Financial Contracting under Imperfect Enforcement*

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Abstract

We develop a model of financial contracting under imperfect enforcement. Financial contracts are designed to keep entrepreneurs from diverting project returns, but enforcement is probabilistic and penalties are limited. The model rationalizes the prevalence of straight debt and common stock, and its predictions are consistent with a host of empirical capital structure regularities – across countries, across firms, and across time.

1 Introduction

The paper builds a tractable corporate finance model upon the assumption that contract enforcement is imperfect. Our narrow goal is to explain the empirical relationships between legal environment and business finance that have been documented in a rich recent literature; see, for example, Djankov et al. (2008) and the references therein. Our broader goal is to explain the widespread use of plain debt and common stock in the funding of businesses around the world, and thereby provide a new theory of capital structure.

Like Shleifer and Wolfenzon (2002), we assume that entrepreneurs may divert project returns (ex post moral hazard) and that legal protection of investor rights is only an imperfect guard against such diversion.1 For example, suppose the entrepreneur attempts to leave the

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1As discussed by Shleifer and Vishny (1997), diversion takes many different forms. For specific examples, see e.g. Johnson et al. (1998) and Atanasov et al. (2007).
firm with valuable resources. Then, the entrepreneur is only punished when apprehended and when the court is able to verify wrongdoing. Our key assumption is that either the apprehension or the verification is uncertain.\textsuperscript{2}

In reality, diversion sometimes occurs. Even in the United States, where legal protection of investors is arguably better than anywhere else in the world, looting of companies by their insiders is a considerable problem.\textsuperscript{3} Still, the average transfer from investors to entrepreneurs taking the form of illegal diversion is probably small, because it is in the joint interest of investors and entrepreneurs to design financial contracts that keep entrepreneurs honest. For simplicity, we therefore abstract from factors that would induce positive expected diversion in equilibrium – such as leniency towards small crimes and uncertainty about future enforcement possibilities – and focus squarely on the contractual implications of out-of-equilibrium diversion threats.

In our model, the optimal financial contract gives the entrepreneur enough money in each state to keep her from diverting. The required amount is determined by properties of the legal system, notably the probability of apprehension and the severity of punishment. As it turns out, the entrepreneur is sometimes willing to behave honestly even without any monetary compensation. When the project returns are sufficiently small, the punishment for diversion exceeds the maximum gains from diversion, and honesty is the best policy. In bad states, the optimum financial claim is therefore debt-like. In better states, diversion is more tempting, and the entrepreneur must be compensated financially in order not to succumb to the temptation. We provide conditions under which this compensation is a linear function of total returns. A key condition is that the apprehension probability does not increase too rapidly in the diverted share, so that diversion becomes an all-or-nothing proposition.

Since the entrepreneur’s compensation is zero for all sufficiently bad states and linear in all better states, the entrepreneur’s claim has the property of common stock. It follows that the sum of external investors’ claims corresponds to a combination of a standard debt claim and common stock. If all external investors are passive and identical, the simplest optimal security structure is thus that all external investors hold convertible debt.\textsuperscript{4} However, when investors

\textsuperscript{2}See Daughety and Reinganum (2000), Bull and Watson (2004), MacLeod (2007), and Kvaly and Olsen (2009) for discussions of probabilistic enforcement of claims in courts.

\textsuperscript{3}Besides well known corporate scandals involving large firms, such as Enron, there is a vast number of smaller firms whose insiders have cheated outside investors; see for example the evidence reported by Akerlof and Romer (1993).

\textsuperscript{4}Formally, convertible debt is a debt claim that can be converted into a predetermined number of shares at the will of the debt owner.Convertible debt is therefore a combination of debt and equity.
can increase the apprehension probability through monitoring, we show that equity provides stronger monitoring incentives than debt. In this case, plain debt and common stock constitute the unique smallest set of optimal securities.

Among other things, the model predicts that external funding is increasing in the entrepreneur’s wealth, the project’s expected profitability, asset tangibility, and the strength of investor protection, and decreasing in project risk. It predicts that the fraction of equity that is held by external investors is increasing in investor protection. It furthermore generates the following predictions concerning variation in financial leverage: Leverage ought to be decreasing in the entrepreneur’s wealth (past profits), project risk, and expected profitability.5

We also study entrepreneurs’ self-financing decisions, that is, their choice between (i) liquidating assets up front in order to increase own investment, and (ii) keeping the assets and use them as collateral for outside funds. The model predicts that assets with a low value to external investors – illiquid assets – are kept outside the project. As liquidity increases beyond a critical value, some assets are being pledged as collateral for debt, and as liquidity increases beyond a second critical value, some of the assets are sold upfront. The model explains why only debt is collateralized, and predicts that both diversion exposure and market liquidity influence to what extent outside assets are sold upfront to finance new projects or used as collateral.

A limitation of our model is that we confine attention to entrepreneurial firms. While the conflict between corporate insiders and outside investors is similar in managerial firms, the resolution is different. In the entrepreneurial firm, the insider decides on the investment volume subject to outsider participation; in a managerial firm, outsiders decide on investment. Relatedly, our model is probably a better depiction of small and medium-sized firms than of large corporations. In the latter case, diverting a substantial fraction of the firm’s resources may require collusion among several insiders and possibly by outside board members as well.

Let us now locate our contribution within the theoretical corporate finance literature. Of course, we do not originate the idea that financial contracts reflect the temptation of entrepreneurs to abscond with project returns. Allen (1981) is the seminal contribution. In Allen’s model, the project is infinitely lived. The entrepreneur is unable to divert any capital, but can strategically default on interest payments. After a default, investors can terminate the contract. If the entrepreneur has no own funds, a share contract is optimal, because a contract with substantial fixed payments induces strategic default in too many (low payoff) states of the

5For relevant capital structure evidence, see Harris and Raviv (1991), Rajan and Zingales (1995), Booth et al. (2001), and Giannetti (2003).
world. A wealthy entrepreneur, on the other hand, will also be pledging some fixed repayments, since this wealth is kept as outside collateral. Although the existence of outside collateral is an artifact of the assumption that capital cannot be diverted, Allen’s analysis anticipates our fundamental argument for the coexistence of debt and equity: Debt exists whenever there is significant punishment for small crimes, and equity exists whenever there is an affine relationship between the magnitude of the attempted diversion and the magnitude of the punishment.

Emphasizing the incentive effects of the contract termination threat, Allen (1981) abstracts from the role that the legal system plays in controlling managerial theft and other forms of self-dealing (but see Allen (1980)). The impact of the legal system on financial contracts is more pronounced in Lacker and Weinberg (1989), who assume that diversion is feasible, but that the entrepreneur has to forgo a constant fraction of any diverted value in order to evade punishment. While Lacker and Weinberg consider an insurance problem rather than a financing problem, Castro et al. (2004) consider a similar model in which firms raise capital. However, they restrict attention to two possible outcome states and focus on general equilibrium effects rather than on security design.

In a dynamic security design model with a fixed one-time investment and privately observable i.i.d. return realizations, DeMarzo and Fishman (2007) combine the contract termination threat with diversion costs. In equilibrium, the entrepreneur is willing to pay out all revenues up to a certain (state-dependent) amount in order to prevent contract termination, but retains a fixed fraction of returns above this amount in order to resist the temptation to divert. Thus, the model rationalizes a mixture of outside debt and equity in combination with inside equity.

Fundamentally, our model has much in common with DeMarzo and Fishman’s. Our non-pecuniary penalty plays the same role as their continuation payoff, and our apprehension probability corresponds to their diversion costs. However, since our static model relieves us of the need to analyze a complicated dynamic game, we are able to enrich it in other dimensions and thereby more fully address the evidence on financial constraints and capital structure.

The role of imperfect contract enforcement in shaping financial contracts moved sharply into focus with the rise of corporate governance research in the 1990s (Hart (1995); Shleifer and Vishny (1997)). Our model is particularly closely related to Shleifer and Wolfenzon (2002). Inspired by Becker (1968), they assume that a diverting entrepreneur is caught sometimes but not always. However, Shleifer and Wolfenzon assume that the punishment is close to zero when the diverted amount is small. As a consequence, their enforcement technology is compatible

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6See also DeMarzo and Sannikov (2006) and Biais et al. (2007).
with equity finance but incompatible with debt finance.\(^7\)

Our rationale for the existence of debt is distinct both from the “costly state verification” models of financial contracting by Townsend (1979) and Gale and Hellwig (1985), in which debt has the feature of minimizing verification costs, and from the ex ante moral hazard approach due to Innes (1990), in which debt provides maximum effort incentives for the manager. Instead, as in Diamond (1984), it is the presence of non-pecuniary punishment that drives the use of debt. However, in other respects our analysis is quite different from Diamond’s. There, the state is unverifiable and the optimal contract inflicts punishment in proportion to the shortfall whenever repayment is below a target. Although everyone understands that default is non-strategic, there may be substantial punishment in equilibrium. In our model, on the other hand, the state is verifiable, and punishment is only triggered if default is strategic. It is also the verifiability of the state that supports outside equity in our model, a security that is empirically common yet suboptimal both in Diamond’s model and in the costly state verification models.

Our model is also related to previous work on the optimality of issuing multiple types of securities. It is well understood that multiple securities can be optimal for a variety of reasons, including the provision of risk sharing facilities (Allen and Gale (1988)), information acquisition incentives (Boot and Thakor (1993); Fulghieri and Lukin (2001)), and investor intervention incentives (Berglöf and von Thadden (1994); Dewatripont and Tirole (1994); see also Hart and Moore (1995)). However, apart from the above mentioned works of Allen (1981) and DeMarzo and Fishman (2007), the literature has largely failed to rationalize the coexistence of debt with inside and outside equity in models with many outcome states. To the extent that uncertainty is involved at all, the models usually either allow for many outcome states and predict a plethora of different securities or allow only two states, in which case two securities may be optimal, but the link to debt and equity becomes tenuous. Chang (1992) is noteworthy for generating debt and outside equity, but his model predicts that insiders should have “nat claims. Ravid and Spiegel (1997) also rationalize debt and outside equity, but only under the strong assumption that all external investors are completely uninform ed about the projects that they fund.

By providing a simple explanation for the co-existence of external debt and internal and external equity, our model helps to bridge the gap between the theoretical literature on optimal securities and the largely empirical literature on capital structure. In the absence of such a model, the capital structure literature has mostly taken the co-existence of debt and equity for

\(^7\) The focus of Shleifer and Wolfenzon is on market-wide phenomena rather than on the form of optimal financial contracts. Therefore, their model does not need to provide a role for debt.
granted and proceeded to discuss the relative importance of the two claims. Another source of tension is that the theoretical financial contracting literature has increasingly come to emphasize moral hazard problems, whereas the empirical literature has continued to interpret data in view of tax wedges or adverse selection problems. Our model demonstrates that the moral hazard approach is capable of coherently addressing the capital structure evidence.

2 The basic model

We consider the funding of a single project idea that cannot be traded. The originator of the idea is called the entrepreneur. She is risk neutral and has initial wealth $\omega$. For now, we assume that all initial wealth is in the form of cash or other assets that can be costlessly liquidated.

2.1 The project

The project transforms monetary investments, $I$, into output. The output generates cash flow $\Pi = p\eta q(I)$, where $p$ is stochastic with a continuous and differentiable probability density function $f(\cdot)$. For simplicity, we assume that $p$ is always non-negative. The cumulative distribution function associated with $f$ is denoted $F$, and the expectation of $p$ is denoted $\bar{p} = \int_0^\infty p f(p) dp$. We initially assume that $q(I)$ is an increasing, twice differentiable and concave function satisfying $q(0) = 0$, $q'(0) > 1/\bar{p}$ and $\lim_{I \to \infty} q'(I) < 1/\bar{p}$, but we will also consider cases in which $q(I)$ is not concave. One natural interpretation of the model is that $q$ is the quantity of a homogeneous and perishable good, $p$ is the price, and the firm is a price taker in a competitive market with production to stock and demand uncertainty. The parameter $\eta$ is a measure of the firm’s productivity, and to save on notation we shall set $\eta = 1$ except when studying variation in productivity. We assume that both the distribution function $f$ and the realization $p$ are commonly observable and verifiable with the caveat described in Section 2.3.

We abstract from discounting. The first-best level of investment, call it $I^{fb}$, thus maximizes

\[ \text{maximize } \int_0^\infty \int_0^\infty \Pi(I, p) \eta q(I) f(p) dp dI \]
expected net returns

\[ E_p[\Pi(I, p) - I] = \int_0^{\infty} pq(I) f(p) dp - I = \bar{pq}(I) - I. \]

Given our assumptions, \( I^{fb} \) is the unique solution to the first-order condition

\[ \bar{pq}'(I^{fb}) - 1 = 0. \]

### 2.2 Diversion

Investments into the project are fully contractible; they may be used for no other purpose. However, repayment is only imperfectly enforceable. Thus, the entrepreneur may attempt to divert (steal) project returns.\(^{10}\) For simplicity, we assume that an entrepreneur who diverts gives up on any legal claims. (In Section 4.6, we briefly discuss the case in which successful diversion does not preclude the entrepreneur from also obtaining part of the non-diverted returns.)

### 2.3 Enforcement

If the entrepreneur diverts an amount \( S \), we assume that she is apprehended with probability \( \tilde{\varphi}(S) \). With probability \( 1 - \tilde{\varphi}(S) \), the diversion attempt succeeds and the entrepreneur can fully enjoy the stolen returns.\(^{11}\) If the entrepreneur is apprehended, all her financial assets are confiscated.\(^{12}\) In addition, there is a non-pecuniary punishment, such as jail or reputational loss, corresponding to a utility cost \( \tilde{\gamma}(S) \). We assume that at most \( \tilde{S}(p, I) \leq pq(I) \) may be diverted. Define relative diversion as \( s = S/\tilde{S} \).

The enforcement technology is characterized as follows:

(i) The apprehension probability satisfies \( \tilde{\varphi}(S) = \varphi(s) \geq \varphi_0 + \varphi_1 s \), for \( s \in (0, 1) \); \( \varphi(s) = \varphi_0 + \varphi_1 \), for \( s = 1 \), with parameters \( \varphi_0 > 0, \varphi_1 \geq 0 \).

\(^{10}\)As discussed by Shleifer and Vishny (1997) (and many others since then) such stealing can take many forms. For example, the entrepreneur may leave the firm together with the project idea and customer base, and tunnel funds through excessively high input prices or excessively low output prices.

\(^{11}\)While our main interpretation is that \( p \) always is verifiable and that \( \varphi(S) \) reflects the probability that the entrepreneur is apprehended, our model allows for an alternative interpretation; investors observe \( p \) but can only verify \( p \) in courts with probability \( \varphi(S) \). If \( p \) cannot be verified, the entrepreneur’s diversion attempt succeeds.

\(^{12}\)Under our assumption that entrepreneurs are not punished for crimes that they did not commit, we know from Becker (1968) that it is optimal to impose the harshest possible punishment. Therefore, our assumption, that all legal claims are forfeited, is consistent with optimal enforcement.
(ii) Analogously, the non-monetary punishment satisfies $\tilde{\gamma}(S) = \gamma(s) \geq \gamma_0 + \gamma_1 s$; for $s \in (0, 1)$; $\gamma(s) = \gamma_0 + \gamma_1$, for $s = 1$, with parameters $\gamma_0 > 0$, $\gamma_1 \geq 0$.

(iii) The parameters satisfy the inequality

$$\varphi_1 \leq \frac{\varphi_0 \gamma_0 (1 - \varphi_0)}{\gamma_1 + 2 \varphi_0 \gamma_0}.$$ 

While the assumptions are not innocuous, they represent a reasonable approximation to actual enforcement technologies: Assumption (i) says that the probability of apprehension is always positive. It may depend in complex ways on the diverted fraction $s$, but for a given fraction it does not depend directly on the diverted amount $S$. Assumption (ii) says that an entrepreneur who is caught diverting is always punished, no matter how little is diverted. The punishment might depend in complex ways on $s$, but does not depend directly on $S$. (Actually, this latter requirement can be relaxed; under a suitable reformulation of Assumption (iii), all our results go through if there is an additional linear effect of $S$ on the punishment.) Finally, the bound on $\varphi_1$ says that the apprehension probability should not increase too fast in the diverted fraction.

Together, the three assumptions ensure that if the entrepreneur diverts, then she diverts all available resources; for a proof, see the Appendix. We can thus focus merely on the cases of no diversion ($s = 0$) and full diversion ($s = 1$).

Accordingly, we simplify our notation and define $\varphi = \varphi_0 + \varphi_1$ and $\gamma = \gamma_0 + \gamma_1$ from now on. For ease of exposition, we also restrict attention to the case $S(p, I) = pq(I)$, except when otherwise noted.

### 2.4 Outside investors

There are many potential outside investors, all of whom are risk neutral. Let $\omega_j$ denote the wealth of investor $j$. Investors’ joint wealth is large relative to the entrepreneur’s project, and we normalize investors’ required rate of return to zero.

A financial contract specifies the repayment $r_j$ that outside investor $j$ is entitled to in each

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13To be precise, our main results generalize to the case $\tilde{\gamma}(S) = \gamma(s) + \gamma_2 S$ if the parameters satisfy the inequality

$$\varphi_1 \leq \frac{\varphi_0 \gamma_0 (1 - \varphi_0 (1 + \gamma_2))}{\gamma_1 + 2 \varphi_0 \gamma_0 (1 + \gamma_2)}.$$
state $p$. The set of contracts that we consider comprises all possible functions $r : \mathbb{R}_+ \to \mathbb{R}$.

Since investments are fully contractible, the entrepreneur will not divert funds ex ante. Letting $A$ denote the entrepreneur’s financial investment into the project, and letting $L_j$ denote the funding by investor $j$ and $\sum_j L_j = L$ the total outside funding, the total investment level is thus $I = L + A$.

### 2.5 Timing

The basic version of the model has two main stages: The funding stage and the repayment stage. In between these two stages, the state $p$ realizes (Figure 1).

\[
\begin{align*}
t = 0 & \quad \text{The entrepreneur finances an investment project ($I$) with a combination of own resources ($A$) and outside financing ($L$).} \\
t = 1 & \quad \text{The state $p$ realizes.} \\
t = 2 & \quad \text{The entrepreneur decides on repayment and diversion.}
\end{align*}
\]

Figure 1: Timing

Intuitively, financial constraints are only binding if the project is not too profitable, the entrepreneur is not too wealthy, and enforcement is not too strong. In particular, it does not pay to divert any money if the expected diversion payoff $(1 - \bar{\varphi}(s))pq(I)$ is smaller than the expected penalty $\bar{\varphi}(s)\bar{\gamma}(s)$ for all feasible $s$.$^{14}$ Using the definitions of $\varphi$ and $\gamma$, we may define the threshold state given by the equation $(1 - \bar{\varphi}(1))pq(I) = \bar{\varphi}(1)\bar{\gamma}(1)$ as

\[
\hat{p}(I) = \frac{\gamma}{q(I)} \frac{\varphi}{1 - \varphi},
\]

and make the assumption

\[
I^{f^b} - \omega > q(I^{f^b}) \left[ \bar{p} - (1 - \varphi) \int_{\hat{p}(I^{f^b})}^{\infty} (p - \hat{p}(I^{f^b}))f(p)dp \right]. \quad (1)
\]

$^{14}$We will later show that the entrepreneur invests all own funds, i.e., $\omega = A$ and hence the diversion payoff does not involve funds kept outside the project.
The assumption will imply that the entrepreneur is financially constrained. The left-hand side is the external funding that is required for first-best investment, and (as will become clear) the right-hand side is the expected total return minus the entrepreneur’s expected earnings. Deferring a detailed interpretation, we note that if $\varphi = 1$, the right-hand side is equal to the expected gross return, $pq(I^{fb})$. Therefore, $\varphi < 1$ is a necessary but not sufficient condition for the entrepreneur to be financially constrained.

3 Analysing the basic model

We already know that partial diversion is sub-optimal. Therefore, the entrepreneur’s problem is to propose an investment level, $I$, and a set of contracts, where each contract consists of a pair $(L_j, r_j(p))$, together with a level of self-investment $A$, and a state contingent diversion probability $\Delta(p)$, so as to maximize her expected payoff,

$$U = E_p[(1 - \Delta(p))(pq(I) + \omega - A - \sum_j r_j(p)) + \Delta(p)((1 - \varphi)(pq(I) + \omega - A) - \varphi\gamma)],$$

subject to the feasibility constraint,

$$I \leq A + \sum_j L_j,$$

the investors’ participation constraints,

$$L_j \leq E_p[r_j(p)],$$

and the ex ante and ex post wealth constraints of the entrepreneur and the investors respectively,

$$A \leq \omega,$$

$$L_j \leq \omega_j,$$

$$pq(I) + \omega - A - \sum_j r_j(p) \geq 0 \text{ for all } p,$$

$$\omega_j - L_j + r_j(p) \geq 0 \text{ for all } p.$$

The problem’s solution is also potentially constrained by the entrepreneur’s optimal diversion choice, which we already know to be either zero diversion or full diversion. Thus, the entrepreneur refrains from diversion if and only if

$$pq(I) + \omega - A - \sum_j r_j(p) \geq (1 - \varphi)(pq(I) + \omega - A) - \varphi\gamma.$$
A key step in the analysis is to observe that, because of the utility loss \( \gamma \), diversion is inefficient and never occurs under optimal financial contracting.\(^{15}\)

**Lemma 1** Any set of financial contracts that induces a financially constrained entrepreneur to divert is not optimal.

**Proof.** See Appendix. \( \square \)

Provided that external funding is constrained below the first-best level, it follows that the entrepreneur will maximize external funding subject to all the constraints listed above. Indeed, for there to be a funding shortage, the no-diversion constraint

\[
pq(I) + \omega - A - \sum_j r_j(p) \geq (1 - \varphi)(pq(I) + \omega - A) - \varphi \gamma
\]

must be binding in some states. (If the condition were always slack, the entrepreneur could ask for some additional funding that would generate a positive expected net return after repayment.) Since the no-diversion constraint is binding in some states, the entrepreneur optimally invests all own liquid wealth in the project: Suppose to the contrary that \( \omega > A \). Any decrease in \( \omega - A \) (increase in \( A \)) can be matched by an identical decrease in external funding \( L \), and therefore by corresponding reduction in total repayment obligations \( r(p) = \sum_j r_j(p) \) in all states \( p \). Such a move from external to internal funding keeps the left hand side of the incentive constraint constant while strictly decreasing the right hand side. All other constraints that were previously satisfied, remain satisfied. Thus, if the incentive constraint sometimes binds, the additional internal investment allows total funding to increase.

Likewise, in order to maximize external funding, the entrepreneur will always increase expected repayment until the no-diversion constraint binds, except when already constrained by the ex post wealth constraint. Denote the constrained optimal investment level \( I^* \). How should the financial contracts be designed in order to raise \( I^* - \omega \) from external investors? We have already noted that the entrepreneur is never tempted to divert when the realized state is low enough, that is when \( p < \hat{p}(I^*) \). Consequently, the entrepreneur should pledge all returns in these states, yielding a total repayment \( r(p) = pq(I^*) \). When \( p \geq \hat{p} \), the no-diversion constraint

\(^{15}\)In order to have diversion occurring with positive probability in equilibrium, we could have assumed that there is some uncertainty about \( \gamma \). For example, suppose that everything is as described above, except with a small probability some entrepreneurs will privately learn that \( \gamma = 0 \) after making their investment. If the probability is small enough, it is better to have such entrepreneurs divert in equilibrium rather than offering a contract that prevents diversion completely.
is binding. Using Equation (2) to solve for the total repayment in these high states, we have $r(p) = \varphi (pq(I^*) + \gamma)$. Given the repayment schedule, the maximum external funding $L$, and hence the investment level $I^*$, are obtained by integrating over $p$ to get

$$L = I^* - \omega = \int_{0}^{\hat{p}(I^*)} pq(I^*) f(p) dp + \int_{\hat{p}(I^*)}^{\infty} \varphi (pq(I^*) + \gamma) f(p) dp,$$

which can be rewritten as

$$L = I^* - \omega = q(I^*) \left[ p - (1 - \varphi) \int_{\hat{p}(I^*)}^{\infty} (p - \hat{p}(I^*)) f(p) dp \right]. \tag{3}$$

The former expression says that all surplus can be pledged for states that are worse than $\hat{p}(I^*)$; in better states, the entrepreneur can pledge $\varphi \gamma$ plus a constant fraction of the return. Equation (3) makes the equivalent statement that maximum external funding equals the expected gross surplus less the surplus that must, in expectation, be given to the entrepreneur in order to prevent diversion.\footnote{The linearity of the contract depends on our assumption that the entrepreneur is risk neutral. With a risk-averse entrepreneur, $r(p)$ would typically be strictly convex. Conversely, the entrepreneur’s payout would be then concave over some range. In reality, concave compensation of entrepreneurs has the drawback that insurance contracts become attractive. If we were to expand the entrepreneur’s set of actions to allow stabilization of the firm’s revenues across states, a linear contract might become preferable even in the case of a risk averse entrepreneur, despite the no-diversion constraint then being slack in some states (Diamond (1998)).}

Although our assumptions guarantee that the first-best investment level $I^{fb}$ is defined by a unique solution to a first-order equation, these assumptions do not guarantee that Equation (3) produces a unique solution for $I^*$. The investment level is nonetheless uniquely determined: Since no funds are diverted in equilibrium, and external investors are only compensated according to the entrepreneur’s own cost of funds, the entrepreneur prefers the highest solution. Our main proposition summarizes.

**Proposition 1** Optimal financial contracting yields a total investment level described by the
largest solution to the equation

\[ I^* = \omega + q(I^*) \left[ \bar{p} - (1 - \varphi) \int_{\bar{p}(I^*)}^{\infty} (p - \hat{p}(I^*)) f(p) dp \right], \]  

(4)

a total level of external funding,

\[ L = I^* - \omega, \]

and a total state-contingent repayment,

\[ r^*(p) = \begin{cases} 
    pq(I^*) & \text{if } p < \hat{p}(I^*); \\
    \varphi (pq(I^*) + \gamma) & \text{if } p \geq \hat{p}(I^*). 
\end{cases} \]  

(5)

The optimal repayment schedule is piecewise linear and corresponds exactly to a combination of straight debt and common stock. The straight debt contract is a claim on a fixed amount that is paid out in all states, except if the total return falls short of covering the claim. In the latter case, all available funds are paid out. Common stock is claim on a constant fraction of the surplus that remains after fixed claims are covered. The payouts are illustrated in Figure 2.
Let

\[ L_D = q(I^*) \int_0^{\hat{p}(I^*)} pf(p)dp + \frac{\varphi \gamma}{(1 - \varphi)} (1 - F(\hat{p}(I^*))) \]  

(6)

denote the market value of the external investors’ debt claims and let

\[ L_E = q(I^*) \left[ \varphi \int_{\hat{p}(I^*)}^{\infty} (p - \hat{p}(I^*))f(p)dp \right] \]

(7)

denote the market value of outside investors’ equity claims. Note that \( L_D + L_E = L = I^* - \omega \).

We are now ready to discuss how variation in the underlying parameters affect the volume and composition of external funding. Since the comparative static analysis is fairly mechanical, we relegate most of it to the Appendix.
3.1 The volume of external funding

As one would expect, both the availability of internal funds and the quality of legal institutions facilitate external funding.

**Proposition 2** Total external funding $L$, as well as total debt $L_D$, are increasing in the apprehension probability $\varphi$, the severity of non-pecuniary punishment $\gamma$, and in the entrepreneur’s wealth $\omega$. The effect of legal parameters on outside equity is ambiguous.

**Proof.** See Appendix.

Intuitively, more efficient legal enforcement reduces the entrepreneur’s temptation to divert returns. Since the entrepreneur can credibly pledge a greater fraction of returns to external investors, the investors are willing to provide more funds when investor protection is better. Of course, this finding corresponds to a core belief in the law and finance literature, a belief that is by now well documented empirically (La Porta et al. (1997,1998), and La Porta et al. (2008)).

The reason why external funding increases when the entrepreneur becomes more wealthy is that the added wealth will be invested in the project, which thereby generates higher returns in all states. Keeping the nominal payout to investors constant, the temptation to divert diminishes. As the incentive constraint becomes slack, the entrepreneur can raise more external funds.

The impact of legal institutions on debt is immediate. An increase in $\varphi$ or $\gamma$ both increases the face value of debt $\varphi \gamma / (1 - \varphi)$ and the investment level, the investment increase making it more likely that the debt will be fully repaid. The positive effect of legal institutions on debt is in line with the empirical findings of Djankov et al. (2007).

Perhaps surprisingly, the impact of the legal parameters on the value of outside equity is indeterminate in general. The reasons are twofold. First, for a given investment volume, better enforcement favors debt holders. As we have just seen, this composition effect is part of the reason why debt increases. Second, decreasing returns to scale means that the additional investment demand due to better enforcement may be too small to undo the composition effect.

Even considering the sum of internal and external equity, we have to make additional assumptions (guaranteeing that the entrepreneur is sufficiently heavily financially constrained) in order to definitely predict the positive association between legal enforcement and the value of equity markets documented by Djankov et al. (2008).
Features of the return distribution $F(p)$ also affect funding opportunities. For example, it is immediate that a uniform increase in profitability (a proportional increase in payoffs in all states) allows a higher level of investment. Slightly less obviously, in view of our risk neutrality assumption, an increase in risk always tightens the funding constraint.

**Proposition 3** Let $f_H(p)$ be a mean-preserving spread of $f(p)$. Then, ceteris paribus, external funding $L$ is weakly smaller under $f_H(p)$ than under $f(p)$. The funding reduction is strict if $\int_0^\beta F_H(p)dp > \int_0^\beta F(p)dp$.

**Proof.** See Appendix. ■

Increasing risk entails (weakly) increasing payoffs for the entrepreneur because the entrepreneur’s return is a convex function of the project’s return. Since the entrepreneur gets a larger average fraction of the more risky returns, investors suffer and are less willing to provide funding.\(^{17}\) At one level, this result is entirely familiar. Finance textbooks often argue that risky projects are difficult to fund because creditors bear a disproportionate fraction of the downside risk. However, the textbook argument merely assumes the use of a debt contract, with the entrepreneur owning stock; the argument would be false if external investors held stock only. In our model, the claim structure is endogenous.\(^{18}\)

Since low risk facilitates funding, several independent projects will be easier to fund than several correlated projects. This insight echoes Diamond (1984). As Tirole (2006) (pages 158-163) observes, models in which diversification facilitates funding can potentially explain both the prevalence of conglomeration and the apparent reduction in average returns that are associated with conglomeration (the diversification discount). Tirole’s point is that marginal returns are decreasing, so even if total returns are larger, average returns will be smaller in better funded firms. Our model suggests the complementary view that conglomeration could be optimal even if it is associated with some cost (unmodelled here) if agency problems are large. Stand-alone firms then stand alone precisely because agency problems are smaller, and they are able to save on the costs that conglomerates incur.

\(^{17}\)The reason why the relationship is not always strict is that the whole increase in risk may be concentrated on a subset of states where the entrepreneur’s payoff is either constant (below $\bar{p}$) or linearly increasing (above $\bar{p}$). If so, the distribution of expected payoffs between the entrepreneur and the investors is unaffected.

\(^{18}\)Incidentally, increasing risk does not reduce funding in the influential model of Holmstrom and Tirole (1997). There, the project risk as such is of no consequence for the availability of external funds. (What matters is merely the relative probability of success under good versus bad behavior.)
3.2 The composition of funding

An immediate consequence of our analysis is that the ratio of external equity to internal equity is greater in countries with better legal protection. According to Proposition 1, this ratio is simply $\varphi/(1 - \varphi)$. The positive relationship is consistent with the cross-country evidence of Denis and McConnell (2003).

Empirical work on capital structure often focuses on leverage – that is, on the ratio of the value of debt to the value of total assets.\(^{19}\) In principle, it matters whether one considers book leverage,

$$l = \frac{L_D}{\omega + L_D + L_E},$$

or market leverage

$$l_M = \frac{L_D}{\bar{q}(I)},$$

but as it turns out, our comparative static results are insensitive to this distinction. Since book leverage is perhaps the most commonly used measure in empirical studies, we focus on this measure here.

**Proposition 4** Leverage $l$ is decreasing in the entrepreneur’s wealth $\omega$.

**Proof.** See Appendix. ■

While an increase in the entrepreneur’s wealth, $\omega$, facilitates external funding (both $L_D$ and $L_E$ increase), decreasing returns to scale in production imply that external funds increase less than proportionally. This result may help to explain why leverage tends to be declining in past profitability (Harris and Raviv (1991); Rajan and Zingales (1995)). Past profits raise the amount of internal equity as long as most earnings are retained rather than paid out. Note how our explanation for this correlation differs from the pecking-order story of Myers and Majluf (1984). Their view is that an entrepreneur with a sound project prefers to use internal funds, because adverse selection makes external funds too costly. Our view is that external funds tend to be fairly priced, but that moral hazard makes external funds too scarce. Additional internal funds relax the external funding constraint, but the availability of external funds increases less

\(^{19}\)In empirical work, it is not trivial to decide how to define leverage, as noted by Rajan and Zingales (1995) (page 1427-1429), but in our simple model it is uncomplicated.
than proportionally.20

Next, let us consider the impact of project riskiness on leverage. Our model replicates the conventional wisdom (Harris and Raviv (1991)) that higher return volatility is associated with smaller leverage.

**Proposition 5** Let \( f_H(p) \) be a mean-preserving spread of \( f(p) \). Then, ceteris paribus, leverage \( l \) is weakly smaller under \( f_H(p) \) than under \( f(p) \). The leverage reduction is strict if \( \int_0^\beta F_H(p)dp > \int_0^\beta F(p)dp \).

**Proof.** See Appendix. ■

The intuition is straightforward. For a fixed level of investment, an increase in risk increases the value of equity and decreases the value of debt. Thus, the only way in which leverage could go up is if investment were to go up. However, as we know (Proposition 3) investment goes down, further accentuating the drop in leverage. Note that our argument does not invoke bankruptcy costs, which is the traditional explanation for the negative impact of risk on leverage.

The model also provides a simple explanation for why the market-to-book ratio \( m = \overline{\eta}q(I^*)/I^* \) is negatively related to leverage, as found by Rajan and Zingales (1995) among others: Consider a multiplicative shift in productivity, recalling that the parameter \( \eta \) shifts \( q(I) \).21 (We can show that \( m = \overline{\eta}q(I^*)/I^* \) is increasing in \( \eta \).) Since an increase in investment that is caused by a productivity shift rotates up the gross return curve, but does not change the nominal value of full debt repayment, it is immediate from Figure 2 that there is an increase in the set of states in which equity repayments are positive, and the share of equity repayment grows in all states where the share was already positive. Therefore, the proportional increase in the value of repayments on equity, and hence in \( L_E \), are greater than the proportional increase in the value of debt repayments \( L_D \).

**Proposition 6** Leverage \( l \) is a decreasing function of productivity \( \eta \).

---

20The theoretical functional form relationship between wealth and leverage that we derive is typically non-linear and also in other respects quite different from the reduced forms studied in the empirical literature. There, it is common to control for firm size, for example through the inclusion of sales as a regressor. Such a correction is appropriate in our framework if size is driven primarily by factors such as diversification; if size is instead primarily driven by insider wealth, it is an endogenous variable that does not belong on the right hand side of the regression.

21Observe that we cannot change \( \overline{\eta} \) without simultaneously changing the whole distribution \( f \). The model does not provide clear-cut results regarding the effects of a spread-preserving change in the mean, because the payoffs to the various claims are driven by other features of the spread than those that are captured by the variance.
Proof. See Appendix. ■

To put it simply, debt is a preferred instrument for small investments, since the entrepreneur is not tempted to divert small amounts of money. As investments grow, the face value of debt stays constant, and it becomes increasingly likely that there is a surplus after debt is repaid. A fraction of this surplus can be pledged to investors through the issuance of stocks. Thus, external equity becomes an increasingly important source of funds as the project’s prospects improve. Even if there is no increase in the supply of internal equity, the force is strong enough to entail a reduction in leverage.

Note that our intuition for the negative relationship between growth potential and leverage is similar to the pecking-order argument that equity issuance is a last resort after debt issuance has been exhausted, to be used primarily when the value of investment is large, except our model suggests that the two instruments should be used in tandem; ideally, every dollar of increased investment ought to be funded through an increase in debt as well as equity.

Proposition 7 Leverage \( l \) is increasing in severity of punishment \( \gamma \).

Proof. See Appendix. ■

The severity of punishment contributes to raising the face value of debt in our model and even if the increased investment also raises the value of equity, the value of debt increases by more.

On the other hand, the impact on leverage from better enforcement (higher \( \varphi \)) is ambiguous. The source of ambiguity is that improved quality of enforcement give outside equity owners a larger share of project returns after the debt is repaid, and this positive effect on equity financing may dominate the effect on debt financing.\(^{22}\)

4 Extensions

While the simple model allows us to address a wide range of regularities, it has limitations. For example, it gives the impression that the face value of debt is independent of firm characteristics (unless there are entrepreneur-specific or project specific elements to \( \varphi \) or \( \gamma \)). Let us therefore consider some extensions.

\(^{22}\)Perhaps as a reflection of the model’s ambiguity, the empirical literature has not yet reached a consensus regarding the relationship between leverage and enforcement quality. Recently, de Jong et al. (2007) conclude that leverage is indeed higher in countries with stronger enforcement, but a similar study by Fan et al. (2006) concludes that leverage is lower in countries with a better legal environment.
4.1 Choice of enforcement quality

There are several ways in which entrepreneurs can affect the enforcement quality. For example, the form of incorporation, the choice of auditor, and whether or not to list the firm on one or several stock exchanges all affect the financial reporting requirements and thereby the ease with which resources may be diverted.

Let us return to the one-period model, and suppose that $\varphi$ can be affected at a cost $c(\varphi)$. Let $\varphi$ denote the baseline level of enforcement available to all firms. The problem is then to choose $\varphi$ to maximize $\tilde{p} \eta q(I^*(\varphi)) - I^*(\varphi) - c(\varphi)$. We assume that $c(\varphi)$ is increasing, twice differentiable and a convex function satisfying $c(\varphi) = 0$ and $\lim_{\varphi \to 1} c(\varphi) = \infty$. The optimal apprehension probability is thus given by the first-order condition

$$\frac{\partial I^*}{\partial \varphi} (\tilde{p} \eta q(I^*(\varphi)) - 1) - \frac{\partial c}{\partial \varphi} = 0. \tag{8}$$

Differentiation of this condition provides a characterization of how the apprehension probability depends on the fixed parameters. For example, we show in the Appendix that $\varphi$ is an increasing function of productivity $\eta$ under constant returns to scale (and more generally when $-q''(I^*(\varphi))$ is not too large). Intuitively, improved enforcement quality is worth more when the return to investment is greater. This finding is consistent with the cross-listing evidence that firms having better growth opportunities are more inclined to cross-list (see Doidge et al. (2004) and Pagano et al. (2002)).

Note that the previous model by Durnev and Kim (2005) can address the same evidence. Durnev and Kim also offer a more detailed discussion of endogenous corporate governance as well as an empirical cross-country analysis.

4.2 Active investors

Monitoring by external investors offers another important mechanism for deterring entrepreneurs’ self-dealing; see, e.g., Gomes and Novaes (2006) and the references therein. In reality, investor activism is difficult to describe ex ante and the right mix of (potentially unobservable) actions may depend on many unforeseen contingencies. We therefore assume that investor activism is not directly contractible. However, the structure of financial claims clearly affects incentives for being active, and this is the mechanism that we explore here. More precisely, we provide conditions under which it is optimal that active investors hold common stock – and strictly suboptimal for active investors to hold debt – without imposing any exogenous constraints on
the set of available financial contracts. The basic intuition is that debt holders have relatively less to gain by improving enforcement quality, since they receive all the returns in the bad states when the entrepreneur is never tempted to divert anyway.\textsuperscript{23}

Specifically, consider the following extension of the one-period model. After the project is financed but before the outcome of the project is observable, any investor may take costly actions that increase $\varphi$. Let $c(\varphi)$ be the investor’s private costs associated with apprehension probability $\varphi$, and let this function have the same properties as in the previous subsection.

For simplicity, we assume that $\varphi$ is determined by the maximum of the investors’ actions. That is, monitoring by different investors are perfect substitutes. From the literature on voluntary provision of public goods (e.g., Bergstrom et al. (1986)) it follows that, given sufficient heterogeneity, only the investor with the largest stake will monitor in a non-cooperative equilibrium.

The main change to the model is that the entrepreneur not only proposes contracts to investors, but also nominates one investor to take a large stake in the firm and to monitor actively, resembling the nomination of a Chairman of the Board.\textsuperscript{24} We call this investor the active investor, although activity of course is an equilibrium phenomenon rather than an exogenous investor trait. Although the model is very stylized, it captures a well-known mechanism behind the concentration of outside ownership and the relation between ownership concentration and monitoring: Only outsiders with a substantial financial interest will engage in costly monitoring, and the need for monitoring in turn provides a reason for the concentration of financial claims.

The active investor must be given incentives both to buy the proposed claims (participation constraint) and to take costly actions in order to improve the enforcement quality (incentive compatibility constraint). All other constraints in the basic model (Section 2) take the apprehension probability as given and remain the same as before. Hence the structure of total repayment to external investors will be the same as in the basic model (see equation (5)). For simplicity, we abstract from the possibility of inflicting non-monetary punishment on the active investor.

Let $\varphi^*$ denote the apprehension probability that the entrepreneur’s contract offer seeks to

\textsuperscript{23}Although this intuition seems to suggest that it may be optimal to let passive investors hold non-monotonic claims that only pays out in low states, it turns out that such claims do not offer an improvement over standard debt and common stock as long as active investors have too little wealth to hold all the common stock.

\textsuperscript{24}By letting the entrepreneur propose the board member, we remove all bargaining power the active investor may have.
induce. Let \( r_a(p, \Delta) \) denote the state-contingent repayment promised to the active investor, where \( \Delta \) is an indicator variable taking the value 1 if the entrepreneur attempts diversion and 0 otherwise. Let \( A_a \) denote the active investor’s investment.

Given an optimal contract, we know that the entrepreneur is indifferent between diverting and not diverting in all states above a threshold. Thus, if monitoring falls short of the level that the entrepreneur’s contract is designed for, there will be a diversion attempt whenever the state is above the implied threshold level. That is,

\[
\Delta = \begin{cases} 
1 & \text{if } \varphi < \varphi^* \text{ and } p > \hat{p}(\varphi, I); \\
0 & \text{otherwise.} 
\end{cases}
\]

We have already shown that it is always better to reduce the repayment to investors than to trigger a diversion attempts by the entrepreneur (Lemma 1). The entrepreneur has no desire to leave unnecessary rents to the active investor. The optimal repayment \( r_a^*(p, \Delta(\varphi^*, p)) \) thus minimizes the payout to the active investor

\[
\int_0^\infty r_a(p, \Delta(\varphi^*, p)) f(p) dp,
\]

subject to the entrepreneur’s no-diversion constraint

\( \Delta(\varphi^*, p) = 0, \)

the feasibility constraint

\( r_a(p, \Delta(\varphi^*, p)) \leq r^*(p), \)

the active investor’s incentive constraint

\( \varphi^* \in \arg\max_{\varphi} \left[ \int_0^\infty r_a(p, \Delta(\varphi, p)) f(p) dp - c(\varphi) \right], \)

the active investor’s participation constraint

\( \int_0^\infty r_a(p, \Delta(\varphi^*, p)) f(p) dp - c(\varphi^*) \geq A_a, \)

and the active investor’s wealth constraint

\( A_a \leq \omega_a. \)
Four implications are immediate. First, it is clear from the active investor’s incentive constraint that the active investor should receive no payout if the entrepreneur diverts; penalizing \( \Delta = 1 \) is the cheapest way to provide monitoring incentives. The interpretation is that the largest investor ought to have fiduciary duties, with severe remedies in case of breach.\(^{25}\)

Second, the participation constraint implies that the active investor is compensated for his private efforts by receiving more state-contingent repayment than justified by the monetary investment \( A_a \) alone. (For evidence that large shareholders are offered shares at a discount, see for example Hertzel and Smith (1993).)

Third, it is weakly suboptimal to have positive payouts in low states, that is to let \( r_a(p, \Delta(\varphi, p)) > 0 \) for \( p < \tilde{p}(\varphi^*) \). The reason is that the entrepreneur will not divert in these low states even if the active investor reduces monitoring to bring the apprehension probability marginally below \( \varphi^* \). Thus, on the margin, these low-state payoffs have no impact on the active investor’s willingness to monitor. Unless the feasibility constraint binds, all payouts should therefore be concentrated to the states \( p \geq \tilde{p}(\varphi^*) \) in which a reduction in monitoring would tempt the entrepreneur divert.

Fourth, it is never necessary to give a rent to the active investor. Regardless of which monitoring level is optimal, the incentive compatible payout should exactly satisfy the participation constraint. Hence, we know that \( \varphi^* \) ought to solve

\[
\tilde{p}(q'(I^*(\varphi)) - 1) \frac{dI^*}{d\varphi} = c'(\varphi).
\]

Beyond these characterizations, the form of the active investor’s payout is indeterminate. Since the entrepreneur’s incentive to deviate is the same in all states above a cut-off level \( \tilde{p} \), it is only the expected payout conditionally on the state exceeding \( \tilde{p} \) that matters for the monitoring incentive.

However, we seek to minimize the number of different claims. One claim for all external investors, convertible debt, only suffices if it gives the active investor a sufficient incentive to monitor. Let

\[
\sigma = \frac{A_a + c(\varphi^*)}{L(I^*(\varphi^*))}
\]

\(^{25}\)Here is one case in which conclusions might be different under more realistic informational assumptions. If investors are unsure about the entrepreneur’s probity, draconian punishment of active investors for sins perpetrated by the entrepreneur could inefficiently reduce overall willingness to invest.
denote the active investor’s share of expected payouts. Since only the share of payouts in high states matter, with a single claim we can write the incentive compatibility condition as \( c(\varphi^*) \leq \sigma L_E(I^*(\varphi^*)) \). Inserting for \( \sigma \) and using the identity \( L = L_E + L_D \), we conclude that convertible debt provides adequate monitoring incentives only if

\[
c(\varphi^*) \leq A_n \frac{L_E(I^*(\varphi^*))}{L_D(I^*(\varphi^*))}.
\]

That is, convertible debt is only an optimal claim for active investors whose investment is sufficiently large relative to the cost of monitoring. If this condition is violated, for example because the active investor’s wealth is small, the active investor must hold a claim that is different from those of passive investors: There must then be at least two claims – one that pays out relatively more in low states and another that pays out relatively more in high states.

Since the entrepreneur should hold common stock, we thus seek conditions under which common stock and plain debt is optimal. The only reason not to rely on plain debt is that the payout to debtholders in high states reduces the payout that can be pledged to the active investor in these states. Thus, in principle, it could be attractive to have a claim with a non-monotonic payout. However, even with plain debt the availability of high state payouts is sufficient to incentivize the active investor as long as \( c(\varphi^*) \leq L_E(I^*(\varphi^*)) \).

**Proposition 8** Suppose \( c(\varphi^*)/L_E(I^*(\varphi^*)) \in (\omega_n/L_D(I^*(\varphi^*)), 1) \). Then, the unique smallest set of optimal securities is plain debt and common stock.

Since the entrepreneur holds common stock, and outside investors must hold at least two separate claims, any other optimal security structure involves at least three different claims. Active investors holds common stock only or a combination of common stock and some debt. Note that it is by no means necessary that the active investor holds all the firms stock in order to have optimal monitoring incentives in our model.

Needless to say, our analysis here provides only the first steps towards understanding the contracting with active investors. Extensions of the model might address issues such as restrictions on active investors’ securities trading and the problem of collusion between entrepreneurs and active investors (?).

### 4.3 Asset exposure

So far, we have assumed that all the project’s gross returns are equally exposed to diversion. However, many assets are naturally protected from diversion (difficult to move) or can be
protected by contract. For example, by signing a contract saying that the entrepreneur cannot sell or collateralize buildings without investors’ explicit consent, diversion of value tied up in buildings can be largely prevented, as the entrepreneur can no longer find the necessary buyer or lender to sustain the diversion attempt. In some cases, the entrepreneur can thus be costlessly controlled; in other cases, controlling the manager may be more costly – but if financial constraints are severe, control costs could still be outweighed by the increased investment volume.

Assume that a fraction \( (1 - \lambda) \) of project returns is protected from diversion, or equivalently that a fraction \( \lambda \) is exposed to it. Since the benefits from successful diversion is reduced from \( pq(I) \) to \( \lambda pq(I) \) while the utility loss from an unsuccessful diversion attempt, \( \gamma \), remains the same, the entrepreneur is less prone to divert project returns. Apart from the reduction of the entrepreneur’s expected gain from diversion, the structure of the entrepreneur’s financing problem is identical to the one analyzed in Section 3. Consequently, the the shape of the financial claims remains the same. However, since only a fraction of returns are exposed to diversion, the project’s gross returns can be larger before the entrepreneur is tempted to divert. Conversely, the entrepreneur can raise less debt financing as exposure increases.

**Proposition 9**  
As the exposure \( \lambda \) increases, the entrepreneur will raise less outside funds, especially less debt.

**Proof.** See Appendix. ■

The value of outside equity may go either down or up. On the one hand, increased exposure implies that the entrepreneur can pledge a smaller fraction of residual returns after debt payments are made; on the other hand, the reduction in the face value of debt means that there may be equity payout in more states of the world.

Asset exposure also interacts in non-trivial ways with the other parameters of the model. However, we relegate the study of such interaction effects to future work.

\(^{26}\)The work of Almeida and Campello (2007) is suggestive of the interaction effects that we have in mind. They find that, conditionally on being financially constrained, firms’ investment–cash flow sensitivity is greater in firms with more tangible assets (see Gan (2007) for a similar result). In our model, the analogous exercise is to consider the sign of the cross-derivative \( \frac{d^2L}{d\omega d\lambda} \) (which should be negative, as \( \lambda \) is the opposite of tangibility).
4.4 Outside collateral

Outside collateral, such as homes and cars, play an intriguing role in the entrepreneur’s maximization problem. The liquidation of such assets is costly, as the revenue falls short of the value that the entrepreneur attaches to them. Indeed, the inefficiencies associated with liquidation explain precisely why the entrepreneur may not want to sell the assets and invest the proceeds in the project to begin with. As we shall now see, various parameter constellations give rise to (i) cases in which it is optimal to liquidate assets upfront, (ii) cases in which assets instead are used as collateral for debt— but not for equity—and (iii) cases in which assets are neither sold nor collateralized.

Consider a situation in which the entrepreneur owns outside assets, such as family homes and heirlooms, that are more valuable to her than to others. Let $v$ denote the entrepreneur’s valuation of the outside asset, and let $\beta v$ be the market price (where $\beta \leq 1$). Due to financial constraints, it may nonetheless be optimal for some of the entrepreneur’s assets to be sold ex ante to increase the entrepreneur’s self-investment. Let $(1 - K)$ denote the fraction of outside assets sold ex ante. Unsold outside assets might be transferred to the investors conditional on the outcome of the project. Let $1 - x(p)$ denote the conditional collateralization rate, that is, the fraction of (remaining) outside assets that are transferred to investors in state $p$, and let $\bar{x} = E[x(p)]$ be the expected fraction of assets not transferred to investors ex post.

Since project returns and outside assets might be differentially exposed to diversion, we let $(1 - \theta)$ denote the probability that an attempt to divert outside assets will succeed. As before, diversion of project returns succeeds with probability $(1 - \varphi)$. Collateralization may also affect an outside asset’s exposure to diversion. For now we assume that it does not— so the entrepreneur may fully divert a collateralized assets. (If collateralized assets are more difficult to divert, there will be less upfront asset sales and more use of collateral.)

As before, it is always optimal to have the entrepreneur invest all her cash, and never optimal to let her divert project returns in equilibrium (proof omitted). To simplify the notation, we assume that there are many affluent investors who are able to buy all outside claims on the firm. The entrepreneur thus maximizes her expected payoff,

$$U = E_p[pq(I) - r(p) + x(p)Kv],$$

by choosing a repayment profile, $r(p)$, an amount of outside assets sold ex ante, $(1 - K)$, and
a collateralization rate, 1 - \( x(p) \), subject to the no-diversion constraint

\[
pq(I) - r(p) + x(p)Kv \geq (1 - \varphi) pq(I) + (1 - \theta) Kv - \varphi\gamma,
\]

the investor’s participation constraint

\[
L \leq E_p [r(p)] + E_p [(1 - x(p)) \beta Kv],
\]

and the feasibility constraints

\[
L + \omega + (1 - K)\beta v \geq I, \\
pq(I) + (1 - x(p)) \beta Kv - r(p) \geq 0 \quad \text{for all } p, \\
0 \leq K \leq 1, \\
0 \leq x(p) \leq 1 \quad \text{for all } p.
\]

Inspection of the no-diversion constraint reveals that it is never optimal to transfer the entitlement to outside assets in states where the entrepreneur receives a positive cash flow from the project. It is better to reduce the entrepreneur’s cash flow and simultaneously reduce the inefficient transfer of outside assets to investors. By implication, only debt holders’ claims are ever collateralized.

Comparing the maximization problem with outside assets to the analogous problem without outside assets (Section 3), they have a closely similar structure. It remains optimal to pay investors all project returns for low \( p \) values (the penalty for diversion is considerable even for a small amount of diverted funds). More precisely, for \( p \leq \hat{p}(I) \equiv (\varphi\gamma + \theta Kv) / (q(I)(1 - \varphi)) \) all project returns are paid to investors (debt holders).\(^27\) We have already showed that it is optimal to let the entrepreneur keep outside assets when she receives a cash return on the investment, i.e., \( x(p) = 1 \) if \( p > \hat{p}(I) \). Since \( x(p) = 1 \) whenever the no-diversion constraint binds, it follows that the repayment profile \( r(p) \) remains linear and that the optimal contract can still be implemented through a combination of debt and and common stock.

\(^{27}\)If \( \theta = 0 \) or \( K = 0 \), then \( \hat{p} = \hat{p} \). In all other cases, the existence of outside assets enables the entrepreneur to promise higher repayment to debt holders and raise more debt financing.
Let
\[ M(I^*(\bar{x}, K)) = \frac{\bar{p}q'(I^*(\bar{x}, K)) - 1}{1 - q'(I^*(\bar{x}, K))\left(\bar{p} - (1 - \varphi)\int_{\rho}^\infty pf(p)dp\right)} \] (10)
denote the entrepreneur’s expected profit from a marginal increase in self-investment, taking into account the additional outside funding. The numerator is the marginal return to investment and the denominator an inverse investment multiplier due to increased funding.

**Proposition 10** A financially constrained entrepreneur pledges unsold outside assets as collateral to debt holders ($\bar{x} < 1$) if the assets’ liquidity satisfies the condition
\[ \beta > \beta \equiv \frac{1}{M(I^*(1, 1))}. \]
The entrepreneur sells a fraction of outside assets ($K < 1$) if the assets’ liquidity satisfies the condition
\[ \beta > \bar{\beta} \equiv \frac{1}{M(I^*(x_{\min}, 1))} + \frac{\theta \left(1 - \bar{F}(\bar{p})\right)}{x_{\min}} \geq \beta, \] (11)
where $x_{\min} \in (0, 1)$ is defined in the Appendix.

**Proof.** See Appendix. □

We see that the entrepreneur prefers to liquidate assets ex post rather than ex ante. Since outside assets can be confiscated if the entrepreneur make a diversion attempt, outside assets serve as a disciplining device which prevents the entrepreneur from diversion and thereby enables her to credibly promise a higher share of the return to investors. Hence, outside assets facilitates increased financing, particularly so if the outside assets are difficult to divert. However, since outside assets pledged as collateral are transferred only if $p < \hat{p}$, the expected transfer of these assets, $(1 - \bar{x})$ will always be strictly lower than 1. Letting $x_{\min}$ denote the highest expected fraction of outside assets that the entrepreneur can promise to transfer to investors (see Appendix for a precise definition), the use of outside assets is illustrated in Figure 3.
Figure 3: Use of outside assets.
The figure describes what fraction of outside assets will be sold upfront, $1 - K^*$, and
the expected fraction of (unsold) outside assets that will be liquidated as collateral,
$1 - \bar{x}^*$, as functions of the liquidity of outside assets, $\beta$.

Under decreasing returns to scale, it is suboptimal to pledge the maximum level of collateral
if $\beta$ exceeds the cut-off $\bar{\beta}$ only slightly. Similarly, it is not optimal to sell all outside assets ex
ante if $\beta$ only slightly exceeds the cut-off $\bar{\beta}$.28

If the entrepreneur holds different types of outside assets, Proposition 10 implies that she
should sell assets that are liquid and exposed to diversion, and keep assets that have low sales
values and are difficult to divert. However, some assets may be valuable both when kept and
when sold. For instance, the entrepreneur might own an office building in a city centre. The
building may have a high market value (high $\beta$) and be difficult to divert for the entrepreneur
(high $\theta$). In this case, it might be better to keep the building, using it to raise more external
financing, and instead sell assets with lower liquidity (low $\beta$) that are more exposed to diversion
(low $\theta$).29

28On the other hand, if the technology exhibits constant-returns-to-scale ($q'(I)$ is constant) it follows that
the benefits from raising an additional unit of capital is constant. In this case, the entrepreneur would either
pledge the maximum level of collateral or nothing. Similarly for ex ante sale of assets, the entrepreneur would
either sell all outside assets or none.

29Note that while pledging outside assets as collateral always increases external funding and thereby the size
of the project, this may not be the outcome if assets are sold ex ante. Although ex ante sale of assets make
Proposition 10 is related to Myers and Rajan (1998), who also investigate how a firm’s debt capacity is affected by assets’ liquidity and their exposure to management misbehavior. Like us, they show that if assets become very liquid or easy to divert, the firm’s capacity to raise funds may decline. We build on their analysis by studying outside assets that are non-essential for the success of the particular project and by examining how liquidity of assets drives the optimal combination of equity and debt.

4.5 Technology choice

A weakness of the model that we have considered so far is that it fails to account for cross-country differences in average returns. As documented by La Porta et al. (2002), firms in better legal regimes produce higher average returns on capital. As it stands, our model implies the opposite. Since firms in worse legal regimes are more heavily credit rationed, their total value is small due to limited investment, but their average return on investment is large. In their own theoretical model, La Porta et al assume that production technologies are the same everywhere and yield constant returns to scale. However, their diversion technology is such that entrepreneurs will typically divert some returns in equilibrium. The value of the firm, as measured by the market value of assets held by external investors, is thus decreasing in the amount of diversion – which in turn is decreasing in the extent of investor protection.

We do not deny the possibility that actual diversion, legal or illegal, has a detrimental impact on investors’ average returns. Yet, we believe that another factor may be as important: Financial constraints prevent entrepreneurs from taking advantage of technologies with increasing returns to scale. In order to substantiate our point, suppose that the entrepreneur has the choice between two technologies. One technology, call it technology 1, has constant returns to scale. Let \( q_1 = k_1I \), with \( k_1 > 1 \). The other technology entails a fixed cost, but has lower variable costs, and thus has increasing returns. For example, let \( q_2 = \max\left\{0, k_2(I - \tilde{I})\right\} \), where \( \tilde{I} > 0 \) and \( k_2 > k_1 \). Note that technology 2 is preferable to technology 1 if and only if \( k_2(I - \tilde{I}) > k_1I \), or equivalently, \( I > \tilde{I}k_2/(k_2 - k_1) \equiv \tilde{T} \).

In this example, firms in a country with sufficiently weak institutions will invest less than \( \tilde{T} \) and have an average rate on return on invested capital of \( k_1 - 1 \), whereas firms in countries with sufficiently strong institutions will invest more than \( \tilde{T} \) and have an average return on invested capital more than \( k_1 - 1 \). The entrepreneur able to invest more own funds, this effect might be dwarfed by the reduced ability to attract outside financing.
capital above $k_1 - 1$ (but always below $k_2 - 1$).

### 4.6 Other extensions

It is easy to think about other extensions of the model.

(i) First, like Allen (1981) and DeMarzo and Fishman (2007), we could assume that the project generates returns over many periods. An immediate consequence is that the entrepreneur will be disciplined by delayed payouts. Preliminary investigations suggest that late vesting stock is an optimal instrument for rewarding the entrepreneur under a variety of assumptions concerning the return dynamics. However, the exact shape of financial contracts appears to hinge on the correlation of returns across periods.

(ii) Relatedly, the utility loss from being punished, $\gamma$, is likely to vary across entrepreneurs as well as across time. Suppose for example that the cost of being punished is a reflection of reduced future entrepreneurial opportunities. Then, entrepreneurs with a short horizon or whose productivity is declining will find it hard to raise funds in general and debt in particular. Moreover, as indicated in footnote 15, uncertainty about the future value of $\gamma$ offers a natural way in which to have diversion in equilibrium. Entrepreneurs who face an unusually low $\gamma$ after the investment stage would divert instead of repaying (except if the return is very low), consistently with evidence that diversion increases in bad times (e.g., Johnson et al. (2000)). Uncertainty about $\varphi$ has similar implications.

(iii) We have considered the case in which diversion involves leaving the firm. Another relevant case is when entrepreneurs divert without leaving the firm. In this case the entrepreneur stays on and keeps her claims on the firm unless investors are able to verify in courts that the entrepreneur has diverted funds. If investors can verify that illegal diversion has taken place with a fixed probability smaller than 1, the ensuing analysis yields results closely similar to those we have presented above. The financial contracts remain linear but the compensation to the entrepreneur has to be increased in order to reflect that it is more attractive to make a diversion attempt.

(iv) Our analysis focuses on entrepreneurial firms. However, in many firms the investors rather than the manager decide on the scale of investments. If the manager has little own wealth, an immediate difference between the two regimes is that projects will be smaller under investor control. Since investors give no weight to the manager’s rents, they continue investing only until the investors’ marginal return equals the marginal cost of funds. Under entrepreneur control, on the other hand, investment continues until investors’ average return equals the marginal
cost of funds. Another difference between large corporations and a typical entrepreneurial firm is that the top manager has more circumscribed powers in a large corporation, so that diversion requires collusion with other influential managers.

(v) Perhaps the most urgent next step is to investigate to what extent the model can match data quantitatively. As noted above, the model straightforwardly pins down the ratio between external to internal equity (it is \( \varphi/(1-\varphi) \)), but the debt/equity ratio depends on many parameters, including differences in assets’ vulnerability to diversion and collateralizability. Not all these parameters are available off-the-shelf, so this step is challenging.

5 Conclusion

The paper presents a new model of financial contracting, based on the idea that legal enforcement is imperfectly suited to deter managerial diversion of project returns. We use the model to explain the coexistence of plain debt and common stock, to make predictions about ex ante asset sales and the use of collateral, and to analyze cross-sectional and intertemporal variation in the capital structure of owner-managed firms.

Since the model is analytically tractable, we hope that it will provide a useful vehicle for subsequent research as well.

6 Appendix

We begin by proving that the entrepreneur only considers \( S = 0 \) and \( S = \overline{S} \). Specifically, we derive the conditions under which the choice \( \overline{S} \) dominates all \( S \in (0, \overline{S}) \). Since diversion entails giving up all legal returns, the expected utility associated with diversion is

\[
D = (1 - \overline{\varphi}(S))S - \overline{\varphi}(S)\overline{\gamma}(S).
\]

Defining \( s = S/\overline{S} \), consider the entrepreneur’s problem to maximize \( D \) with respect to \( s \) subject to the feasibility constraint \( S \leq \overline{S} \). A sufficient condition for any solution \( s > 0 \) to be at the corner \( s = 1 \), is that \( \partial D/\partial s > 0 \) for all \( s \leq 1 \). Consider first the case in which \( \varphi(s) = \varphi_0 + \varphi_1 s \) and \( \gamma(s) = \gamma_0 + \gamma_1 s \). We then have

\[
D(s, \overline{S}) = (1 - (\varphi_0 + \varphi_1 s))s\overline{S} - (\varphi_0 + \varphi_1 s)(\gamma_0 + \gamma_1 s).
\]
Thus,

\[ \frac{\partial D(s, \overline{S})}{\partial s} = -2\varphi_1 S + (1 - \varphi_0) \overline{S} - \varphi_1 (\gamma_0 + 2\gamma_1) - \varphi_0 \gamma_1. \]

Since \( \frac{\partial^2 D}{\partial s^2} < 0 \), a sufficient condition for a corner solution at \( s = 1 \) is that \( \frac{\partial D(1, \overline{S})}{\partial s} \geq 0 \), or equivalently that

\[ (1 - (2\varphi_1 + \varphi_0)) \overline{S} - \varphi_1 (\gamma_0 + 2\gamma_1) - \varphi_0 \gamma_1 \geq 0. \]

It is straightforward to check that the coefficient on \( \overline{S} \) is positive by Assumption (iii). Thus, it is easier to have a corner solution the larger is \( \overline{S} \). The smallest level of \( \overline{S} \) that entails any diversion threat is given by the equation \( D(1, \overline{S}) = 0 \), which has the solution

\[ \overline{S} = \frac{(\varphi_0 + \varphi_1)(\gamma_0 + \gamma_1)}{1 - (\varphi_0 + \varphi_1)}. \]

Inserting this value for \( \overline{S} \) into our sufficient condition and solving for \( \varphi_1 \) yields

\[ \varphi_1 \leq \frac{\varphi_0 \gamma_0 (1 - \varphi_0)}{\gamma_1 + 2\varphi_0 \gamma_0}, \]

which is precisely Assumption (iii).

Consider next the remaining cases \( \varphi(s) \geq \varphi_0 + \varphi_1 s \) and \( \gamma(s) \geq \gamma_0 + \gamma_1 s \), with at least one strict inequality when \( s \) lies in some interval of \((0, 1)\) (but with equality when \( s = 1 \)). Note that these functions, whether linear or not, always lie weakly above the corresponding linear functions that end at \( \varphi_0 + \varphi_1 \) and \( \gamma_0 + \gamma_1 \) respectively. That is, they yield weakly lower utility. Since \( s = 1 \) is the dominant choice for the corresponding linear function, it is thus also the dominant choice now.

### 6.1 Proof of Lemma 1

Consider a set of contracts with a total repayment profile \( r^1(p) \) that induces the entrepreneur to divert in some state \( p^d \). Under these contracts, the outside investors’ total expected repayment in state \( p^d \) is \( r^1(p^d) = \varphi \left(p^d q(I) + \omega - A\right) \). Consider next an alternative set of contracts with the same total repayment profile in all states, except in state \( p^d \) where the repayment is \( r^2(p^d) = \varphi(p^d q(I) + \omega - A + \gamma) \). Note that \( r^2(p^d) \) is the largest repayment that does not induce the entrepreneur to divert in state \( p^d \). Moreover, since \( \omega \geq A \) and \( \gamma > 0 \), it follows that \( r^2(p^d) > \)
$r^1(p^d)$. Since the entrepreneur does not have to leave rents to external investors, it is possible to raise more external funds under the alternative set of contracts, and since the original investment is below $I^*$, the expected net return on a small additional investment is positive.

### 6.2 Proof of Proposition 2

Recall first that $I^*$ is the solution to

\[ I = \omega + L(I, \overline{p}), \tag{12} \]

where

\[
L(I, \overline{p}) = q(I) \left[ \overline{p} - (1 - \varphi) \int_{\hat{p}(I)}^\infty (p - \hat{p}(I)) f(p) dp \right],
\]

and

\[
\hat{p}(I) = \frac{\gamma}{q(I)} \frac{\varphi}{1 - \varphi}.
\]

Taking the derivative of $L$ and arranging terms, we have

\[
\frac{\partial L(I, \overline{p})}{\partial I} = q'(I) \left[ \overline{p} - (1 - \varphi) \int_{\hat{p}(I)}^\infty p f(p) dp \right] > 0.
\]

Since $q$ is increasing and the bracketed term is positive, the whole expression is positive. Moreover, since $I^*$ is the largest solution to (12), it follows that

\[
\frac{\partial L(I^*)}{\partial I} < 1.
\]

In order to identify the effect of parameter changes on $I^*$, we totally differentiate (12). Let us first consider a change in the entrepreneur’s wealth, $\omega$. Differentiation with respect to $\omega$ yields

\[
dI^* = d\omega + \frac{\partial L(I^*)}{\partial I} dI^* + \frac{\partial L(I^*)}{\partial \omega} d\omega.
\]
The last term is zero. Thus, we have
\[ \frac{dI^*}{d\omega} = \frac{1}{1 - \partial L(I^*)/\partial I} > 1. \]

Consider next a change in the apprehension probability, \( \varphi \). Differentiation yields
\[ \frac{dI^*}{d\varphi} = \frac{dL}{d\varphi} = \frac{\partial L(I^*)}{1 - \partial L(I^*)/\partial I}. \]

We have already shown that the denominator is positive. The numerator can be computed as
\[ q(I^*) \left[ \int_{\hat{p}(I^*)}^{\infty} (p - \hat{p}(I^*))f(p)dp + \hat{p}(I^*)(1 - F(\hat{p}(I^*)))/\varphi \right], \]
which is also positive. Finally, consider an increase in the penalty parameter \( \gamma \). Differentiation yields
\[ \frac{dI^*}{d\gamma} = \frac{dL}{d\gamma} = \frac{\partial L(I^*)}{1 - \partial L(I^*)/\partial I}. \]

As has been shown, the denominator is positive. The numerator can be computed as \( \varphi(1 - F(\hat{p}(I^*))) \), which is also positive. The numerator is
\[ q(I^*) \left[ \varphi \int_{\hat{p}(I^*)}^{\infty} (p - \hat{p}(I^*))f(p)dp + (1 - F(\hat{p}(I^*))) \frac{\varphi^2}{1 - \varphi} \right], \]
which is again positive.

Using (6) we are now ready to consider the impact of the same four parameters on debt, \( L_D \). Differentiation with respect to \( \omega \) yields
\[ \frac{dL_D}{d\omega} = \frac{\partial L_D \partial I^*}{\partial I \partial \omega} + \frac{\partial L_D}{\partial \omega}, \]
which is positive: We have already proved that \( dI^*/d\omega > 0 \), and it is straightforward to show that
\[ \partial L_D/\partial I = q'(I^*) \int_{\hat{p}(I^*)}^{\infty} pf(p)dp, \]
and that \( \partial L_D/\partial \omega = 0 \).
Analogously, differentiation of $L_D$ with respect to $\varphi$ yields
\[
\frac{dL_D}{d\varphi} = \frac{\partial L_D}{\partial I} \frac{dI^*}{d\varphi} + \frac{\partial L_D}{\partial \varphi},
\]
which is again positive: We have already proved that $dI^*/d\omega > 0$ and that $\partial L_D/\partial I > 0$, and it is easy to check that
\[
\frac{\partial L_D}{\partial \varphi} = \frac{\gamma}{(1 - \varphi)^2} (1 - F(\hat{p})).
\]

Next, differentiate $L_D$ with respect to $\gamma$ to get
\[
\frac{dL_D}{d\gamma} = \frac{\partial L_D}{\partial I} \frac{dI^*}{d\gamma} + \frac{\partial L_D}{\partial \gamma},
\]
which is positive, because $dI^*/d\gamma > 0$ and $\partial L_D/\partial I > 0$ as already shown, and so is
\[
\frac{\partial L_D}{\partial \gamma} = \frac{\varphi}{(1 - \varphi)^2} (1 - F(\hat{p})).
\]

### 6.3 Proof of Propositions 3 and 5

Let the probability distribution $f_H(p)$ represent a mean-preserving spread (MPS) of the initial probability distribution $f(p)$. Formally, for all $x \geq 0$, we have
\[
\int_0^x F_H(p)dp \geq \int_0^x F(p)dp,
\]
with strict inequality for some $p$. It suffices to show that, for any investment level $I$, any MPS improves the expected payoff to the entrepreneur, since the expected payoff to investors must decrease correspondingly. In other words, we want to show that
\[
q(I)(1 - \varphi) \int_{\hat{p}}^{\infty} (p - \hat{p}) f_H(p)dp \geq q(I)(1 - \varphi) \int_{\hat{p}}^{\infty} (p - \hat{p}) f(p)dp. \tag{13}
\]
Observe that
\[
\int_{\hat{p}}^{\infty} (p - \hat{p}) f(p) dp = \int_{0}^{\infty} (p - \hat{p}) f(p) dp - \int_{0}^{\hat{p}} (p - \hat{p}) f(p) dp
\]
\[
= (\bar{p} - \hat{p}) - \int_{0}^{\hat{p}} (p - \hat{p}) f(p) dp
\]
\[
= (\bar{p} - \hat{p}) + \int_{0}^{\hat{p}} F(p) dp,
\]
where the last equality is established through integration by parts (noting that continuity of \(f\) implies that \(F(0) = 0\)).

Deriving the analogous expression for the distribution function \(f_H\), it follows that (13) holds if
\[
\int_{0}^{\bar{p}} F(p) dp \leq \int_{0}^{\hat{p}} F_H(p) dp,
\]
which is a consequence of the definition of \(F_H\). Note finally that if the latter inequality is strict, the payoff to investors is strictly smaller under \(f_H\) than under \(f\).

To prove Proposition 5, recall that the value of outside equity, \(L_E\), is proportional to the value of the entrepreneur’s inside equity. We have just proven that the value of inside equity is increasing in riskiness. With the value of \(L_E\) thus also increasing in the project’s riskiness for any \(I\), and the entrepreneur’s investment staying constant at \(\omega\), the decline in \(I = \omega + L_D + L_E\) must be matched by an even larger decline in \(L_D\). Consequently, \(L_D/(\omega + L_D + L_E)\) must also fall.

### 6.4 Proof of Proposition 4

By definition,
\[
\frac{dl}{d\omega} = \frac{\partial l}{\partial \omega} + \frac{\partial l}{\partial L_D} \frac{dL_D}{d\omega} + \frac{\partial l}{\partial L_E} \frac{dL_E}{d\omega}
\]
\[
= \frac{1}{(\omega + L_D + L_E)^2} \left[ (\omega + L_E) \frac{dL_D}{d\omega} - L_D \left( 1 + \frac{dL_E}{d\omega} \right) \right].
\]

Thus,
\[
\text{sign} \left( \frac{dl}{d\omega} \right) = \text{sign} \left[ (\omega + L_E) \frac{dL_D}{d\omega} - L_D \left( 1 + \frac{dL_E}{d\omega} \right) \right]
\]
\[
= \text{sign} \left[ (I - L_D) \frac{dL_D}{d\omega} - L_D \left( 1 + \frac{dL_D}{d\omega} + \frac{dL_E}{d\omega} \right) \right]
\]
\[
= \text{sign} \left[ \frac{dL_D}{d\omega} - L_D \left( 1 + \frac{dL_D}{d\omega} + \frac{dL_E}{d\omega} \right) \right]
\]
\[
= \text{sign} \left[ \frac{dL_D}{d\omega} - L_D \frac{dI}{d\omega} \right]
\]
\[
= \text{sign} \left[ I \left( \frac{\partial L_D}{\partial \omega} + \frac{\partial L_D}{\partial I} \frac{dI}{d\omega} \right) - L_D \frac{dI}{d\omega} \right].
\]

Since (6) implies that \( \partial L_D/\partial \omega = 0 \), the equilibrium leverage at \( I = I^* \) can be written

\[
\text{sign} \left( \frac{dl(I^*)}{d\omega} \right) = \text{sign} \left[ \frac{dI^*}{d\omega} \left( \frac{\partial L_D(I^*)}{\partial I} I^* - L_D(I^*) \right) \right]
\]
\[
= \text{sign} \left[ \frac{dI^*}{d\omega} \left( q(I^*) \int_0^{\bar{p}(I^*)} pf(p)dp - \frac{L_D(I^*)}{I^*} \right) \right]
\]
\[
< 0,
\]

where the inequality follows from the facts that \( dI^*/d\omega > 0 \) (established in Proposition 2), that \( q'(I) < q(I)/I \) (from our assumptions on \( q \)), and that \( L_D(I^*) > q(I^*) \int_0^{\bar{p}(I^*)} pf(p)dp \).

### 6.5 Proof of Proposition 6

By definition,

\[
\frac{dl}{d\eta} = \frac{\partial l}{\partial L_D} \frac{dL_D}{d\eta} + \frac{\partial l}{\partial L_E} \frac{dL_E}{d\eta}
\]
\[
= \frac{1}{(\omega + L_D + L_E)^2} \left[ (\omega + L_E) \frac{dL_D}{d\eta} - L_D \frac{dL_E}{d\eta} \right].
\]
Thus,

\[
\text{sign} \left( \frac{dl}{d\eta} \right) = \text{sign} \left[ (\omega + L_E) \frac{dL_D}{d\eta} - L_D \frac{dL_E}{d\eta} \right]
\]

\[
= \text{sign} \left[ (I - L_D) \frac{dL_D}{d\eta} - L_D \left( \frac{dI}{d\eta} - \frac{dL_D}{d\eta} \right) \right]
\]

\[
= \text{sign} \left[ I \frac{dL_D}{d\eta} - L_D \frac{dI}{d\eta} \right]
\]

\[
= \text{sign} \left[ I \left( \frac{\partial L_D}{\partial I} \frac{dI}{d\eta} + \frac{\partial L_D}{\partial \eta} \right) - L_D \frac{dI}{d\eta} \right]
\]

\[
= \text{sign} \left[ \frac{dI}{d\eta} \left( \frac{\partial L_D}{\partial I} - \frac{L_D}{I} \right) + \frac{\partial L_D}{\partial \eta} \right].
\]

Evaluating at \( I^* \) we have

\[
\text{sign} \left( \frac{dl(I^*)}{d\eta} \right) = \text{sign} \left[ \frac{\partial L_D(I^*)}{\partial \eta} + \frac{\partial L_E(I^*)}{\partial \eta} \left( \frac{\partial L_D(I^*)}{\partial I} - \frac{L_D(I^*)}{I^*} \right) + \frac{\partial L_D(I^*)}{\partial \eta} \right]
\]

\[
= \text{sign} \left[ \frac{\partial L_D(I^*)}{\partial \eta} \left( \frac{1}{1 - \frac{L_D(I^*)}{I^*}} - \frac{L_D(I^*)}{I^*} \right) \right]
\]

\[
= \text{sign} \left[ q(I^*) \int_0^{\hat{\rho}(I^*)} pf(p)dp \left( 1 - q'(I^*) \int_0^{\hat{\rho}(I^*)} pf(p)dp - \frac{L_D(I^*)}{I^*} \right) \right]
\]

\[
+ q(I^*) \phi \int_0^{\hat{\rho}(I^*)} pf(p)dp \left( q'(I^*) \int_0^{\hat{\rho}(I^*)} pf(p)dp - \frac{L_D(I^*)}{I^*} \right)
\]

\[
= \text{sign} \left[ q(I^*) \int_0^{\hat{\rho}(I^*)} pf(p)dp \left( 1 - \frac{L_D(I^*)}{I^*} \right) \right]
\]

\[
- q(I^*) \phi \int_0^{\hat{\rho}(I^*)} pf(p)dp \left( \frac{L_D(I^*)}{I^*} \right)
\]

\[
= \text{sign} \left[ \int_0^{\rho(I^*)} pf(p)dp - \frac{L_D(I^*)}{I^*} \right]
\]

\[
< 0,
\]

where the inequality follows from the facts that \( L_D(I^*) > q(I^*) \int_0^{\rho(I^*)} pf(p)dp \) and \( \bar{p}q(I^*) > I^* \).
6.6 Proof of Proposition 7

The proof is analogous to that of Proposition 6 up to the point that

\[
\operatorname{sign}\left(\frac{dl(I^*)}{d\gamma}\right) = \operatorname{sign}\left[\frac{\partial L_D(I^*)}{\partial \gamma} \left(1 - \frac{\partial L_E(I^*)}{\partial I} - \frac{L_D(I^*)}{I^*}\right) + \frac{\partial L_D(I^*)}{\partial I} \left(\frac{\partial L_D(I^*)}{\partial I} - \frac{L_D(I^*)}{I^*}\right)\right].
\]

Taking the partial derivatives of \(L_D\) and \(L_E\) with respect to \(\gamma\) and inserting, we have

\[
\operatorname{sign}\left(\frac{dl(I^*)}{d\gamma}\right) = \operatorname{sign}\left[\frac{\varphi(1-F(p_0))}{(1-\varphi)^2} \left(1 - q'(I^*)\varphi \int_{\hat{p}(I^*)}^{\infty} pf(p)dp - \frac{L_D(I^*)}{I^*}\right) \right.

- \frac{\varphi(1-F(p_0))}{(1-\varphi)^2} \left(1 - q'(I^*)\varphi \int_{\hat{p}(I^*)}^{\infty} pf(p)dp - \frac{L_D(I^*)}{I^*}\right) \bigg] = \operatorname{sign}\left[1 - q'(I^*)\varphi \int_{\hat{p}(I^*)}^{\infty} pf(p)dp - \varphi q'(I^*) \int_{0}^{\hat{p}(I^*)} pf(p)dp - (1 - \varphi) \frac{L_D(I^*)}{I^*}\right].
\]

From the fact that \(q'(I^*) < q(I^*)/I^*\) we have that

\[
1 - q'(I^*)\varphi \int_{\hat{p}(I^*)}^{\infty} pf(p)dp - \varphi q'(I^*) \int_{0}^{\hat{p}(I^*)} pf(p)dp - (1 - \varphi) \frac{L_D(I^*)}{I^*} > 1 - \frac{q(I^*)}{I^*} \varphi \int_{\hat{p}(I^*)}^{\infty} pf(p)dp - \varphi \frac{q(I^*)}{I^*} \int_{0}^{\hat{p}(I^*)} pf(p)dp - (1 - \varphi) \frac{L_D(I^*)}{I^*}
\]

\[
= \frac{1}{I^*} \left[ I^* - q(I^*)\varphi \int_{\hat{p}(I^*)}^{\infty} pf(p)dp - q(I^*)\varphi \int_{0}^{\hat{p}(I^*)} pf(p)dp - (1 - \varphi)L_D(I^*) \right] = \frac{1}{I^*} \left[ I^* - L_D(I^*) - L_E(I^*) \right] \geq 0.
\]

The last inequality is strict if \(\omega > 0\), but because of the strict inequality in the second line, \(dl/d\gamma\) is unambiguously positive even if the entrepreneur has no wealth.
6.7 Choice of enforcement quality

By introducing a productivity parameter $\eta$ in front of $q(I)$ it follows that the investment level $I^*$ is the largest solution to

$$I = \omega + L(I),$$

where

$$L(I) = \eta q(I) \left[ \bar{p} - (1 - \varphi) \int_{\hat{p}(I)}^{\infty} (p - \hat{p}(I)) f(p) dp \right],$$

and

$$\hat{p}(I) = \frac{\gamma \varphi}{\eta q(I) (1 - \varphi)}.$$

Differentiating the first-order condition (8), we have

$$\frac{d^2 I^*}{d \varphi d \eta} (\hat{p} q'(I^*) - 1) + \frac{d I^*}{d \varphi} \hat{p} q'(I^*) + \frac{d I^*}{d \eta} \eta \hat{p} q''(I^*) \left[ \frac{d I^*}{d \varphi} \right]^{\gamma} - \frac{\partial^2 c}{\partial \varphi^2} d \varphi = 0.$$  

(15)

Given that the second order condition is satisfied, the last bracket is negative. From (15) it follows that

$$\frac{d \varphi}{d \eta} = -\frac{\frac{d^2 I^*}{d \varphi d \eta} (\hat{p} q'(I^*) - 1) + \frac{d I^*}{d \varphi} \hat{p} q'(I^*) + \frac{d I^*}{d \eta} \eta \hat{p} q''(I^*)}{\frac{d^2 I^*}{d \varphi^2} (\hat{p} q'(I^*) - 1) + \left( \frac{d I^*}{d \varphi} \right)^2 \eta \hat{p} q''(I^*) - \frac{1}{\varphi^2 \varphi}}.$$  

To find the sign of $d \varphi/d \eta$ we differentiate (14) to obtain

$$\frac{d I^*}{d \varphi} = \frac{\partial L(I^*)}{\partial \varphi} > 0, \quad \frac{d I^*}{d \eta} = \frac{\partial L(I^*)}{\partial \eta} > 0,$$

where $I^*$ is the largest solution to (14). Consequently, $\partial L(I^*)/\partial I < 1.$
Differentiating $dI^*/d\varphi$ yields

$$\frac{d^2 I^*}{d\varphi d\eta} = \frac{d^2 L}{d\varphi d\eta}$$

$$= \left[ \frac{\partial^2 L(I^*)/\partial \varphi \partial \eta \cdot (1 - \partial L(I^*)/\partial I)}{\partial L(I^*)/\partial \varphi \cdot \partial^2 L(I^*)/\partial I \partial \eta} \right]$$

Furthermore,

$$\frac{\partial^2 L(I^*)}{\partial \varphi \partial \eta} = q(I^*) \int_0^{\infty} p f(p) dp + \frac{\gamma}{1 - \varphi \eta} \tilde{p}^2 f(\tilde{p})$$

$$- \frac{\gamma}{1 - \varphi \eta} (1 - F(\tilde{p}(I^*)))$$

$$\frac{\partial^2 L(I^*)}{\partial I \partial \eta} > 0.$$

The sign of (16) is not obvious. However, by observing that

$$\frac{dI^*}{d\varphi} = \frac{\partial L(I^*)/\partial \varphi}{1 - \partial L(I^*)/\partial I}$$

$$= \eta q(I^*) \lambda \int_0^{\infty} (p - \tilde{p}(I^*)) f(p) dp + \frac{\gamma}{(1 - \varphi \eta)} (1 - F(\tilde{p}(I^*)))$$

$$= \frac{\partial^2 I^*}{d\varphi d\eta} (\tilde{p} q(I^*) - 1) + \frac{dI^*}{d\varphi} \tilde{p} q(I^*)$$

is positive. Consequently, $d\varphi/d\eta > 0$ if $-q''(I) = 0$ or not too large.

6.8 Proof of Proposition 9

Illiquid project returns facilitates more debt and outside equity financing. By setting $(1 - \varphi)\lambda p q(I) - \varphi \gamma$ equal to 0 we have that the entrepreneur credibly can promise to pay out the
full project returns to external investors if \( p \leq \hat{p}(I, \lambda) \), where

\[
\hat{p}(I, \lambda) = \frac{\gamma}{q(I)} \frac{\varphi}{(1 - \varphi) \lambda}.
\]

The optimal state contingent repayment to investors becomes

\[
r^*(p) = \begin{cases} 
pq(I^*) & \text{if } p < \hat{p}(I^*); \\
(1 - \lambda (1 - \varphi)) pq(I^*) + \varphi \gamma & \text{if } p \geq \hat{p}(I^*). 
\end{cases}
\]

Note that the repayment expression is the same as before when \( \lambda = 1 \) and larger if \( \lambda < 1 \). Since \( r^*(p) \) is monotonically decreasing in \( \lambda \), the total funding result follows. The face value of debt is given by \( \hat{p}q(I^*) \), which is decreasing in \( \lambda \). Since the face value and the investment both decrease in \( \lambda \), so does the value of the debt.

### 6.9 Proof of Proposition 10

First, consider the case in which the firm provides collateral. Let \( X(p) \) be the set of all functions \( x(p) \) satisfying (i) \( 0 \leq x(p) \leq 1 \), (ii) \( x(p) = 1 \) for \( p > \hat{p} \), and (iii) \( pq(I) - r(p) + x(p)Kv \geq (1 - \varphi) q\pi(I) + (1 - \theta) Kv - \varphi \gamma \) for \( p < \hat{p} \) (the last inequality implies that the entrepreneur by pledging collateral does not violate the no-diversion condition). Note that the entrepreneur and the investors are indifferent among all collateral agreements \( x(p) \in X(p) \) having the same expectation \( \bar{x} \), \( \bar{x} = \int_0^{\hat{p}} x(p) f(p) dp \).

Taking into account that outside assets can be sold ex ante or pledged as collateral we have that \( I^* \) is the solution to

\[
I^* = \omega + \beta(1 - K)v + (1 - \bar{x}) \beta Kv + \hat{L}(I^*),
\]

where

\[
\hat{L}(I) = q(I^*) \left( \hat{p} - (1 - \varphi) \int_{\hat{p}(I^*)}^{\infty} \left( p - \hat{p}(I^*) \right) dF \right).
\]

Note also that \( \beta(1 - K)v \) is the value of assets sold ex ante and that \( (1 - \bar{x}) \beta Kv \) is the sales value of assets expected to be transferred to the debt holders as collateral.

Furthermore, note that the entrepreneur is only financially constrained if
\[ I^* - \omega > q(I^*) \left[ \bar{p} - (1 - \varphi) \int_{\hat{p}(I^*)}^{\infty} (p - \hat{p}(I^*)) f(p) dp \right]. \]  

(18)

Since sale or transfer of outside assets is inefficient \((\beta < 1)\), only a financially constrained entrepreneur will use outside assets to raise additional funding. In the following we assume that condition \((18)\) is satisfied.

From \((17)\) we have that

\[
\begin{align*}
\frac{dI^*}{dK} &= \frac{\frac{\partial L}{\partial K} - \bar{x} \beta v}{1 - \frac{\partial L}{\partial I}} = \frac{\theta \left( 1 - F(\hat{p}) - \bar{x} \beta \right)}{1 - q'(I) \left[ \bar{p} - (1 - \varphi) \int_{\hat{p}}^{\infty} p f(p) dp \right]}, \\
\frac{dI^*}{dx} &= \frac{-K \beta v}{1 - \frac{\partial L}{\partial I}} = \frac{-K \beta v}{1 - q'(I) \left[ \bar{p} - (1 - \varphi) \int_{\hat{p}}^{\infty} p f(p) dp \right]}.
\end{align*}
\]

The entrepreneur’s problem is thus to choose \(K\) and \(\bar{x}\) in order to maximize

\[ U(\bar{x}, K) = \bar{p}q(I^*(\bar{x}, K)) - I^*(\bar{x}, K) + \bar{x}Kv, \]

where \(I^*(\bar{x}, K)\) is implicitly defined by \((17)\) and \(\bar{x}Kv\) represents the value the entrepreneur attaches to outside assets kept after the project is completed. All outside assets remain in the hands of the entrepreneur if \(K = \bar{x} = 1\) (unless she makes an unsuccessful diversion attempt). Denote the optimal fraction of outside assets sold up front \(1 - K^*\) and let \(1 - \bar{x}^*\) denote the optimal expected fraction of remaining assets transferred to investors as collateral. The marginal payoff to keeping assets rather than selling them is

\[
\begin{align*}
\frac{dU(\bar{x}^*, K)}{dK} &= [\bar{p}q'(I^*(\bar{x}^*, K)) - 1] \frac{dI^*(\bar{x}^*, K)}{dK} + \bar{x}^* v \\
&= \left[ \frac{\bar{p}q'(I^*(\bar{x}^*, K)) - 1}{1 - q'(I^*(K, \bar{x}^*))} \left( \bar{p} - (1 - \varphi) \int_{\hat{p}}^{\infty} p f(p) dp \right) \left( \theta \left( 1 - F(\hat{p}) - \bar{x}^* \beta \right) + \bar{x}^* \right) \right] v \\
&= \left[ M(I^*(\bar{x}^*, K)) \left( \theta \left( 1 - F(\hat{p}) - \bar{x}^* \beta \right) + \bar{x}^* \right) \right] v,
\end{align*}
\]

(19)
whereas the marginal payoff to keeping assets rather than pledging them as collateral is

\[
\frac{dU(\bar{x}, K^{*})}{d\bar{x}} = \left[ \tilde{p}q(I^{*}(\bar{x}, K^{*}))-1 \right] \frac{dI^{*}(\bar{x}, K^{*})}{d\bar{x}} + K^{*}v
\]

(20)

\[
= \left[ \frac{\tilde{p}q(I^{*}(\bar{x}, K^{*}))-1}{1-\varphi q(I^{*}(\bar{x}, K^{*}))} \left( \tilde{p} - (1-\varphi) \int_{\tilde{p}}^{\infty} p f(p) dp \right) \right] (-\beta) + 1 \right] K^{*}v
\]

\[
\]

Ex ante sale of a fraction or all outside assets is optimal if \(dU(\bar{x}^{*}, 1)/dK < 0\), or equivalently if

\[
M(I^{*}(\bar{x}^{*}, 1)) \left[ \theta \left( 1 - F(\tilde{\bar{p}}) \right) - \bar{x}^{*} \beta \right] + \bar{x}^{*} < 0
\]

(see (19)). Solving for \(\beta\), it follows that ex ante sale of assets is optimal if and only if

\[
\beta > \frac{1}{M(I^{*}(\bar{x}^{*}, 1))} + \frac{\theta \left( 1 - F(\tilde{\bar{p}}) \right)}{\bar{x}}.
\]

(21)

Pledging collateral to debt holders is optimal if \(dU(1, K^{*})/d\bar{x} < 0\), or equivalently

\[
1 - M(I^{*}(1, K^{*}))\beta < 0
\]

(see (20)). Solving for \(\beta\), it follows that pledging collateral is optimal if

\[
\beta > \frac{1}{M(I^{*}(1, K^{*}))}.
\]

(22)

Since

\[
\frac{\theta \left( 1 - F(\tilde{\bar{p}}) \right)}{\bar{x}} \geq 0,
\]

it is always optimal to exhaust the possibility to increase investment by pledging collateral before assets are sold ex ante.

Since only debt holders’ claims are secured by outside assets, the highest feasible expected transfer of outside assets is

\[
(1 - x^{\min})v = \int_{0}^{\tilde{\bar{p}}} \min \left[ v, \frac{1}{\beta} \left( \frac{\varphi \gamma + \theta v}{1-\varphi} - pq(I) \right) \right] f(p) dp.
\]
To understand the right-hand side expression, note that the face value of debt is \( \varphi \gamma + \theta v / 1 - \varphi \) while the project yields \( pq(I) \), and that the collateral can be used to cover the difference. In order to repay the debt holders 1 dollar the entrepreneur has to sell assets worth \( 1 / \beta \) dollars to him. The min- function follows from the fact that the entrepreneur can at most sell all outside assets, \( v \).

Substituting \( x \) with \( x^{\text{min}} \) in (21) and with 1 in (22), Proposition 10 follows.
References


