Assessing the Impact of Alternative Fair Value Measures on the Efficiency of Project Selection and Continuation*

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Abstract

This study examines how alternative accounting measures of fair value impact the effectiveness of debt covenants in mitigating inefficiencies in debt-financed investment decisions. In our setting, shareholders make a non-contractible project choice after signing a debt contract. Control over a later decision to continue or abandon depends on fair value information made available at that time. Renegotiation implements first-best continuation decisions; however, fair value-based covenants can substantially reduce both the probability of renegotiation and shareholders’ incentives to engage in costly asset substitution. We find that a conservative fair value measure tends to perform best in reducing the probability of renegotiation and, hence, attendant costs when debt contracts must be structured to deter asset substitution. When renegotiation is ruled out, we obtain similar orderings of fair value covenants in minimizing the expected cost of inefficiencies. Although we cast our analysis in the context of alternative fair value measures per se, the intuition on the ability of conservative accounting values that to render asset substitution unattractive likely carries over to settings where accounting numbers are informative of values relevant to continuation decisions but may not come under the rubric of fair value.
I. Introduction

This study examines the impact of alternative fair value measures on the effectiveness of debt contracts in resolving inefficiencies in debt-financed investment and project continuation decisions. The setting considers a firm that can make non-contractible, post-borrowing investment decisions that result in potential asset substitution problems where borrowers (shareholders) have incentive to choose projects that transfer wealth from lenders (creditors) while lowering net present value (NPV).\(^1\) Fair value enters as the basis for covenants that allocate control rights over future project continuation decisions. Measures employed within our analysis reflect concepts of fair value contained in United States’ Generally Accepted Accounting Principles (GAAP) and International Financial Reporting standards (IFRS). We assess the contracting benefits of fair value measures in lowering the likelihood of renegotiation in order to achieve efficiencies in both continuation and project selection decisions. We also consider orderings in reducing the expected cost of inefficiencies when renegotiation is ruled out.

Debt covenants that trigger transfers of control rights based on accounting variables are common in practice,\(^2\) especially for small, high growth, firms more susceptible to asset substitution problems (e.g., Bradley and Roberts 2004). Studies by Chava and Roberts (2008) and Nini, Smith, and Sufi (2009) find evidence that covenant violations grant creditors significant influence over subsequent investments, often leading to reductions in capital spending. Our study investigates the effectiveness of covenants in contributing to the resolution of agency conflicts between shareholders and creditors that arise due to shareholders’ discretion over investment policy after debt contracts are in place. Specifically, we examine how the

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1 There is a substantial literature on asset substitution problems and the potential role of covenants to debt contracts in mitigating such problems made prominent by Jensen and Meckling (1976) and Smith and Warner (1979).

2 Typical variables include net worth, tangible assets, and other balance sheet items that encompass the investment values that are the object of our analysis.
effectiveness of covenants is impacted by the accounting measures upon which they are based.

Fair value measures proposed by GAAP, through Statement of Financial Standards (SFAS) No. 157 (FASB 2008), and IFRS, through International Accounting Standards (IAS) No. 32 (IASB 2003) and IAS No. 36 (IASB 2004), include value “in use” and value “in exchange”, defined in our analysis as firm value if the project is continued (continuation value) and if the project is abandoned (abandonment value), respectively. SFAS No. 157’s “highest and best use” corresponds to a fair value equivalent to the greater of the value in use or the value in exchange, a measure we define as the maximum of continuation value and abandonment value. Our analysis considers these fair value measures and adds a measure not contained in existing accounting standards, defined as the minimum of continuation value and abandonment value. This last measure represents a form of accounting conservatism. Inclusion of a conservative fair value measure is apt given the oft-heard claim that conservatism contributes to contracting efficiency.\(^3\) While, for technical reasons, we do not exhaust the universe of possible covenants, this set of covenants provides sufficient variety for us to demonstrate how different types of covenants impact renegotiation and shareholders’ incentives to engage in asset substitution.

The novelties in our study are the introduction of a post-contracting project selection problem and an explicit emphasis on the role of debt covenants based on fair value measures in implementing optimal project selection, as well as continuation, decisions. The importance of such an inquiry is evident in the substantial latitude firms in practice have over projects and investments made subsequent to the issuance of debt, especially general obligation debt, and the current attention being given to the properties of accounting valuation concepts in the design of contracts. With respect to the former, covenants that implement first-best continuation decisions

\(^3\) Recent studies that suggest conservatism may lower contracting costs and otherwise contribute to contracting efficiency include Watts (2003) and Bushman and Piotroski (2006).
in the absence of an asset substitution problem may no longer do so given post-contracting
discretion over projects. As for the latter, we will show that the effectiveness of covenants in
resolving inefficiencies at both the project selection and continuation stages vary, sometimes
sharply, depending on the measures upon which covenant thresholds are based.

In support of the claim that accounting conservatism furthers debt contracting efficiency,
we find that the minimum value covenant best balances the deterrence of asset substitution
against the likelihood of renegotiation. Intuitively, the incentive of shareholders to pursue risky,
low NPV projects is curbed by covenants based on conservative accounting measures that are
more likely to convey control rights to creditors who then use renegotiation to extract the value
of risky projects that happen to payoff. This value extraction occurs because high risks diminish
the value of the creditors’ claim under the initial debt contract, which allows them to credibly
threaten to abandon the project even if continuing is efficient. Shareholders anticipate the
consequences of renegotiation ex ante when choosing what project to pursue. In sum, by making
high risk, low NPV projects less attractive, covenants based on conservative accounting serve as
a credible commitment device to avoid such projects and attendant renegotiation.

Moreover, suppressing renegotiation, we find similar covenant orderings when structuring covenants to
deter poor project choices while minimizing the cost of inefficient continuation decisions. This
similarity suggests that similar results obtain as one varies renegotiation costs.

As is in other studies concerned with the properties of covenants in incomplete
contracting settings, we take debt financing as a given. In our model, shareholders obtain
financing for projects from competitive risk-neutral creditors in the form of pure discount debt

\footnote{Notably, we also find that the maximum value covenant reflecting the standard setters’ highest and best use
concept of fair value is dominated by the continuation value covenant.}

\footnote{This similarity is not surprising in that the states at the interim date for which renegotiation is needed are those in
which inefficient continuation decisions would otherwise be made.}

\footnote{Examples include Aghion and Bolton (1992), Décamps and Faure-Grimaud (2002), and Gârleanu and Zwiebel
(2009).}
that matures at a future date coincident with the realization of project payoffs. Shareholders’
choice of projects in which to invest is made post-contracting and is non-contractible. At an
interim date, the firm reports a fair value corresponding to one of the four alternative measures
described earlier. A covenant based on the reported value determines whether a technical default
has occurred, and if so gives creditors control over a decision to continue or abandon the project.
If the project is abandoned, creditors receive the smaller of the discounted maturity value or the
abandonment value. If the project is continued, payoffs are distributed and creditors receive the
smaller of the maturity value of the debt or the project payoff. Shareholders receive any
abandonment or continuation payoff in excess of the payment to creditors.

Shareholders have the opportunity to select an inferior project that transfers wealth from
creditors and lowers the value of the firm as a whole. Because competitive, rational creditors
break even, \textit{ex ante}, shareholders bear the expected loss in firm value due to any \textit{ex post}
incentives to choose inferior projects. We show that debt covenants can render such projects
unattractive to shareholders, enabling them to avoid the expected losses associated with such
projects.

Given a sufficiently tight covenant threshold, each of the fair value based debt covenants
can deter shareholders from choosing an inferior project. For example, an infinite threshold will
ensure that a technical default always occurs and creditors always control the continuation
decision. The downside is that tightening covenants increases the prospect of inefficient
abandonment by creditors, \textit{ceteris paribus}. Because costless renegotiation can resolve \textit{ex post}
inefficiencies from tightening covenants, the structure of covenants is irrelevant absent
renegotiation costs.\textsuperscript{7} However, given costs of renegotiation such as legal fees and executives’

\textsuperscript{7}The Coase (1960) result that renegotiation would ensure efficient decisions irrespective of covenant tightness does
not carry over to the \textit{ex ante} investment decision in our model.

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time are non-trivial, the likelihood of renegotiation has economic substance. Although writing covenants that mitigate the necessity of renegotiation may also be costly, Chava and Roberts (2008) suggest that simple threshold covenants of the type we consider are standard and involve little in the way of compliance monitoring. The current popularity of covenants that transfer control rights is consistent with a transactions cost advantage.

We initially capture the asset substitution problem in a simplified model in which we suppress uncertainty beyond the continuation stage. Similar to the workhorse structure in Tirole (2006), we assume arbitrary private benefits to shareholders from the selection of inferior projects associated with wealth transfers should such projects be continued. This characterization allows us to capture the forces in the conflict that arises between shareholders and creditors when shareholders can engage in asset substitution while preserving analytic tractability. We show that the results in the simplified model carry over to a setting more in keeping with the classic asset substitution problem in which uncertainty persists until debt maturity. This richer setting, which involves compound options with knockout features that depend on the allocation of control rights, requires the use of numerical analysis. We find that the relative effectiveness of covenants in this more complex setting mirrors that in our simple setting, demonstrating that the same economic forces are at work.

The calibrated examples also provide a sense of economic importance. As indicated by our numerical examples, the reduction in probabilities of renegotiation can be substantial. For instance, assuming covenant thresholds sufficient to deter selecting the closest alternative to an optimal project, the minimum value covenant reduces the probability of renegotiation to approximately a quarter of that necessary to deter such a choice under either full creditor control.

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8 Inferior projects may allow greater shirking, perquisite extraction, or diversion of assets, thereby providing benefits while lowering project value.
or the maximum value covenant (from .497 to .132). A similar reduction in expected loss expressed as rates of return obtains when renegotiation is ruled out (from 5.55% to 1.51%). Notwithstanding the limitations of numerical examples, albeit calibrated similarly to other studies of debt contracting, these comparisons suggest that the relative impacts of fair value covenants are potentially important in an economic sense.

Remaining sections are organized as follows: Section II reviews the principal antecedent studies; Sections III and IV present our model and formal results on the use of fair value covenants to deter inferior projects while minimizing the probability of renegotiation or, in the absence of renegotiation, minimizing expected costs; Section V extends our analysis to the less tractable classic asset substitution problem; Section VI concludes. Appendix A identifies several distributions that satisfy restrictions that arise in our proposition on strict orderings with renegotiation in Section IV. Appendix B provides proofs and derivations of results presented in the body of the paper and Appendix C discusses how our results in Sections III and IV would be impacted if we relax an assumption that breaks creditors’ strict indifference in some states, as discussed in Section III.

II. Antecedent Studies

The literature on debt contracting in the presence of agency conflicts is extensive. Grossman and Hart (1986) and Aghion and Bolton (1992) develop models that analyze the allocation of control rights. Grossman and Hart (1986) consider a two firm setting in which one firm makes a relation-specific investment followed by production decisions that are ex ante non-contractible. Renegotiation leads to ex post efficiency with respect to production decisions conditional on the ex ante relation-specific investment. The ex post allocation of surplus impacts the ex ante investment, much as debt covenants impact project choice in our study. In Aghion
and Bolton (1992), investors finance an entrepreneur’s new investment. A contract allocates the right to choose a hidden action that affects both parties’ payoffs. A perfect signal always leads to first-best decisions irrespective of how control rights are allocated. However, the allocation of control rights can reduce inefficiencies given an imperfect signal. While the models are quite different in the tensions driving results, they have in common the notion that control rights as allocated by ex ante contracts impact the efficiency of resolving agency conflicts that arise ex post.9

In a more closely related study involving wealth transfers between shareholders and creditors, Gârleanu and Zwiebel (2009) examine the allocation of control rights between a lender and a borrower in a setting where the borrower has private information about a potential wealth transfer that depends on a future investment. The lender can learn the wealth transfer at a cost and the study focuses on minimizing renegotiation and information acquisition costs. They predict that the likelihood of giving control rights to creditors increases in the uncertainty about the wealth transfer. Accounting plays no role in their model because the optimal allocation of control gives unconditional rights to either shareholders or lenders.

Other studies of how accounting choices affect the project continuation decisions of firms with debt include Gigler, Kanodia, Sapra, and Venugopalan (2009). They also consider value-based covenants; however, in a context in which project selection is fixed and only continuation value is informative with respect to such decisions. As in our study for a fixed project without renegotiation, debt contracting efficiency is assessed in terms of the impact of covenants on continuation and abandonment decisions in reducing costs of inefficient abandonment (“false alarms”) and continuation (“undue optimism”). They show that covenants based on an unbiased

9Dewatripont and Tirole (1994) consider financial contracting in a setting similar to Aghion and Bolton (1992) in which they allow for more than one class of outsiders, debt holders and outside equity, in order to efficiently discipline firm managers.
measure of continuation value implement first-best continuation. They interpret this result as contrary to the notion of conservatism in valuation contributing to debt contracting efficiency.

Li (2009) considers a similar problem in which uncertainty about the borrower’s profitability is resolved at an interim date at which time it is efficient to eliminate unprofitable projects that borrowers prefer to continue. Debt covenants ensure efficient abandonment of unprofitable projects. She shows that the benefit of conservative accounting depends on the likelihood that the borrower is profitable relative to the value obtainable through abandonment, and that accounting is irrelevant in the presence of costless renegotiation. However, the results in both Gigler, et al. (2009) and Li (2009) are sensitive to the assumption of a fixed initial investment.

III. Model

Setting

We consider a firm that finances new investment using zero coupon debt obtained from competitive, risk-neutral lenders. Risk-neutral shareholders control the firm and we abstract from any agency problems between the firm and its shareholders. There are three dates on the timeline as depicted in Figure 1. At Time 0, shareholders offer a take-it-or-leave-it debt contract to creditors. Because creditors are rational and competitive, the contract must satisfy the breakeven constraint that the expected value of the loan obligation equals the loan proceeds.

After the contract is signed, shareholders invest the borrowed funds in a project of their choosing. The project choice is non-contractible. At Time 1 the project may be abandoned or continued. A debt covenant, if any, determines whether creditors rather than shareholders control

10 Gigler, et al.’s (2009) informational inefficiencies correspond to what we term inefficiencies at the continuation stage.
11 Less closely related, are Göx and Wagenhofer’s (2009), study of the use of impairment accounting in mitigating post-contracting moral hazard, and Kwon, Newman, and Suh’s (2001) study of the principal’s choice of conservative or aggressive accounting in a moral hazard setting when limited penalties can be imposed on the agent.
the continuation decision. If the project is abandoned, creditors receive the lesser of the project’s abandonment value or the present value of the debt’s face amount. Shareholders receive any excess of the abandonment value over the present value of the debt’s face amount. If the project is continued, it yields a payoff at Time 2 and creditors receive the lesser of the project’s payoff or the face amount and shareholders receive any excess of the payoff over the face amount.

(Insert Figure 1 about here)

The Time 1 abandonment value $s_1$ and continuation value $v_1$ are unknown at Time 0 with distribution functions over the positive real line $[0, \infty)$ denoted $F_v(v_1)$ and $F_s(s_1)$, respectively. The project requires an initial investment of $s_0 = E[s_1]$. Initially, we assume that all uncertainty is resolved at Time 1. In Section V, we relax this assumption to allow uncertainty at Time 1 about the project’s Time 2 payoff. Without loss of generality, we further assume that the discount rate is zero.\(^\text{12}\) The contract specifies a face amount (maturity value) $M$ payable at Time 2 and may include a covenant applied at Time 1 based on $v_1$, $s_1$, or both. The covenant specifies the set of $(v_1, s_1)$ values for which creditors control the continuation decision and can therefore be viewed as specifying thresholds for a technical default.

Smith and Warner (1979) describe asset substitution as a problem in which shareholders pursue higher risk, lower NPV projects than indicated at the time of debt contracting. We model the asset substitution problem in exactly this way in Section V; however, we noted earlier, this proves to be intractable. In this section, we reflect the forces involved in asset substitution in a simple way by allowing the shareholders the opportunity to engage in an activity that decreases continuation value of the project to creditors by $R > 0$ while increasing value to shareholders by

\(^{12}\) We employ a non-zero risk-free discount rate in Section V in order to calibrate our numerical examples. The timeline in Figure 1 includes an arbitrary discount rate $r$ to encompass the model extension in Section V.
$\beta R, \beta \in [0,1)$, implying a reduction in firm value of $(1-\beta)R$. This modeling device provides a tractable way to incorporate the shareholder creditor conflict and resembles the modeling of private benefits by Holmström and Tirole (1997) and various models in Tirole (2006) in that shareholders may receive the benefit even in the event that creditors do not receive full payment. Shareholders receive $\beta R$ when creditors have not received full payment only in situations where shareholders control the continuation decision or where creditors control the continuation decision but cannot credibly threaten to abandon. If creditors control the continuation decision and can credibly threaten to abandon, they extract $\beta R$ from shareholders in renegotiation.

Broadly speaking, the project selection problem in both our analytical model and the model of Section V reflect the notion that shareholders may seek to advance their interests at the expense of creditors. In equilibrium, creditors rationally anticipate this behavior, implying that shareholders will bear any loss of efficiency in project selection. Because shareholders ultimately bear the expected loss from selecting an inferior project that they might find attractive after borrowing, shareholders benefit from adding a covenant to the debt contract that renders inferior projects unattractive ex post.

**Continuation**

At the Time 1 decision to continue or abandon the project, the first-best choice continues a project if $s_1 < v_1 \equiv s_*$ and otherwise abandons. Holding aside a project selection problem, shareholders receive $\max\{0, v_1 - M\}$ from continuation, versus $\max\{0, s_1 - M\}$ from abandonment. Creditors, in turn, receive $\min\{v_1, M\}$ from continuation versus $\min\{s_1, M\}$ from abandonment. Under the initial contract, both shareholders and creditors prefer to continue (abandon) when $s_1 < \min\{v_1, M\} \equiv s_*$ ($s_1 > \max\{v_1, M\} \equiv s_*$).
If both \( v, s < M \), then the initial contract gives creditors the full value of the firm so that they prefer to abandon only if \( s > v \) - the first-best choice. Both continuing and abandoning give shareholders a payoff of zero under the original contract. In this case, if shareholders control the continuation decision, then they can force renegotiation to extract surplus by threatening to abandon (continue) when \( v < s < M \) \((s < v < M)\). While a threat to continue is robust to any Time 1 uncertainty that \( v \) may exceed \( M \), as is present in Section V, the threat to abandon when \( s < v < M \) is a knife-edge case in that credibility of this threat does not carry over to a setting that allows for such uncertainty, no matter how small.\(^{13}\) Accordingly, in order to avoid a knife edge case and maintain preferences consistent with those in Section V, we assume that shareholders prefer to continue for \( s \leq \max\{v, M\} = s_\epsilon \).\(^{14}\) As we show later, in equilibrium, debt covenants will be structured so that shareholders do not control the continuation decision when \( v, s < M \), implying that their preference in this region has no impact on our predictions.

If \( v, s > M \), then continuing (abandoning) gives shareholders a payoff of \( v - M (s - M) \) so that they prefer to abandon only if \( s > v \) - the first-best choice. Both continuing and abandoning give creditors \( M \) under the initial contract. Similar to the case with shareholders, creditors can threaten to continue (abandon) when \( v, s > M \). In this case, it is the threat to continue when \( M < v < s \) that is a knife-edge case; i.e., such a threat is not credible if there is any Time 1 uncertainty, however small, that \( v < M \).\(^{15}\) Hence, we assume that creditors prefer to continue when \( s \leq s_d \). As we show later, debt covenants that set a covenant threshold

\(^{13}\) This threat is also not credible if shareholders have selected the inferior project, since continuing would give them the benefit \( \beta R \).

\(^{14}\) In Appendix C, we consider the effects of relaxing this assumption on our propositions.

\(^{15}\) Alternatively, the threat is not credible if creditors have a time preference and \( v \) is paid after \( s \). In either case, the assumption yields preferences that carry over to the setting in Section V where there is uncertainty at Time 1.
equal to $M$ can achieve first-best continuation decisions without renegotiation if the asset substitution problem is not too severe. In such cases, the assumption that the continuation threat is not credible has no impact on our results. If the asset substitution problem is severe, it may be necessary to set covenant thresholds greater than $M$, implying such an assumption does have an impact on our results. However, it does not alter the tenor of our predictions.\footnote{In Appendix C we consider the effects of relaxing the assumption that the continuation threat is not credible on our propositions.}

Figure 2 depicts shareholders’ and creditors’ continuation preferences $s_e$ and $s_d$, respectively. Reflecting on the above discussion, the continuation preferences in Figure 2 can be viewed as the limit of preferences in the later analysis where we introduce uncertainty in Time 2 project payoffs conditional on the Time 1 continuation value as the variance of remaining Time 2 uncertainty goes to zero. In particular, the shareholders’ (creditors’) preferences correspond to $M$ plus a long call option (short put option) on an asset with zero volatility or, equivalently, no time left to maturity.

(Insert Figure 2 about here)

The shareholder preference curve $s_e$ is perfectly aligned with the first-best preference curve $s_1$ in the upper right quadrant of Figure 2 ($v_1, s_1 > M$) and the creditor preference curve $s_d$ is perfectly aligned with $s_e$ in the lower left quadrant ($v_1, s_1 < M$). On the other hand, shareholders prefer to continue for all $(v_1, s_1)$ in the lower left quadrant ($v_1, s_1 < M$) while the first-best choice abandons in some cases. Creditors prefer to abandon for all $(v_1, s_1)$ in the upper right quadrant ($v_1, s_1 > M$) while the first-best choice continues in some cases. In the upper right quadrant ($v_1 < M < s_1$), both creditors and shareholders prefer to abandon, consistent with the first-best choice. In the lower left quadrant ($s_1 < M < v_1$), both creditors and shareholders prefer
to continue, consistent with the first-best choice. It follows that, in the absence of a project selection problem, first-best continuation decisions can be implemented without the need for renegotiation by any covenant that allocates control over the continuation decision to shareholders in the upper right quadrant and to creditors in the lower left.

We describe debt covenants in terms of a reported (fair) value and threshold $k$ such that a value below (above) $k$ gives control over the continuation decision to creditors (shareholders). Specifically, we consider covenants based on the following fair value measures: Abandonment value, $s_1$; continuation value, $v_1$; maximum value, $\max\{v_1, s_1\}$; and minimum value, $\min\{v_1, s_1\}$.

Relating these measures to accounting standards as discussed in the introduction, continuation value is value in use (i.e., present value of expected future payoffs from the project$^{17}$); abandonment value is value in exchange (i.e., value of the assets created by the project in the resale market); maximum value is representative of the “highest and best use” criterion; and minimum value reflects the accounting concept of conservatism. We acknowledge that more complex functions of $s_1$ and $v_1$ could improve upon these four functional representations of fair value in lowering the likelihood of renegotiation or expected costs of inefficiencies in the absence of renegotiation. However, the simplicity of threshold covenants in practice suggests that such complexity is prohibitive or unnecessary in terms of contracting and compliance costs. It is also unclear what additional insights we would gain by solving for a covenant as an arbitrary function.

It is easily checked from Figure 2 that sans a project selection problem, setting a threshold of $k = M$ for any of these covenants would allocate decision rights in a manner that

$^{17}$ In Section 4, we introduce uncertainty beyond the continuation stage changing the expression of continuation value to the discounted expected value $e^{-r} \mathbb{E}[v_{2} | v_{1}]$. 


implements first-best continuation.\textsuperscript{18} Accordingly, renegotiation in our model is moot absent shareholder discretion over projects.

IV. Project selection

With Renegotiation

The choice of the project by shareholders at Time 0 alters preferences for continuation at Time 1. Given the presence of a project selection problem (i.e., \( R > 0 \)), the inferior project makes shareholders (creditors) more inclined to continue (abandon) relative to their respective preferences in the case where the optimal project is chosen. Figure 3 depicts these preference changes, with shareholder preferences showing an upward shift from \( s_x \) to

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S_x^\beta = \max\{v_1, M\} + \beta R \text{ due to the extra value that shareholders realize from continuing, and with creditor preferences showing a downward shift from } s_d \text{ to } S_d^\beta = \min\{v_1, M\} - R \text{ due to the value } R \text{ lost from continuing the inferior project. Figure 3 also depicts the first-best rule of continuing the inferior project when } s_1 < v_1 - (1 - \beta)R. \]

(Insert Figure 3 about here)

If shareholders retain full control of the continuation decision, they will bear an expected loss from the opportunity to select the inferior project because the project will be irresistible \textit{ex post} and the creditors will price the debt accordingly. Absent renegotiation, the inferior project yields an additional payoff to shareholders of \( \beta R \) if it is continued, while it does not reduce the shareholders’ payoff from abandonment. Furthermore, from Figure 3 it is clear that shareholders’ continuation preference for the inferior project lies strictly above the first-best continuation line so that shareholders prefer to continue in some states where the first-best choice is to abandon.

\textsuperscript{18} This feature does not extend to the setting with uncertainty at the continuation stage considered in Section V. As we will show, achieving first-best continuation then requires renegotiation under any of the fair value covenants.
This aspect allows shareholders to extract concessions from creditors in renegotiation since creditors expect to lose $R$ if the project is continued. Indeed, even if $\beta = 0$ so that shareholders gain no benefits from continuation absent renegotiation, shareholders will still prefer the inferior project because of the benefits it allows them to extract from creditors in renegotiation. Of course, creditors understand this so that, in equilibrium, creditors receive their required return irrespective of the project selected, implying that shareholders bear the expected loss in firm value from their choice of the inferior project.

Shareholders therefore benefit from using covenants that eliminate the \textit{ex post} benefits of the inferior project. If a covenant allocates greater control rights to creditors in regions of the $(v_1, s_1)$ plane where creditors would abandon the inferior project but would continue the optimal project, it reduces the shareholders’ expected payoff from the inferior project. Because the fair value covenants vary in the extent of control that they allocate to creditors, their effects in deterring shareholders from choosing the inferior project also vary.

Renegotiation at the continuation stage eliminates inefficiencies in continuation decisions. While renegotiation also indirectly influences project selection by affecting shareholders’ expected payoff, renegotiation \textit{per se} may not deter selection of the inferior project. Specifically, if shareholders retain full control (no covenants), renegotiation resolves inefficiencies in continuation decisions, but provides no disincentives to choose the inferior project. However, as we will show, renegotiation in combination with any of the fair value covenants set with an appropriate threshold can implement both the optimal project and first-best continuation decisions.\footnote{Analogous to Gärleanu and Zweibel (2009), in all but restrictive circumstances, thresholds necessary to deter selection of the inferior project are tighter than thresholds set when information is symmetric.} If creditors control the continuation decision, they will have a tendency to inefficiently abandon the project. In such cases, they will renegotiate the debt contract in
exchange for continuing; however, shareholders do not benefit \textit{ex post} from this continuation because creditors have full bargaining power in such situations and will extract all but an arbitrarily small portion of the benefits of renegotiation. Our assumption of a competitive debt market implies that creditors earn their required return which, in turn, implies that shareholders extract this benefit \textit{ex ante}. The only remaining costs are those associated with the inferior project and any costs associated with renegotiation.

In some cases, a fair value covenant, set assuming that the optimal project will be selected, with the threshold \( k \) equal to the maturity value \( M \), will induce shareholders to choose the optimal project. This situation is ideal in the sense that the threshold \( k = M \) allows first-best continuation of the optimal project without the need for renegotiation. The key is for a covenant to allocate sufficient control rights to creditors so that shareholders’ expected gain from continuing the inferior project is less than their expected loss from inefficient creditor abandonments. Necessary conditions for a covenant to have this property are stated below:

\textbf{Proposition 1.} \textit{Necessary conditions for a fair value covenant to deter shareholders from choosing the inferior project are that creditors control the continuation decision for some (positive measure) set of} \((v_1, s_1)\) \textit{such that} \( M - R < s_1 < M < v_1 \).

If creditors are allocated control in the region given by Proposition 1 and shareholders have chosen the inferior project, creditors prefer to abandon under the Time 0 contract and shareholders will receive a zero payoff. If continuation is efficient, creditors will renegotiate to eliminate the \textit{ex post} inefficiency; however, their control over the decision grants them bargaining power that lets them extract all but a trivial amount of the surplus from renegotiation so that shareholders receive a zero payoff even after renegotiation. If shareholders had instead
chosen the optimal project, the Time 0 contract would have induced creditors to continue. This would have yielded shareholders $v_1 - M > 0$, which exceeds the zero payoff from the inferior project implying that the inferior project has a negative incremental payoff to shareholders in this region. There is no other set of $(v_1, s_1)$ values for which shareholders would be worse off for having chosen the inferior project. The following is an immediate corollary:

**Corollary 1.1.** *Given a covenant threshold $k = M$ set assuming the optimal project will be chosen, neither the continuation value nor maximum value covenant suffice to deter shareholders from choosing the inferior project.*

With covenant thresholds set to $k = M$, neither the continuation value nor maximum value covenant allocates control over continuation to creditors in the region identified in Proposition 1, while the minimum value and abandonment value covenants grant the entire $M - R < s_1 < M < v_1$ region to creditors. Although the minimum value and abandonment value covenants give identical control rights to creditors in the inefficient abandonment region identified in Proposition 1, the minimum value covenant has the further advantage of eliminating the shareholder control in the upper left quadrant ($v_1 < M < s_1$) where shareholders expect a higher payoff from the inferior project if they control the continuation decision. Thus, the minimum value covenant dampens shareholders’ incentive to choose the inferior project relative to the abandonment value covenant:

**Proposition 2.** *Given a covenant threshold $k = M$ set assuming the optimal project is chosen, if the abandonment value covenant suffices to deter shareholders from choosing the inferior project, then the minimum value covenant does so as well. However, the reverse is not true.*
If shareholders were indifferent between the optimal and inferior projects under a minimum value covenant with threshold \( k = M \), then the abandonment value covenant with a threshold \( k = M \) would not suffice to implement the optimal project. In this case, the minimum value covenant would strictly dominate all other fair value covenants in lowering the probability of renegotiation.

In cases where a covenant with threshold set to \( k = M \) fails to deter shareholders from choosing the inferior project, the debt contract can include a threshold \( k > M \) as a means for shareholders to render the inferior project unattractive and therefore avoid the expected loss associated with choosing that project. Subject to the proviso that the extreme of full creditor control is sufficient to deter the inferior project, increasing the threshold of a fair value covenant above the maturity value enlarges the region in which creditors are allocated control over continuation, ultimately to the point where the optimal project is preferred by shareholders.\(^{20}\)

Shareholders can set a fair value covenant threshold to implement the optimal project, while relying on renegotiation to achieve first-best continuation. The fair value covenants differ with respect to the regions within which control is transferred and, therefore, amount of covenant threshold tightening needed to deter selection of the inferior project. Accordingly, the covenants also differ in the likelihood of renegotiation necessary to resolve \textit{ex post} inefficient continuation decisions by either shareholders or creditors.\(^{21}\) Figure 4 depicts regions in which renegotiation occurs, assuming that the optimal project is implemented.

(Insert Figure 4 about here)

It is evident from Panels (a) and (c) (Panels (b) and (c)) of Figure 4 that probabilities of

\(^{20}\)The most that a covenant can do toward deterring selection of the inferior project is to allocate full control to creditors raising the prospect of inefficient abandonment. It is conceivable that there would be a sufficient likelihood of continuation of the inferior project and related benefit to shareholders that even this extreme transfer of control rights would not deter shareholders from choosing that project.

\(^{21}\)As considered in the next subsection, in the absence of renegotiation, these differences also imply variation in expected costs of inefficient continuation.
renegotiation in this case are the same for the minimum (continuation) value and abandonment (maximum) value covenants. Our next proposition formalizes this observation and provides a useful lemma as we work toward strict orderings of fair value covenants by probabilities of renegotiation:

**Proposition 3.** Given a covenant threshold $k \geq M$ and selection of the optimal project by shareholders, both maturity values and probabilities of renegotiation are identical under each of the following pairs: i) minimum value and abandonment value covenants, and ii) continuation value and maximum value covenants.

The regions in which renegotiation would take place in order to eliminate inefficiencies are equivalent amongst the pairs of covenants. Thus, analogous to Proposition 2, if the abandonment value covenant suffices to implement the optimal project, then the fact that the minimum value covenant can replicate the same probability of renegotiation while maintaining implementation of the optimal project implies weak dominance of the minimum value covenant in reducing probabilities of renegotiation. Similarly, it can be shown that the continuation value covenant weakly dominates the maximum value covenant. It remains to establish conditions under which strict dominance in renegotiation probabilities occurs when covenant thresholds and maturity values are set to deter shareholders from choosing the inferior project.

Given an assumption that debt value is increasing in maturity value at the equilibrium maturity values, the pair-wise dominance orderings above are strict within the relevant set of parameters. The potential for debt value to be decreasing in maturity value arises because increases in maturity value diminish the credibility of creditors’ threat to abandon in order to extract concessions in renegotiation when they control the project continuation decision. In order for debt value to be decreasing in the maturity value of debt, the situation must be pathological in
the sense that an increase in maturity value would reduce the amount creditors are willing to lend because creditors are lending to the company not primarily for the promised repayment, but for the right to hold up the firm and extract value in renegotiation. In the proposition below, we provide weak conditions on model parameters to eliminate this prospect without need of the assumption that debt value is increasing in maturity value. Appendix A identifies several distributions that satisfy these conditions. Increasing debt covenant thresholds strictly increases the value of debt. Thus, if debt value is increasing in maturity value, then the implicit function theorem implies that covenant thresholds and maturity value are partial substitutes with an inverse relationship determined by creditors’ breakeven condition.

**Proposition 4.** Given parameters such that full creditor control would deter shareholders from choosing the inferior project, and assuming that maturity value \( M \) is decreasing in the covenant threshold \( k \), then the minimum value (continuation value) covenant can implement the optimal project with a strictly lower probability of renegotiation than the abandonment value (maximum value) covenant. Sufficient conditions for such orderings to exist in equilibrium are that the project has a positive expected payoff even absent the abandonment option \( (E[v_1] > s_0) \) and that the expected project return, cum abandonment option, is less than 100% \( (E[\max\{v_1, s_1\}] < 2s_0) \).

The orderings in Propositions 4 suggest that fair value concepts advanced by accounting standards may not be the most appropriate in furthering debt contracting efficiency. As discussed in the introduction, the dominance of continuation value covenants over maximum value covenants challenges the intuition behind the “highest and best use” rationale cited by the FASB in SFAS No. 157 and by the IASB in ED-2009-5. From a debt contracting perspective, fair value reporting based on the presumption that firms would optimally continue or abandon overlooks
the self-defeating incentive of firms to engage in wealth transfers in the presence of debt by investment decisions that reduce firm value. The dominance of minimum value covenants over abandonment value covenants suggests a role for conservatism in fair value reporting in furthering debt contracting efficiency.

The ordering of the minimum value and continuation value covenants with respect to probabilities of renegotiation is ambiguous. It may appear from Figure 4 that the continuation value covenant should dominate since, in that figure, it entails less renegotiation than the minimum value covenant. However, the depiction of regions in which renegotiation takes place is under the assumption that covenant thresholds and maturity values are the same. In equilibrium, this equality will not hold because, for any given covenant threshold, the minimum value covenant gives strictly greater control to creditors than the continuation value covenant. As a result, debt contracts that use minimum value as the basis for covenants will have either a lower covenant threshold, a lower maturity value, or both, vis-à-vis contracts that base debt covenants on continuation value. Thus, consistent with Leftwich’s (1983) observation, debt contracts may need to be “customized” with respect to the accounting (fair value) measure employed; an argument for flexibility in financial reporting.

**Without Renegotiation**

Until this point, we have emphasized the relative effects of alternative fair value covenants on the likelihood of renegotiation to resolve inefficiencies. The implicit presumption is that costly renegotiation is preferred to allowing inefficiencies to remain unresolved. In the absence of renegotiation, the objective becomes the minimization of expected costs of inefficiencies while implementing the optimal project.

The expected cost of inefficiencies is defined as the expected loss in firm value from
suboptimal decisions. When creditors make an inefficient choice to abandon, the value loss is 
\( s^a - s_1 \) and the value loss is \( s_1 - s^p \) when shareholders make an inefficient choice to continue.\(^\text{22}\)

Creditors breakeven, \textit{ex ante}, so that shareholders ultimately bear the expected cost of these inefficiencies.

Because renegotiation occurs in the same regions where shareholders or creditors would otherwise make inefficient choices at the continuation stage, ordering covenants based on minimizing the expected cost of inefficient decisions (equivalently maximizing shareholders’ \textit{ex ante} expected payoff) closely resembles the orderings based on minimizing probabilities of renegotiation.\(^\text{23}\)

\textbf{Proposition 5.} \textit{Given a covenant threshold} \( k \geq M \) \textit{and selection of the optimal project by shareholders, the maturity value for all four covenant types equals the value assuming first-best continuation and is independent of} \( k \). \textit{The costs of continuation inefficiencies are identical under each of the following pairs: i) minimum value and abandonment value covenants, and ii) continuation value and maximum value covenants. Given parameters such that full creditor control would deter shareholders from choosing the inferior project, then the minimum value (continuation value) covenant implements the optimal project with strictly lower expected inefficiency costs than the abandonment value (maximum value) covenant.}

In the next section, we extend the model to allow uncertainty beyond the continuation stage and to redefine the effects of project selection in a manner that corresponds to the familiar asset substitution problem in which inferior projects increase the variance of payoffs from

\(^{22}\) Shareholders (creditors) never make an inefficient choice to abandon (continue) because their preference curves lie strictly above (below) the first-best curve \( s^p \).

\(^{23}\) This proposition places less restrictions than Proposition 4 because the absence of renegotiation eliminates complications in determining whether debt value is increasing in maturity value.
continuation while reducing NPV. While the minimum value covenant does best in an overall sense in limiting the necessity of renegotiation in our numerical examples, it is possible to find examples in which continuation value covenant prevails.

V. Numerical analysis

Uncertainty at Time 1

In this section, we introduce uncertainty at Time 1 about the continuation value. This extension demonstrates that the suppression of such uncertainty in the previous section does not drive the results in a qualitative sense. Moreover, by calibrating parameters used in the examples from antecedents employed in the literature for plausibility, the examples provide a sense of the economic significance of the differing impact of covenants on either the likelihood that renegotiation would be necessary to eliminate inefficiencies or the expected loss from inefficiencies when renegotiation is ruled out.

The model structure remains similar to that in Sections III and IV. The changes include specifying the distributions for continuation values and abandonment values, uncertainty in continuation values at Time 2, and fixing project parameters. The time line and related depiction of decisions and payoffs to parties remains the same as in Figure 1.

In the previous section, given a fixed project, fair value covenants could be set to implement first-best continuation decisions without renegotiation. However, this property no longer holds when we allow for uncertainty at Time 1. Because uncertainty at Time 1 shifts shareholders’ and creditors’ continuation preferences away from the first-best cutoff, there are regions of the \((v_1, s_1)\) plane for which neither shareholders’ nor creditors’ continuation preferences are aligned with the first-best rule. While covenants in Sections IV varied solely with respect to the likelihood of renegotiation necessary to deter shareholders from choosing the
inferior project, in this section covenants also differ in the likelihood of renegotiation even absent project choice.

We model uncertainty similar to Décamps and Faure-Grimaud (2002) by assuming normally distributed log returns for continuation and abandonment values \( v_t, s_t \), respectively, at \( t \in \{0,1,2\} \):²⁴

\[
\begin{pmatrix}
\log(v_t / v_{t-1}) \\
\log(s_t / s_{t-1})
\end{pmatrix} \sim N\left( \begin{pmatrix}
\mu_{vi} - \frac{1}{2} \sigma_{vi}^2 \\
\mu_{si} - \frac{1}{2} \sigma_{si}^2
\end{pmatrix},
\begin{pmatrix}
\sigma_{vi}^2 & \rho_{vi} \sigma_{vi} \sigma_{si} \\
\rho_{si} \sigma_{vi} \sigma_{si} & \sigma_{si}^2
\end{pmatrix}\right),
\]

where \( i \) denotes the project selected. Under this structure, \( E_t[v_{t+1}] = e^{\mu v} v_t \) and \( E_t[s_{t+1}] = e^{\mu s} s_t \).

The Time 2 payoff from continuing, \( v_2 \), is unknown at Time 1. Given pure discount debt, the value of equity at Time 1 resembles a long position in a call option. As mentioned earlier, this structure captures the classic problem in finance in which shareholders may choose drift and diffusion parameters in a manner that increases the value of their option position while reducing NPV. However, because of the combination of exchange, knockout, and default options, the project selection problem becomes analytically intractable. Alternative distributions that we considered are either not rich enough (Bernoulli) or likewise intractable (uniform).

**Continuation**

As before, Time 1 preferences of shareholders and creditors along with the allocation of control rights determine whether a project will be continued. A first-best continuation rule continues a project if \( s_1 < e^{-r} E_t[v_2 | v_1] \equiv s_c \). The appendix shows that shareholders prefer to continue if and only if \( s_1 < e^{-r} E_t[\max\{v_2, M\} | v_1] \equiv s_c \) while creditors prefer to continue if and

³⁴ Equivalently, \( v_t \) and \( s_t \) satisfy the stochastic differential equations \( dv_t = v_t \mu v dt + v_t \sigma v d z_v \) and \( ds_t = s_t \mu s dt + s_t \sigma s d z_s \) where \( z_v \) and \( z_s \) are correlated Brownian motions with \( d z_v d z_s = \rho d t \).
only if $s_1 < e^{-r} \mathbb{E}_t[\min\{v_2, M\} \mid v_1] \equiv s_{fd}$. Figure 5 depicts these preferences.

(Insert Figure 5 about here)

Shareholders prefer to continue for $(v_1, s_1)$ values that lie below the bold line $s_{x}$, which lies strictly above the first-best threshold $s_x$, reflecting the shareholders’ incentives to take risk in light of their payoff resembling a long call option. Creditors prefer to continue for $(v_1, s_1)$ values below the thin line $s_{fd}$, which lies below $s_x$ reflecting creditors’ incentives to avoid risk on account of their position resembling a short put option. The shareholder preference function $s_x$ is bounded below by both $s_x$ and by the maturity value $e^{-r} M$. Shareholders never prefer to abandon when $s_1 < e^{-r} M$ because that guarantees them a zero payoff whereas continuing always offers a strictly positive expected payoff. The creditor preference function $s_{fd}$ is bounded above by $s_x$ and $e^{-r} M$. Creditors never prefer to continue when $s_1 > e^{-r} M$ because abandoning guarantees that they receive their maximum payoff. The function $s_x$ approaches $s_x$ for large values of $v_1$ while $s_{fd}$ approaches $s_x$ for small values of $v_1$. Thus, unlike the model with no uncertainty at Time 1, renegotiation cannot be avoided by giving creditors control over the continuation decision for values of $(v_1, s_1)$ in the lower left quadrant of Figure 5 and giving shareholders control for values in the upper right quadrant. However, holding project selection aside, renegotiation can resolve all inefficiencies at the continuation stage.

The intuition is as follows: Suppose that creditors control continuation at Time 1. Absent

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25 A covenant that gives creditors control if and only if $s_1 > s_x$ will implement first-best continuation decisions without renegotiation; however, we show later that this covenant is a less effective deterrent to inferior projects than the continuation and minimum value covenants.

26 Proof of a proposition to this effect is available from the authors upon request. This result is similar to a result of Grossman and Hart (1986).
renegotiation, they will inefficiently abandon the project if \( s_I \in (s_L, s_U) \). In such cases, they can make a take-it or leave-it offer to shareholders to increase the maturity value of debt in exchange for continuing. Because creditors control the decision, they will demand a maturity value such that shareholders are only negligibly better off from continuing under the renegotiated contract than they would have been abandoning under the original contract. This allows creditors to recover the loss in expected value that would have occurred had they made an inefficient choice to abandon. Of course, creditors would have anticipated the opportunity to benefit from such renegotiations at the time of writing the initial debt contract so that the shareholders extract the value \textit{ex ante} of the creditors’ \textit{ex post} ability to demand concessions in renegotiation. An analogous argument can be made if shareholders control continuation.

As in the previous section, the detraction of allocating full control rights to either party relates to the likelihood of renegotiation. Fair value covenants substantially reduce the regions in which shareholders or creditors have control but make suboptimal continuation decisions, thereby reducing the need for renegotiation.\(^{27}\) We consider the same fair value covenants as before: Abandonment value, \( s_I \); continuation value, \( e^{-\tau} \mathbb{E}[v_2 \mid v_1] = s_E \); maximum value, \( \max\{s_E, s_I\} \); and minimum value, \( \min\{s_E, s_I\} \).\(^{28}\) Figure 6 depicts regions of shareholders’ and creditors’ control and regions where renegotiation would be needed to resolve inefficiencies.

(Insert Figure 6 about here)

For a given covenant threshold, the minimum value covenant (Panel (c)) allocates the greatest control rights to creditors consistent with yielding the most conservative measure of fair value. As for regions in which renegotiation would occur, it is evident from comparing Panels (a)

\(^{27}\) In the previous section, any of the fair value covenants eliminate inefficiencies in continuation absent a project selection problem.

\(^{28}\) As noted earlier, we provide for a non-zero discount rate for calibration in numerical examples and employ the expectations operator in defining continuation value consistent with the introduction of uncertainty beyond Time 1.
and (c) (Panels (b) and (d)) that for a given threshold, the minimum (continuation) value covenant results in less renegotiation than the abandonment (maximum) value covenant. In both cases, the covenant with less renegotiation allocates greater incremental control rights to creditors in regions where they would efficiently abandon, but shareholders would inefficiently continue. However, there is the caveat that covenants also affect maturity values necessary to meet creditors’ required return, and maturity values shift preferences for continuation. Hence, orderings of fair value covenants by probabilities of renegotiation cannot be inferred from the figure.

**Project selection**

For purposes of our numerical analysis, we define a set of alternative projects in terms of the continuation value’s drift and diffusion parameters $\mu_v$ and $\sigma_v$, respectively, from (1). In comparison with an optimal project, inferior projects considered in our examples have lower drift, which determines the expected value of continuation, and higher diffusion, which determines the risk of continuation.\(^{29}\) As we elaborate below, the greater risk implied by a higher diffusion parameter gives shareholders an incentive to choose such a project consistent with the option characteristic of their equity in project payoffs. Choosing a project with a higher diffusion parameter increases the expected value of continuation to shareholders at Time 1 given the option to default at Time 2, with the result that they inefficiently continue for some more realizations of $v_1$ and $s_1$ than they would have with a lower risk project. A lower drift parameter implies a reduction in NPV.\(^{30}\) The tension in play at the project selection stage is similar to before in that shareholders have incentive to engage in wealth transfers that lower firm value.

\(^{29}\) We also considered a wasting assets problem in which inferior projects have a negative abandonment value drift parameter lower abandonment value volatility parameters. Results for this case are available from the authors upon request.

\(^{30}\) A higher diffusion parameter also implies a higher value of the option to abandon at Time 1 which, *ceteris paribus*, increases firm value. Thus, for a project to be inferior it must also lower the drift parameter.
This similarity is made evident in Figure 7, which depicts the shift in preferences coincident with selection of an inferior project in the structure used in the current section, Panel (a), and the structure used in the previous sections, Panel (b).

(Insert Figure 7 about here)

Specifically, we consider a project set containing an optimal project, denoted Project $A$, for maximizing firm NPV and four alternative projects, denoted Projects $S_1, S_2, S_3,$ and $S_4$. The parameters of Project $A$ are calibrated as follows: The risk free rate is 0.05 as in Décamps and Faure-Grimaud (2002) which is between 0.045 used by Goldstein, Ju, and Leland (2001) and 0.06 used by Leland (1998). The drift parameters for the project’s continuation value $\mu_c$ and abandonment value $\mu_s$ are double and half the risk free rate, respectively. The continuation value’s diffusion parameter $\sigma_c$ is 0.40 as in Décamps and Faure-Grimaud (2002), while the abandonment value’s diffusion parameter $\sigma_s$ is half at 0.20, consistent with less volatile values in abandonment. The correlation is set at 0.40 based on the correlation between log asset and log market value returns of Compustat firms. The initial continuation and abandonment values are set at 100. These parameters imply that the ex ante value of the project, inclusive of the abandonment option, is 120.1, which is equivalent to a 20% return.

The volatility for all four of the alternative projects is set at 0.70, which is higher than Project $A$’s volatility of 0.4. The drift parameters of Projects $S_1, S_2, S_3,$ and $S_4$ are -0.05, -0.03, -0.003, and 0.005, respectively. These parameters imply that the first-best ex ante values of the projects, inclusive of the abandonment option, are 113.2, 115.0, 117.6, and 118.5, respectively. Hence, in the absence of renegotiation costs or inefficiencies in continuation and project choice, Project $A$ would be preferred to all substitutes and Project $S_4$ would be the best of the substitutes.
Table 1 summarizes project parameters.

(Insert Table 1 about here)

As a benchmark, we initially set covenant thresholds and debt maturity values to minimize the probability of Time 1 renegotiation subject to meeting the creditors’ breakeven condition but without regard to the potential for shareholders to choose projects other than Project A. Figure 8 illustrates shareholders’ indifference curves over projects, represented by $(\mu_v, \sigma_v)$ combinations, under the various fair value covenants. Figure 8 also plots the special cases in which either creditors or shareholders have unconditional control over the Time 1 continuation decision and a renegotiation proof, “first-best” covenant that gives creditors control if and only if $s_1 < s_2$. Relative to Project A, shareholders prefer any alternative project that lies above and to the right of the indifference curves except for the thick gray dashed indifference curve that represents full creditor control, in which case shareholders prefer projects below and to the right.

(Insert Figure 8 about here)

As indicated in Figure 8, the probabilities of renegotiation associated with full control by shareholders (0.433) or creditors (0.497) are both substantially higher than those associated with any of the fair value covenants (ranging from 0.076 to 0.159). Accordingly, if renegotiation is costly, then there is a demand for covenants that reduce the necessity of renegotiation to achieve efficiency in project continuation decisions.

Turning to project selection, when shareholders have full control over continuation, there is a large region within which they have incentive to engage in asset substitution by choosing projects with lower drift and higher volatility than Project A. When creditors have full control, shareholders have no incentive to engage in asset substitution in the form of higher volatility
since creditors can use renegotiation to extract much of the value from such selections. The shareholders’ preference for projects with lower volatility and lower NPV can be likened to the underinvestment problem of Myers (1977).

Focusing on the four alternative projects, Projects $S_1, S_2, S_3,$ and $S_4$, representative of the asset substitution problem, we observe that among these projects, all of the covenants except the abandonment covenant deter shareholders from choosing Project $S_1$. In order for covenants to deter shareholders from choosing alternative projects, it is necessary to reset covenant thresholds and maturity values in a manner that cedes greater control to creditors. As creditors gain more control, they extract more gains from high volatility projects, thereby dampening the incentives of shareholders to choose such projects. Table 2 displays the thresholds and maturity values required to deter the choice of each alternative project while minimizing the probability of renegotiation.$^{31}$

(Insert Table 2 about here)

As the covenants are modified to successively deter the choice of Projects $S_2, S_3,$ and $S_4$, the minimum value covenant does best in minimizing renegotiation probabilities across the sets of projects. The continuation value covenant implements Project $A$ over $S_2$ with less renegotiation than the minimum value covenant, but requires substantial debt covenant tightening, implying more renegotiation, in order to deter shareholders from pursuing projects

$^{31}$ The maturity values and covenant thresholds for the abandonment value covenant given in Table 2 exhibit a sudden shift when adjusting the contract to deter project $S_3$. This occurs because there are multiple maturity values that satisfy the creditors’ breakeven constraint when the covenant threshold of an abandonment value covenant is between about 119.5 and 121.1 under the parameters used in this section. In such cases, we report the contract that minimizes the probability of renegotiation subject to the constraint that shareholders prefer Project $A$. In particular, there are three maturity values that satisfy creditors’ breakeven constraint when the covenant threshold is set to the 119.6 level that deters Project $S_3$; however, although a contract with a higher maturity value would satisfy creditors’ breakeven constraint and would result in less renegotiation than a contract with the 118.9 maturity listed in Table 2, that contract would not deter Project $S_3$. This issue does not arise with the other types of covenants. The large shifts in the maturity values for continuation and maximum value covenants when deterring $S_3$ do not arise from such discontinuities, but instead arises from the need for high covenant thresholds in order to deter projects close to $A$. 

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$S_3$ and $S_4$. To deter choices of Projects $S_3$ and $S_4$, both the continuation value and maximum value covenants must cede virtually full creditor control implying much higher probabilities of renegotiation. Figure 9 depicts the effects of modified covenants on shareholders’ indifference curves for covenants set to deter $S_4$. The thin lines plot indifference curves for the minimum value and continuation value covenants assuming selection of Project $A$ as in Figure 8, while the thick lines plot the corresponding curves assuming covenants are set to so that shareholders prefer Project $A$ to Project $S_4$.

(Insert Figure 9 about here)

The effect of modifying covenants to deter selection of Project $S_4$ is to shift indifference curves to the right, thereby reducing the set of inferior projects preferred by shareholders to the optimal project. We observe that the minimum value covenant entails less renegotiation and deters selection of higher volatility projects in the neighborhood of Project $S_4$ that would not be deterred by the continuation value covenant. Although the minimum value covenant tends to perform best in our examples, there are cases where the continuation value covenant is best, suggesting that some flexibility in the choice of fair value measures may be desirable for lowering costly renegotiation.

Last, we consider the relative effectiveness of the fair value covenants in minimizing the expected cost of inefficiencies in the context of the examples in our extension. Absent renegotiation, inefficient continuation decisions will occur in the shaded regions shown on Figure 6; i.e. the same regions in which renegotiation may occur when feasible. As before, we first plot indifference curves in the case where covenant thresholds and maturity values are set to minimize the expected cost of inefficiencies assuming that Project $A$ is chosen. Figure 10 depicts the results.
The consequences of full shareholder control and full creditor control differ from the case where renegotiation is in play. The absence of renegotiation dampens the incentive of shareholders to choose high volatility projects for which they could extract higher gains through renegotiation from creditors for whom the risk of default at Time 2 is increased. It also reduces the effect of creditor control in deterring selection of such projects since the lack of renegotiation reduces creditors’ expected value from the contract and must be compensated with a higher maturity value. The higher maturity value reduces the likelihood that shareholders will receive a high continuation payoff because it increases the payout to creditors in the event that continuation produces a high payoff.

The orderings of fair value covenants in this case are similar to those with renegotiation. The exception is the abandonment value covenant which is considerably more effective in deterring selection of inferior projects than the other fair value covenants, ignoring the cost of continuation inefficiencies. This is because the covenant threshold based on selection of Project A lies below the discounted maturity value of the debt and increasing volatility has a greater effect in expanding the region in which creditors would inefficiently abandon in this case. With renegotiation, shareholders could lower maturity value since creditors would anticipate extracting greater gains concomitant with the greater threat of abandonment.

In Table 3, we consider the case where covenant thresholds and maturity values are set to minimize the expected cost of inefficiencies subject to the constraint that they prefer not to engage in asset substitution. Similar to the orderings of renegotiation probabilities, the minimum value covenant is most effective in minimizing expected costs associated with deterring the

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32 Shareholders always receive the no-renegotiation payoff when creditors have full control of the continuation decision because creditors appropriate all but a trivial portion of any gains from renegotiation.
choice of Projects $S_2, S_3,$ and $S_4$ when renegotiation is ruled out. As when we allow for renegotiation, allocating full control to creditors deters shareholders from pursuing an inferior project; however, the expected cost of the resulting inefficiencies in the continuation decisions more than offset the expected benefit of improved project choice. Similarly, neither the continuation value nor maximum value covenants are cost effective deterrents to deter asset substitution when Projects $S_3$ or $S_4$ are available. Both covenants must be tightened to the point where they essentially give full control to creditors resulting in expected costs of inefficient continuation that more than offset the expected benefit of shareholders choosing Project $A$.

(Insert Table 3 about here)

VI. Conclusion

We study how different fair value measures impact the effectiveness of debt covenants in deterring suboptimal investment and continuation decisions. In our setting, shareholders make a non-contractible project choice after negotiating a debt contract. The firm also has a subsequent opportunity to abandon the project prior to debt maturity. Shareholders have incentive to continue projects that should be abandoned while creditors have incentive to abandon projects that should be continued. Although renegotiation ensures the efficiency of ex post continuation decisions, the allocation of control rights at the continuation stage through covenants influences the shareholders’ ex ante investment decision, thereby affecting the need for renegotiation. Granting full control to creditors essentially removes incentives for inferior ex ante investments, but comes at the cost of a contract that will very likely require renegotiation in order for creditors to agree to continue projects. The key tension in our model lies in balancing the need to induce efficient ex ante investment choices, which tends toward granting control rights to creditors, against the likelihood of renegotiation.
We consider several fair value measures based on the values of continuing and abandoning the project. Of the set we consider, a conservative fair value covenant, defined as the minimum of the continuation and abandonment values, best serves to deter inefficient *ex ante* project choices while diminishing the necessity of renegotiation associated with giving extensive control rights to creditors. This finding may help explain the commonly held view that conservatism in accounting valuation contributes to debt contracting efficiency. Depending on the projects available to shareholders, a covenant based solely on continuation value may also deter inefficient projects and with a lesser likelihood of renegotiation than a minimum value covenant. We also find that the “highest and best” use measure of fair value contained in accounting standards is dominated by other measures of fair value. This occurs because such a measure implicitly assumes that shareholders will make best use of the firm’s assets even though this is not the case in the presence of an asset substitution problem.

Having employed a simplified structure for analytic tractability that preserves the key tension present in the asset substitution problem to obtain formal results, we then extend the analysis to the richer setting in which shareholders may choose high risk projects that lower NPV in order to exploit the option characteristic of equity in the presence of pure discount debt.\(^{33}\) Our numerical results closely parallel findings contained in our earlier propositions, implying that our formal results are not driven by the additional structure imposed. The examples also serve to convey a sense of potential economic consequences among alternative fair value measures that may be employed in the construction of covenants.

One limitation of our study is the suppression of other financing instruments that may also serve to resolve inefficiencies in project selection and continuation decisions; not an

\(^{33}\) As discussed earlier, because there are multiple options in this setting, analytic results are unattainable under characterizations of multi-period uncertainty, a familiar issue in the literature. Accordingly, we turn to numerical examples to illustrate the properties of fair value covenants in reducing probabilities of renegotiation or in minimizing expected costs of inefficiencies when renegotiation is ruled out.
uncommon modeling choice in studies of debt contracting. While consideration of such instruments lies outside the scope of our inquiry, it is useful to understand what is achievable through debt contracts as a benchmark and the extent to which effectiveness of covenants contained in those contracts may depend on the accounting measures employed. Another limitation is the assumption that debt covenants have a simple threshold form. This restriction suggests scope for further efficiencies in resolving conflicts over project selection and continuation decisions, albeit, with the prospect of higher contracting and compliance costs due to greater contract complexity. Notwithstanding these limitations, the effectiveness of debt covenants in mitigating conflicts of interests between shareholders and creditors almost surely depends on the properties of accounting measures upon which such covenants are based.
References


Appendix A – Distribution examples

This appendix provides examples of the sufficient conditions for debt value to be such that there is a maturity value that satisfies creditors’ breakeven constraint and debt is increasing in maturity value. In the examples, we normalize the expected value $s_0 = E[s_1]$ to be one. An assumption that $E[v_1] > s_0$ guarantees a strict inequality $E[\max\{v_1, s_1\}] > s_0$.

Uniform distributions

If $v_1$ is uniformly distributed over $[0, \bar{v}]$ and $s_1$ is uniformly distributed over $[0, 2]$, the sufficient conditions listed in Proposition 4 correspond to $1 < E[v_1] < 1 + \sqrt{\frac{2}{3}} \approx 1.81$. In other words, the project with the abandonment option yields an expected return less than about 81% and has a positive expected return even without the abandonment option.

Exponential distributions

If $s_1$ has an exponential distribution with $E[s_1] = 1$ and $v_1$ has an exponential distribution with parameter determined by $E[v_1]$, then the sufficient conditions from Proposition 4 correspond to $1 < E[v_1] < \frac{1}{2} (1 + \sqrt{5}) \approx 1.62$.

Lognormal distributions

If $s_1$ and $v_1$ are jointly and independently lognormally distributed as in (1) with $v_0 = s_0 = 1$ and $\mu_s = 0$, the conditions involve a restriction on volatilities of the form

$$\sigma_s < \sqrt{2f(\mu_v) - \sigma_v^2}$$

where $f$ is decreasing in $\mu_v$. For example, if $\mu_v = 0.69$, $(\sigma_v, \sigma_s)$ must lie within the quarter-circle with origin $(0,0)$ and radius 0.25, allowing a maximum volatility of 0.25 for either $v_1$ or $s_1$. If $\mu_v = 0.25$, the corresponding circle has a radius of about 2.29,
allowing for virtually any plausible level of volatility.
Appendix B – Proofs and derivations

Proof of Proposition 1

For any \((v_1, s_1)\) where shareholders control the continuation decision, it is straightforward to show that the inferior project always has greater value for them than the optimal project because the inferior project increases the value of the shareholders’ option to continue by \(\beta R\). If \(\beta = 0\), shareholders still value the inferior project because it increases the amount that they can extract in renegotiation since creditors still lose \(R\) if the project is continued.

If creditors control the project, the differences in values are:

<table>
<thead>
<tr>
<th>Range of (s_1)</th>
<th>Value of inferior project – Value of optimal project, given creditor control</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_1 &gt; s_d)</td>
<td>No difference because creditors abandon either project</td>
</tr>
<tr>
<td>(s_d^\beta &lt; s_1 &lt; s_d)</td>
<td>Creditors prefer to abandon the inferior project, yielding a zero payoff to shareholders, whereas they prefer to continue the optimal project. The resulting difference in payoffs is (M - s_1 \leq 0). The difference is strictly negative for (v_1 &gt; M).</td>
</tr>
<tr>
<td>(s_1 &lt; s_d^\beta)</td>
<td>Creditors continue either project so that the inferior project yields an incremental payoff of (\beta R &gt; 0).</td>
</tr>
</tbody>
</table>

From (B1), the only region where the inferior project yields a negative incremental value is when \(s_d^\beta < s_1 < s_d\) and \(v_1 > M\). When \(v_1 > M\), \(s_d^\beta = M - R\) and \(s_d = M\). Thus, the inferior project yields a negative payoff when \(v_1 > M\) and \(s_1 \in (M - R, M)\).

Proof of Proposition 2

Under both the minimum value and abandonment value covenants with threshold \(k = M\), creditors control the continuation decision when \(s_1 < M\) and shareholders control the continuation decision when both \(v_1\) and \(s_1\) are greater than \(M\). Control only differs across the
two covenants for \( v_1 < M < s_1 \).

Under a minimum value covenant, creditors control this region. This region lies above both their indifference curve \( s_d \) for the optimal project and the curve \( s_d^\beta \) for the inferior project, so that they abandon either and the inferior project yields no incremental payoff in this region.

Under an abandonment value covenant, shareholders control this region. When \( v_1 < M \), their preference curves are \( s_e = M \) and \( s_e^\beta = M + \beta R \) for the optimal and inferior projects, respectively. For \( s_1 > M + \beta R \), shareholders abandon either project yielding no incremental payoff. For \( s_1 \in (M, M + \beta R) \), continuing the inferior project yields an incremental payoff of \( M + \beta R - s_1 \); furthermore, shareholders can extract the value \( s_1 - s_e^\beta = s_1 - [v_1 - (1 - \beta)R] \) in renegotiation to induce the optimal decision to abandon. This yields a net incremental payoff of \( M - v_1 + R > 0 \) since \( v_1 < M \). While this payoff does not depend on \( \beta \), its contribution to the shareholders’ \textit{ex ante} payoff is \( E[I_{v_1 < M} I_{M < s_1 < M + \beta R} (s_1 - v_1 + R)] \), which does depend on \( \beta \). If \( \beta = 0 \), shareholders abandon either project, just as they do with a minimum value covenant.

The incremental payoffs to the inferior project are strictly greater with the abandonment value covenant so long as \( \beta > 0 \). If the abandonment value covenant renders the inferior project unattractive, the minimum value covenant does so \textit{a fortiori}.

\[ \text{Proof of Proposition 3} \]

For a generic debt covenant with the indicator variable \( I_s \) that equals one for \((v_1, s_1)\) where shareholders control the Time 1 continuation decision, the value of debt assuming the optimal project equals:
\[
\text{Debt value} = E[I_{s_1 < s_2} \min\{s_2, M\}] + E[I_{s_1 > s_2} \min\{s_1, M\}] - E[I_s I_{s_2 < s_1 < s_2} (\min\{s_1, M\} - \min\{s_2, M\})] + E[(1 - I_s) I_{s_2 < s_1 < s_2} (s_2 - s_1)].
\]

The third term in (B2), gains from inefficient abandonment, equals zero. If \( s_2 > M \), then
\[
\min\{s_1, M\} - \min\{s_2, M\} = M - M = 0 \text{ in the region given by the indicator variable } I_{s_2 < s_1 < s_2}. \]
If \( s_2 < M \), then \( s_2 = s_2 \) so that the indicator \( I_{s_2 < s_1 < s_2} = 0 \). If \( s_2 > M \), then \( s_2 = s_2 \) and the second term in (B2), losses from inefficient continuation, equals zero. If the debt covenant threshold is set at \( k \geq M \), then any of the four debt covenants will set the indicator variable \( I_s I_{s_2 < s_1 < s_2} = 0 \).

These observations yield the following debt value for covenant thresholds \( k \geq M \):

\[
\text{Debt value} = E[I_{s_1 < s_2} \min\{s_2, M\}] + E[I_{s_1 > s_2} \min\{s_1, M\}] + E[(1 - I_s) I_{s_2 < s_1 < s_2} (s_2 - s_1)].
\] (B3)

Any difference in debt value therefore depends on the renegotiation component. The renegotiation probability also depends on shareholder-initiated renegotiations; however, the assumption that the covenant threshold \( k \geq M \) implies that the renegotiation indicator for shareholders, \( I_s I_{s_2 < s_1 < s_2} = 0 \) for any \((v_1, s_1)\) where shareholders control the continuation decision as shown in Figure 4. From Figure 4, it is also clear that the continuation (abandonment) and maximum (minimum) value covenants share the same renegotiation regions for a given maturity value \( M \) and covenant threshold \( k \geq M \). Thus the debt value and renegotiation probabilities are equal between continuation (abandonment) value and maximum (minimum) value covenants.

The following lemma is used in the proof of Proposition 4:

**Lemma B1.** If shareholders can only pursue the optimal project, there is a unique maturity
value $M^*$ such that a covenant threshold $k^* = M^*$ satisfies the creditors’ breakeven constraint.

This covenant implements first-best continuation. If $E[\max\{v_1, s_1\}] < 2s_0$, then for any $k > k^*$ there exists a maturity value $M < M^*$ that satisfies the creditors’ breakeven constraint and such that debt value is increasing in $M$.

Proof

For the first part of the lemma, if the covenant is set such that $k = M$, then the discussion in Section III shows that the resulting continuation is first-best. The debt value will then equal:

$$\text{Debt}^* = E[1_{s_1 < v_1} \min\{s_1, M\}] + E[1_{s_1 \leq v_1} \min\{s_1, M\}],$$

(B4)

and satisfies:

$$\lim_{M \to 0} \text{Debt}^* = 0 \quad \lim_{M \to \infty} \text{Debt}^* = E[\max\{s_1, s_2\}]$$

(B5)

$$\frac{\partial \text{Debt}^*}{\partial M} = \int_M^\infty F_{s_1} (s_1 | s_1) dF_{s_1} (s_1) + \int_M^\infty F_{v_1} (v_1 | v_1) dF_{v_1} (v_1) > 0.$$ 

Because the breakeven point $E[s_1] = s_0$ lies between zero and $E[\max\{s_1, s_2\}]$, (B5) implies that there exists a unique $M^*$ that satisfies the breakeven condition.

Given an arbitrary $k > k^*$, a contract with threshold $k$ and maturity value $M^*$ exceeds the creditors’ breakeven condition because debt value is strictly increasing in creditor control (See (B3)). If $M = 0$, then $k > M = 0$ and the value of debt equals the value of the exchange option when creditors have control over the project:

$$E[(1 - I_s) 1_{s_1 < v_1} (v_1 - s_1)] = E[(1 - I_s) \max\{0, v_1 - s_1\}].$$

(B6)

This is worth at most $E[\max\{0, v_1 - s_1\}] = E[\max\{v_1, s_1\} ] - s_0$. Thus, if $E[\max\{v_1, s_1\} ] < 2s_0$, then any debt contract with $M = 0$ fails to satisfy the creditors’ breakeven constraint. Because the covenant with threshold $k$ and maturity $M^*$ exceeds the breakeven condition, the covenant
with threshold $k$ and maturity zero fails the breakeven condition, and debt value is continuous in $M$, there must be a maturity value $M < M^*$ that satisfies the breakeven condition and where debt value is increasing in $M$.

\[ \square \]

**Proof of Proposition 4**

The condition that full creditor control deters selection of the inferior project is necessary because all of the debt covenants are equivalent to full creditor control when their threshold is unbounded.

For a generic debt covenant, the probability of renegotiation is:

\[ P(\text{Renegotiate}) = E[I_{s_I \leq v_I \leq k}] + E[(1 - I_{s_I})I_{s_I < v_I}]. \]  

(B7)

Referring to Proposition 3 for the equivalence of renegotiation probabilities for continuation (abandonment) value (abandonment) and maximum value (minimum value) covenants, this gives the following for $k > M$:

\[ P\left( \text{Renegotiation for continuation/maximum} \right) = E[I_{v_I < v_1}I_{s_I < v_1}] = E[I_{M < v_I < v_1}I_{M < v_1}v_I] = \int_M^k [F_s(v_1 | v_I) - F_s(M | v_I)]dF_v(v_I), \]  

(B8)

\[ P\left( \text{Renegotiation for abandonment/minimum} \right) = E[I_{v_I < v_1}I_{M < v_1}] + E[I_{v_I > v_1}I_{M < v_1}] \]  

(B9)

\[ = P\left( \text{Renegotiation for continuation/maximum} \right) + \int_k^v [F_s(k | v_1) - F_s(M | v_1)]dF_v(v_I). \]

The change in the probability of renegotiation with respect to $k$ is:

\[ \frac{dP(\text{Renegotiation})}{dk} = \frac{\partial P(\text{Renegotiation})}{\partial k} + \frac{\partial P(\text{Renegotiation})}{\partial M} \frac{dM}{dk}. \]  

(B10)

Expression (B8) is clearly increasing in $k$ and decreasing in $M$ so that $dM / dk < 0$ is a sufficient condition for $dP(\text{Renegotiate}) / dk > 0$. For (B9), we have:
\[
\frac{\partial P_{\text{Reneg.}}}{\partial k} = \int_{-\infty}^{\infty} f_s(k | v_1) d F_v(v_1) > 0 \quad \frac{\partial P_{\text{Reneg.}}}{\partial M} = -\int_{M}^{\infty} f_s(M | v_1) d F_v(v_1) < 0, \tag{B11}
\]
which gives the same conclusion. Thus, if the conditions of Lemma B1 are satisfied, the probability of renegotiation is increasing in \( k \).

For the case of the minimum and abandonment value covenants, assume that there exists a debt contract with threshold \( k_a \) and maturity value \( M_a \) such that shareholders weakly prefer the optimal project to the inferior project under an abandonment value covenant. Referring to Proposition 3, a minimum value covenant with \( k_a \) and \( M_a \) yields an identical debt value and probability of renegotiation. The incremental value of the inferior project is decreasing in creditor control (see discussion in Proposition 2) so that shareholders strictly prefer the optimal project with this minimum value covenant. The covenant threshold can therefore be lowered, which will reduce the probability of renegotiation under the assumptions of Lemma B1, while maintaining the shareholders’ preference for the optimal project. The line of argument to compare the continuation and maximum value covenants is identical and we therefore omit it. ■

**Proof of Proposition 5**

The first part of the proposition, that \( k \geq M \) implies that the maturity value for all covenant types equals the maturity value corresponding to first-best implementation, follows directly from (B3) because the debt value with renegotiation equals the value with first-best implementation plus the creditors’ renegotiation gains when \( k \geq M \).

The expected cost of inefficiencies equals:

\[
E[I_s I_{v_1 < s_1 < \bar{s}} (s_1 - v_1)] + E[(1 - I_s) I_{s_1 < v_1 < s_1} (v_1 - s_1)]
\]

\[
= E[I_s I_{v_1 < M} I_{v_1 < s_1 < M} (s_1 - v_1)] + E[(1 - I_s) I_{v_1 > M} I_{M < s_1 < v_1} (v_1 - s_1)]. \tag{B12}
\]

Because any of the fair value covenants require at least either \( v_1 > M \) or \( s_1 > M \) to give
shareholders control, the first term in the second line of (B12) equals zero. This gives:

\[
\text{Inefficiency costs for } k \geq M = E[(1 - I_s)I_{v_1 > M}I_{M < v_1 < s_1} (v_1 - s_1)] \quad (B13)
\]

All four covenants give creditors control when \( v_1 < k \) and \( s_1 < k \). The continuation value covenant gives incremental control to creditors, vis-à-vis the maximum value covenant, when \( v_1 < k < s_1 \). This region does not impact the costs of inefficiencies because (B13) includes an indicator that requires \( s_1 < v_1 \), which equals zero where \( v_1 < k < s_1 \). Thus, the continuation and maximum value covenants have identical inefficiency costs since they have identical maturity values. The abandonment and minimum value covenants give creditors control when \( s_1 < k \). The minimum value covenant also gives control when \( v_1 < k < s_1 \). As when comparing the continuation and maximum value covenants, the indicator \( I_{M < s_1 < v_1} \) equals zero in this region.

The last part of the proposition follows because, from (B13), the costs of inefficiencies are decreasing in the extent of shareholder control. Therefore, a lower covenant threshold reduces the costs of inefficiencies.\(^{34}\) Direct computations show that, for a given \( k \geq M \), the incremental value of the inferior project is strictly less for a minimum value (continuation value) covenant than for an abandonment value (maximum value) covenant with the same \( k \). Furthermore, the incremental value of the inferior project is decreasing in \( k \) so that the threshold can be lowered while still deterring the inferior project. This reduces the inefficiency costs. ■

**Continuation preferences from Section V**

The overall value of continuing at Time 1 is:

\[
e^{-r} E_t[v_2 | v_1, s_1] = e^{-r} E_t[v_2 | v_1] = e^{\mu t - r} v_1 \equiv S_e. \quad (B14)
\]

\(^{34}\) The maturity value does not vary with \( k \) for \( k \geq M \), so that only the direct effect of the change in \( k \) affects the inefficiency costs.
Shareholders prefer to continue when \( \max\{0, s_1 - e^{-r} M\} < e^{-r} E_i [\max\{0, v_2 - M\} | v_1] \), which holds if and only if \( s_1 \) is below the following threshold where \( \Phi(\bullet) \) denotes the standard normal distribution:\(^{35}\)

\[
\begin{align*}
\bar{s}_e & \equiv e^{-r} E_i [\max\{v_2, M\} | v_1] \\
& = e^{\mu_{vi} - r} v_1 \Phi \left( \frac{\log(v_1/M) + \mu_{vi} + \sigma_{vi}^2/2}{\sigma_{vi}} \right) + e^{-r} M \Phi \left( -\frac{\log(v_1/M) + \mu_{vi} - \sigma_{vi}^2/2}{\sigma_{vi}} \right). 
\end{align*}
\]

The function \( \bar{s}_e \) asymptotically approaches \( e^{-r} M \) as \( v_1 \to 0 \) and approaches \( \bar{s}_e \) as \( v_1 \to \infty \).

Creditors prefer to continue when \( \min\{s_1, e^{-r} M\} < e^{-r} M E_i [\min\{v_2, M\} | v_1] \), which holds if and only if \( s_1 \) is below the threshold:

\[
\begin{align*}
\bar{s}_d & \equiv e^{-r} E_i [\min\{v_2, M\} | v_1] \\
& = e^{\mu_v - r} v_1 \Phi \left( -\frac{\log(v_1/M) + \mu_v + \sigma_v^2/2}{\sigma_v} \right) + e^{-r} M \Phi \left( \frac{\log(v_1/M) + \mu_v - \sigma_v^2/2}{\sigma_v} \right). 
\end{align*}
\]

The function \( \bar{s}_d \) asymptotically approaches \( \bar{s}_e \) as \( v_1 \to 0 \) and approaches \( e^{-r} M \) as \( v_1 \to \infty \).

Direct comparison of expressions (B14) through (B16) shows that \( \bar{s}_d < \bar{s}_e < \bar{s}_e \) for all \( v_1 \).

The formulas for the Black-Scholes option value’s sensitivity to volatility give the following where \( \phi(\bullet) \) denotes the density for the standard normal distribution:

\[
\frac{\partial \bar{s}_e}{\partial \sigma_{vi}} = -\frac{\partial \bar{s}_d}{\partial \sigma_{vi}} = e^{\mu_{vi} - r} v_1 \phi \left( \frac{\log(v_1/M) + \mu_{vi} + \frac{1}{2} \sigma_{vi}^2}{\sigma_{vi}} \right) > 0, \tag{B17}
\]

which shows that increases in risk make shareholders more likely to continue and make creditors less likely to continue.

---

\(^{35}\) Expression (B15) follows from adding \( e^{-r} M \) to the Black-Scholes value of a call option and using the symmetry of the normal distribution, \( 1 - \Phi(x) = \Phi(-x) \).
Appendix C – Impact of relaxing assumption on credibility of threats

The manuscript and the proofs in Appendix B utilize the assumptions that shareholders cannot credibly threaten to abandon when \( s_1 < v_1 < M \) and creditors cannot credibly threaten to continue when \( M < v_1 < s_1 \). The former assumption is relatively innocuous since shareholders never control the continuation decision when \( s_1 < v_1 < M \) given that the covenant threshold exceeds maturity value. The latter assumption holds if there is any probability, however small, that \( v_1 \) could be less than \( M \).\(^{36}\) In this Appendix, we list the propositions from Section IV and the effects of relaxing these assumptions.\(^ {37}\)

Proposition 1

If shareholders can credibly threaten to abandon the good project when \( s_1 < v_1 < M \), then there is a region where the inferior project has a negative incremental value under shareholder control because the extra payoff \( \beta R \) from the inferior project diminishes the credibility of shareholders’ threat to abandon, and therefore diminishes their gains from renegotiation. Comparisons of the shareholders’ value from the two projects show that this impacts the region where \( v_1 < M \) and \( s_1 < v_1 - \max\{\frac{1}{2}, \beta\}R \). In this case, the set given for the necessary condition in Proposition 1 would be expanded to be the union of \( \{(v_1, s_1) : M - R < s_1 < M < v_1\} \) and \( \{(v_1, s_1) : v_1 < M, s_1 < v_1 - \max\{\frac{1}{2}, \beta\}R\} \).

Corollary 1.1

This result is unaffected because a covenant with \( k = M \) does not give shareholders

\(^{36}\) It would also hold if \( v_1 \) would be paid after \( s_1 \) and creditors have a time preference if \( v_1 \) is realized even a very short time after \( s_1 \) would be paid. In other words, it is fairly easy to develop plausible circumstances that break the indifference that would sustain a creditor’s threat to continue a project that should be abandoned.

\(^{37}\) Given that our results in a qualitative sense are unaffected by such relaxation, we keep the analyses of effects as parsimonious as possible. Further details are available from the authors upon request.
(creditors) control in the regions where \( s_1 < v_1 < M \ (M < v_1 < s_1) \).

**Proposition 2**

If shareholders (creditors) can credibly threaten to abandon (continue) when \( s_1 < v_1 < M \ (M < v_1 < s_1) \), the result still obtains because a covenant with \( k = M \) does not give shareholders (creditors) control in the regions where \( s_1 < v_1 < M \ (M < v_1 < s_1) \).

**Proposition 3**

If \( k \geq M \) and creditors can credibly threaten to continue when \( M < v_1 < s_1 \), then the debt value includes an additional renegotiation component due to the creditors’ ability to threaten continuation when \( M < v_1 < s_1 \). The additional renegotiation gains equal:

\[
E[(1 - I_s)I_{M < v_1 < s_1}(s_1 - v_1)].
\]  

(B18)

The region where additional renegotiation occurs corresponds to the overlap of the creditor control region with the cone in Figure 4 to the right of the vertical \( v_1 = M \) line and left of the diagonal \( s_1 \) line. All four covenants result in renegotiation in the triangle where \( v_1 \in [M, k] \) and \( s_1 \in [v_1, k] \) or, equivalently, \( M < v_1 < s_1 < k \). Only the minimum and continuation value covenants result in renegotiation in the rectangle where \( v_1 \in [M, k] \) and \( s_1 > k \) or, equivalently, \( M < v_1 < k < s_1 \). Because the minimum value (continuation value) covenant gives additional renegotiation benefits to creditors vis-à-vis the abandonment value (maximum value) covenant, creditors value it more. Thus, for a given threshold \( k \) and maturity value \( M \), the debt value and likelihood of renegotiation are greater with the minimum value (continuation value) than with the abandonment value (maximum value) covenant.
Lemma B1

If creditors can credibly threaten to continue when \( M < v_1 < s_1 \), the first part of the lemma follows because the contract with \( k^* = M^* \) does not give creditors control in this region. Because debt value is increasing in creditor control, a debt contract with maturity \( M^* \) and covenant threshold \( k > k^* \) will exceed the creditors’ breakeven condition. If the maturity value is zero, the expression corresponding to (B6) adds the value of the option to abandon:

\[
\begin{align*}
E[(1 - I_s)(\max\{0, v_1 - s_1\} + \max\{0, s_1 - v_1\})] \\
= E[(1 - I_s)(2E[\max\{v_1, s_1\}] - E[v_1])] > s_0 E[1 - I_s].
\end{align*}
\]  

(B19)

If creditors have full control \( (I_s = 0) \), then the value (B19) exceeds the breakeven point \( s_0 \). If creditors have no control \( (I_s = 1) \), then the value (B19) equals zero. In cases where \( k \) is such that (B19) is less than \( s_0 \), then there is an \( M > 0 \) such that a debt contract with maturity \( M \) and threshold \( k \) satisfies creditors’ breakeven constraint and such that debt value is increasing in \( M \). If \( k \) is such that (B19) exceeds zero, debt value could exceed the breakeven value \( s_0 \) for all maturity values, in which case the \( k \) is irrelevant because shareholders would never offer it.

Otherwise, if there is an \( M \in (0, M^*) \) that satisfies the breakeven condition, then the debt value must cross the breakeven point somewhere in which debt value is increasing in \( M \).

Propositions 4 and 5

Both of these propositions rely on the equivalent renegotiation probabilities and debt values given by Proposition 3. These equalities do not hold when creditors can credibly threaten to continue when \( M < v_1 < s_1 \) so that the analogous orderings of covenants given in Propositions 4 and 5 are ambiguous and dependent on the assumed probability distributions.
Time 0
- Firm raises $s_0$ using zero-coupon debt
- Debt covenant is set
- Shareholders choose project subsequent to issuing debt

Time 1
- Covenant threshold violation assessed
- If violated, creditors control continuation
- If project is abandoned, value $s_1$ is realized; creditors receive $\min\{e^{-r}M, s_1\}$ and shareholders receive $\max\{0, s_1 - e^{-r}M\}$

Time 2
- If project was continued at Time 1, value $v_2$ is realized; creditors receive $\min\{v_2, M\}$ and shareholders receive $\max\{0, v_2 - M\}$

Figure 1: Timeline
Figure 2: Continuation preferences

Figure 2 illustrates shareholders’ and creditors’ Time 1 continuation preferences based on the continuation value $v_1$ and abandonment value $s_1$ given a debt maturity value $M$. The diagonal line $\mathcal{S}_p = v_1$ represents the first-best rule of continuing when $v_1 > s_1$. The line $\mathcal{S}_a$ represents shareholders’ preference to continue when $\max\{0, v_1 - M\} > \min\{0, s_1 - M\}$. The line $\mathcal{S}_d$ represents creditors’ preference to continue when $\min\{v_1, M\} > \min\{s_1, M\}$.
Figure 3: Continuation preferences for inferior project

Figure 3 illustrates a shift in shareholders’ and creditors’ Time 1 continuation preferences from choosing the inferior project based on the continuation value $v_1$ and abandonment value $s_1$ given a debt maturity value $M$. The diagonal lines $s_e = v_1$ and $s_e^\beta = v_1 - (1 - \beta)R$ represent the first-best rule of continuing the optimal and inferior project, respectively, when the continuation value exceeds the abandonment value $s_1$. The lines $s_d$ and $s_d^\beta$ represent shareholders’ preference to continue when their payoff from continuing the optimal and inferior projects, respectively, exceeds the payoff from abandoning. The lines $s_d'$ and $s_d'^\beta$ represent creditors’ preference to continue when their payoff from continuing the optimal and inferior projects, respectively, exceeds their payoff from abandoning.
Figure 4: Debt covenants and renegotiation

Figure 4 illustrates the regions of creditor control given by the different debt covenants for a given maturity value $M$ and debt covenant threshold $k$ under the assumption that shareholders do not engage in asset substitution. The hatched regions denote $(v_1, s_1)$ for which creditors control the continuation decision. The diagonal line $g_\alpha$ denotes the first-best cutoff for continuation, where the project should be continued for $s_1 < g_\alpha$. The thick kinked line represents the shareholders’ continuation preference $g_b$ and the thin kinked line represents the creditors’ continuation preference $g_c$. The gray shaded regions indicate where renegotiation occurs because the initial contract induces creditors to make inefficient choices to abandon. For a given maturity value and covenant threshold, all four debt covenants induce renegotiation in the region given by the dark gray triangle. The abandonment and minimum value covenants entail renegotiation in the region given by the light gray rectangle. No renegotiation occurs for any of the covenants if $k = M$. In equilibrium, this is the only case in which the same covenant threshold and maturity value satisfy creditors’ breakeven constraint for all four debt covenants.
Figure 5: Continuation preferences for the extended model

Figure 5 displays the continuation preferences for the model in Section V given Time 1 abandonment value $s_1$ and the Time 1 present value of continuing $e^{-r} E[v_2 \mid v_1] = e^{\mu_1 - r} v_1 = s_1$. The horizontal line $e^{-r} M$ denotes the Time 1 value of the debt’s promised payment, which is the promised payout to creditors if the project is abandoned. The diagonal line $s_1$ indicates the first-best continuation rule of continuing when $s_1 < s_1$. The thick dark line $s_e$ denotes shareholders’ preference to continue when $s_1 < e^{-r} E[max\{v_2, M\} \mid v_1]$ and the thin dark line denotes creditors’ preference to continue when $s_1 < e^{-r} E[min\{v_2, M\} \mid v_1]$. 

55
Figure 6: Continuation inefficiencies and renegotiation regions for different debt covenants

Figure 6 illustrates the regions of creditors and shareholder control under the different covenant types. The hatched regions denote \((v_1, s_1)\) values for which creditors control the continuation decision. The gray shaded areas denote the \((v_1, s_1)\) values for which inefficient continuation decisions occur absent renegotiation. The horizontal line 

\[ e^{-T} M \]

is the Time 1 discounted value of the debt’s maturity amount and \( \delta_s = e^{-T} E[v_2 \mid v_1] \) is the first-best threshold value of \( s_1 \) above which the project should be abandoned. The bold black line denotes the shareholders’ threshold \( \delta_s \) below which they prefer to continue. The thin black line denotes the creditors’ threshold \( \delta_d \) below which they prefer to continue. The maturity values \( M \) and covenant thresholds are for illustrative purposes only and do not represent optimized values.
Continuation preferences and the effect of inferior high risk/low NPV project

Figure 7 shows the shift in continuation preferences of shareholders and creditors from an increase in risk. Panel (a) displays preferences in the model of Section V that incorporates uncertainty at Time 1. The thick lines demark the values of $s_1$ below which shareholders prefer to continue the project. The increase in risk from the inferior project shifts this curve up from the solid thick line to the dashed thick line, which is a consequence of the effect of risk on the call option nature of the equity claim. The thin lines demark the values of $s_1$ below which creditors prefer to continue the project. The increase in risk from the inferior project shifts this curve down from the solid thin line to the dashed thin line, reflecting the impact of risk on the short put option nature of their debt claim. Panel (b) displays preferences from the model of Sections III and IV, where the inferior project takes value of $R$ from creditors in exchange for a gain of $\beta R < R$ to shareholders.
Figure 8: Shareholder indifference curves for asset substitution problem with renegotiation

Figure 8 illustrates shareholders’ preferences over projects, represented by \((\mu_v, \sigma_v)\) combinations, given debt covenants set to minimize the probability of renegotiation under the assumption that shareholders choose Project A. The lines represent the \((\mu_v, \sigma_v)\) combinations that yield the same equity value. Table 1 gives the parameters of the Projects A and \(S_1\) to \(S_4\). Shareholders prefer projects that lie above and to the right of the indifference curves except for the thick, gray dashed indifference curve for the case where creditors have full control. In the case of full creditor control shareholders prefer projects that lie below and to the right of the thick, gray dashed curve. The figure legend lists the renegotiation probabilities associated with choosing Project A under the different covenant types.
Figure 9: Shareholder indifference curves with covenants set to deter choosing Project S₄

Figure 9 illustrates shareholders’ post-contracting indifference curves over the drift $\mu_v$ and volatility $\sigma_v$ of the continuation value for debt covenants based on continuation and minimum values. The contracts are written to minimize the probability of renegotiation subject to the constraint that, post-contracting, shareholders prefer Project A to project $S_4$. The point $A$ refers to the reference project and the points $S_1$ to $S_4$ refer to alternative projects. The figure legend lists the renegotiation probabilities associated with choosing Project A under the different covenant types. The lines represent the $(\mu_v, \sigma_v)$ combinations that yield the same equity value. Aside from the thick gray dashed curve where creditors have full control over the continuation decision, shareholders prefer values of $(\mu_v, \sigma_v)$ above and to the right of the indifference curves over Project A. When creditors have full control, shareholders prefer projects below and to the right of the thick gray dashed line. The thick gray lines represent the project indifference curves when the contract unconditional control to either the shareholders or the creditors. Shareholders never prefer $(\mu_v, \sigma_v)$ combinations that lie in the bottom left region between these curves and they always prefer $(\mu_v, \sigma_v)$ combinations that lie in the upper right region between these curves.
Figure 10: Shareholder indifference curves for asset substitution problem without renegotiation

Figure 10 illustrates shareholders’ post-contracting indifference curves over the drift $\mu_\nu$ and volatility $\sigma_\nu$ of the continuation value for the different forms of debt covenants. The point $A$ refers to the reference project and the points $S_1$ to $S_4$ refer to alternative projects. The debt contract in each case is priced assuming that the firm pursues Project $A$ and sets the debt covenant threshold optimally conditional on that choice. The figure legend lists the renegotiation probabilities associated with choosing Project $A$ under the different covenant types. The lines represent the $(\mu_\nu, \sigma_\nu)$ combinations that yield the same equity value. Shareholders prefer values of $(\mu_\nu, \sigma_\nu)$ above and to the right of the indifference curves over Project $A$. The thick gray lines represent the project indifference curves when the contract unconditional control to either the shareholders or the creditors. Shareholders never prefer $(\mu_\nu, \sigma_\nu)$ combinations that lie in outside of the upper left region between these curves.
Table 1: Project parameters

Table 1 displays the parameters of the project choices we consider the asset substitution problem.

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Continuation value</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial value $v_0$</td>
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<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
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<tr>
<td>Drift $\mu_v$</td>
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<td>-0.050</td>
<td>-0.030</td>
<td>-0.003</td>
<td>0.005</td>
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<td>Volatility $\sigma_v$</td>
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<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
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<tr>
<td><strong>Abandonment value</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial value $s_0$</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Drift $\mu_s$</td>
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<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
<td>0.025</td>
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<td>Volatility $\sigma_s$</td>
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<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
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<tr>
<td>Correlation $\rho_{vs}$</td>
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<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
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<tr>
<td>Risk-free rate</td>
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<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
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<tr>
<td>Value with first-best implementation</td>
<td>120.1</td>
<td>113.2</td>
<td>115.0</td>
<td>117.6</td>
<td>118.5</td>
</tr>
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</table>
Table 2: Covenant choices and equity values for the asset substitution problem with renegotiation

Table 2 illustrates the parameters of debt contracts that minimize the probability of renegotiation subject to the constraint that shareholders prefer Project $A$ to the Project $S_i$, listed in the columns of the table.

<table>
<thead>
<tr>
<th>Optimized Contract implements $A$ vs. alternative $S_i$</th>
<th>$A$</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
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</thead>
<tbody>
<tr>
<td>Project value</td>
<td>120.1</td>
<td>113.2</td>
<td>115.0</td>
<td>117.6</td>
<td>118.5</td>
</tr>
<tr>
<td>Full creditor control</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Debt face value</td>
<td>91.2</td>
<td>91.2</td>
<td>91.2</td>
<td>91.2</td>
<td>91.2</td>
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<tr>
<td>$\text{P}(\text{Renegotiation}; A)$</td>
<td>0.497</td>
<td>0.497</td>
<td>0.497</td>
<td>0.497</td>
<td>0.497</td>
</tr>
<tr>
<td>Abandonment value $s_1$ covenant</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Debt face value</td>
<td>139.0</td>
<td>136.7</td>
<td>133.5</td>
<td>118.9</td>
<td>118.9</td>
</tr>
<tr>
<td>Covenant threshold</td>
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<td>119.4</td>
<td>120.6</td>
<td>119.6</td>
<td>119.6</td>
</tr>
<tr>
<td>$\text{P}(\text{Renegotiation}; A)$</td>
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<td>0.160</td>
<td>0.165</td>
<td>0.222</td>
<td>0.222</td>
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<tr>
<td>Continuation value $e^{-r} E[v_2</td>
<td>v_1] = s_1$ covenant</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Debt face value</td>
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<td>133.7</td>
<td>133.1</td>
<td>91.2</td>
<td>91.1</td>
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<tr>
<td>Covenant threshold</td>
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<td>111.4</td>
<td>114.1</td>
<td>443.9</td>
<td>460.2</td>
</tr>
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<td>0.076</td>
<td>0.077</td>
<td>0.496</td>
<td>0.497</td>
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<td>Minimum value $\min{s_1, s_2}$ covenant</td>
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<tr>
<td>Debt face value</td>
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<td>135.0</td>
<td>135.0</td>
<td>130.5</td>
<td>128.2</td>
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<tr>
<td>Covenant threshold</td>
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<td>105.0</td>
<td>105.0</td>
<td>114.4</td>
<td>117.0</td>
</tr>
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<td>$\text{P}(\text{Renegotiation}; A)$</td>
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<td>0.095</td>
<td>0.095</td>
<td>0.115</td>
<td>0.132</td>
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<td>Maximum value $\max{s_1, s_2}$ covenant</td>
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<tr>
<td>Debt face value</td>
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<td>131.3</td>
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<tr>
<td>Covenant threshold</td>
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<td>128.1</td>
<td>129.4</td>
<td>443.9</td>
<td>460.2</td>
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<tr>
<td>$\text{P}(\text{Renegotiation}; A)$</td>
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<td>0.111</td>
<td>0.111</td>
<td>0.496</td>
<td>0.497</td>
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</tbody>
</table>
Table 3: Covenant choices and equity values for the asset substitution problem without renegotiation

Table 3 illustrates the parameters of debt contracts that minimize the expected costs of inefficient continuation and abandonment subject to the constraint that shareholders prefer Project A to the Project $S_i$ listed in the columns of the table.

<table>
<thead>
<tr>
<th>Optimized Contract implements $A$ vs. alternative $S_i$</th>
<th>Project value for $A$</th>
<th>$S_1$</th>
<th>$S_2$</th>
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<tr>
<td>Value loss versus $A$</td>
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<td>Full creditor control</td>
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<tr>
<td>Debt face value</td>
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<td>129.0</td>
<td>129.0</td>
<td>129.0</td>
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<td>Inefficiency costs</td>
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<td>6.18</td>
<td>6.18</td>
<td>6.18</td>
<td>6.18</td>
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<tr>
<td>Abandonment value $s_1$ covenant</td>
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<tr>
<td>Debt face value</td>
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<td>136.4</td>
<td>136.4</td>
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<td>Covenant threshold</td>
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<td>123.7</td>
<td>123.7</td>
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<td>2.25</td>
<td>2.25</td>
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<tr>
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<td>v_1 = s_1$ covenant</td>
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<tr>
<td>Debt face value</td>
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<td>133.6</td>
<td>133.5</td>
<td>129.0</td>
<td>129.0</td>
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<tr>
<td>Covenant threshold</td>
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<td>115.0</td>
<td>115.7</td>
<td>242.4</td>
<td>292.4</td>
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<tr>
<td>Inefficiency costs</td>
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<td>0.40</td>
<td>0.40</td>
<td>4.73</td>
<td>5.55</td>
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<tr>
<td>Minimum value min{$s_1, s_2$} covenant</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Debt face value</td>
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<td>135.1</td>
<td>135.1</td>
<td>132.7</td>
<td>131.8</td>
</tr>
<tr>
<td>Covenant threshold</td>
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<td>107.4</td>
<td>107.4</td>
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<tr>
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<td>Maximum value max{$s_1, s_2$} covenant</td>
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<td>132.2</td>
<td>132.2</td>
<td>129.0</td>
<td>129.0</td>
</tr>
<tr>
<td>Covenant threshold</td>
<td>132.1</td>
<td>132.1</td>
<td>132.1</td>
<td>242.4</td>
<td>292.4</td>
</tr>
<tr>
<td>Inefficiency costs</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
<td>4.73</td>
<td>5.55</td>
</tr>
</tbody>
</table>