \theta^2} (1 + \rho)(2m^2 + r (1 + 3k^2 \theta^2 - 2\rho))}{2m(m^2 (1 + k^2 \theta^2) + r (1 + 2k^2 \theta^2 - \rho)(1 + \rho))}.
\]
Differentiating the above with respect to $k$, we obtain

\[
\frac{d}{dk}[e(k)] = bkr^2(1 + \rho) \left[ -2m^4(1 + k^2\theta^2) 
+ m^2r(-1 + 3k^4\theta^4 - 2k^2\theta^2(-2 + \rho) - 2\rho(3 + \rho)) 
+ r^2(1 + \rho)(3 + 6k^4\theta^4 + k^2\theta^2(7 - 5\rho) + \rho(2\rho - 1)) \right] 
/ (2m \sqrt{1 + k^2\theta^2} (m^2(1 + k^2\theta^2) + r(1 + 2k^2\theta^2 - \rho)(1 + \rho))^2). 
\]

Simplifying the notation by using $x = m^2/r$ and $p = k^2\theta^2$, the sign of the above derivative is determined by the simple quadratic function

\[-2(1 + p)x^2 + (-1 - 2\rho(3 + \rho) + 2p(2 - \rho) + 3p^2)x + (1 + \rho)(3 + \rho(2\rho - 1) + p(7 - 5\rho) + 6p^2),\]

which has a positive root, the cutoff $g$, above which the function is negative. The second root is negative, so between the lower bound for $m^2/r$ that guarantees an interior $\alpha(k)$, which is always smaller than the cutoff $g$, and the cutoff $g$, the function is positive. □

References


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