Macro, industry and frailty effects in defaults: the 2008 credit crisis in perspective *

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Abstract

We determine the magnitude and nature of systematic default risk using 1971–2009 default data from Moody’s. We disentangle systematic risk factors due to business cycle effects, common default dynamics (frailty), and industry-specific dynamics (including contagion). To quantify the contribution of each of these factors to default rate volatility we introduce a new and flexible model class for factor structures on non-Gaussian (defaults) and Gaussian (macro factors) data simultaneously. We find that all three types of risk factors (macro, frailty, industry/contagion) are important for default risk. The systematic risk factors account for roughly one third of observed default risk variation. Half of this is captured by macro and financial market factors. The remainder is captured by frailty and industry effects (in roughly equal proportions). The frailty components are particularly relevant in times of stress. Models based only on macro variables may both under-estimate and over-estimate default activity during such times. This indicates that frailty factors do not simply capture missed non-linear responses of defaults to business cycle dynamics. We also find significant differences in the impact of crises on defaults at the sectoral level, implying frailty as well as contagion may play a role in systematic default clustering. Finally, we show that the contribution of frailty and industry factors on top of macro factors is economically significant for assessing portfolio risk.

Keywords: systematic default risk; credit portfolio models; mixed-measurement dynamic factor model; frailty-correlated defaults; state space methods; dynamic credit risk management.

JEL classification: G21; C33.
1 Introduction

In this paper we decompose default risk into its different systematic components using a new methodological framework. Observed corporate defaults are known to cluster in time. For example, aggregate US default rates during the 1991, 2001, and 2008 recession periods are up to five times higher than in intermediate expansion years. It is also well known that default rates depend on the prevailing macroeconomic conditions, see for example Pesaran, Schuermann, Treutler, and Weiner (2006), Duffie, Saita, and Wang (2007), Metz (2008), Figlewski, Frydman, and Liang (2008), and Koopman, Kräussl, Lucas, and Monteiro (2009).

Recent research indicates that conditioning on readily available macroeconomic and firm-specific information, though important, is not sufficient to fully explain the observed degree of default clustering. In a seminal study, Das, Duffie, Kapadia, and Saita (2007) reject the joint hypothesis of (i) well-specified default intensities in terms of observed macroeconomic and firm-specific information and (ii) the doubly stochastic (conditional) independence assumption which underlies virtually all available credit risk models in practice. From this finding, two important separate strands of literature have emerged.

A first line of literature attributes the additional variation in default intensities to an unobserved dynamic component, also known as a ‘frailty’ factor. The discussion of frailty factors in the credit risk literature is fairly recent, see Das et al. (2007), McNeil and Wendin (2007), Koopman, Lucas, and Monteiro (2008), Koopman and Lucas (2008), Koopman, Lucas, and Schwaab (2008), and Duffie, Eckner, Horel, and Saita (2009). The frailty factor captures default clustering above and beyond what can be explained by macroeconomic variables and firm-specific information. The unobserved component can pick up the effects of omitted variables in the model as well as other effects that are difficult to quantify, such as the trust in the accuracy of public accounting information, see (Duffie et al. 2009).

A second line of literature puts forward contagion as a relevant factor for additional default clustering. It refers to the phenomenon that a defaulting firm weakens other firms with which it has business links, see the discussion in Giesecke (2004) and Giesecke and Azizpour (2008). Contagion effects may dominate potentially offsetting competitive effects at the intra-industry level, see Lang and Stulz (1992), and Jorion and Zhang (2007a, 2007b). Lando and Nielsen (2008) screen hundreds of default histories in the Moody’s database for
evidence of default contagion. The presented examples suggest that contagion is mainly an intra-industry effect. Contagion may explain default dependence at the industry level beyond that induced by macro and frailty factors.

Despite earlier research in this area, less is known about the relative contribution of the different sources of systematic default risk to observed default clustering. To address this question, we decompose the systematic variation in corporate default counts into its different constituents as suggested in the literature. For this purpose, we develop a novel methodological framework in which default rate volatility at the rating and industry level is attributed to macro, frailty, and industry effects simultaneously.

In our study of systematic default risk and macroeconomic developments, we address the econometric complication that default events are discrete, whereas macroeconomic and financial variables are continuous. We require a framework where both types of variables can be driven by the same underlying risk factors. Most models in the literature either do not account for frailty effects, or if they do, only allow for a limited number of macro variables to enter the model, see for example McNeil and Wendin (2007), Duffie, Eckner, Horel, and Saita (2009), and Koopman, Kräussl, Lucas, and Monteiro (2009). Even with a limited number of macro variables, the observed data is typically treated as unrelated to the frailty component. Auxiliary models for the observed factors are then needed to assess risk out-of-sample, see also Koopman, Lucas, and Schwaab (2008).

In contrast to this earlier work, our mixed measurement dynamic factor framework explicitly captures the (i) joint variation in (discrete, non-Gaussian) corporate default counts and a large number of (continuous, Gaussian) macroeconomic and financial time series variables, (ii) default clustering due to latent frailty risk unrelated to the business cycle, and (iii) unobserved industry-sector dynamics. The latter may arise as a result of direct default contagion through business links on the sector level, or as industry-specific frailty shocks (like the 9/11 attacks, the burst of the dotcom bubble, etc.). We demonstrate that these model features lead to a realistic fit of default rate dynamics at the industry as well as the economy wide level. Our approach yields an integrated framework for estimation, inference, and forecasting of time varying corporate default rates. In particular, no auxiliary models for macroeconomic data are required. Estimation of parameters and latent risk factors is done in a single step.
As the second contribution of our paper, we propose a method to decompose default risk into its latent constituents and to quantify the relative contribution of each of these components. We do so by introducing a pseudo-$R^2$ measure of fit based on reductions in Kullback-Leibler (KL) divergence. The KL divergence is a standard statistical measure of ‘distance’ between distributions. Changes in the KL divergence, when appropriately scaled, allow one to assess the relative contribution of each systematic default risk component (macro, frailty, industry) to overall default rate volatility. We find that on average across industries and time about 70% of total default risk is idiosyncratic, or diversifiable. About half of the remaining 30% share of systematic default risk can be attributed to common variation with the business cycle and financial markets data. The other half of systematic credit risk is driven by a frailty factor and industry-specific factors (roughly in equal proportions). The frailty component cannot be diversified in the cross-section, whereas the industry effects can only be diversified up to a specific extent.

Our reported risk shares vary considerably over industry sectors, rating groups, and time. For example, we find that the frailty component tends to explain a higher share of default rate volatility before and during times of crisis. The inclusion of a frailty factor leads to more probability mass in the right tail of the model-implied portfolio loss distribution. Model-implied economic capital buffers are higher as a result. We demonstrate that industry-specific factors are of key importance for a good fit to observed defaults at the industry level and for capturing industry concentrations in credit portfolios. We also find that the frailty factors do not capture trivial missed non-linear responses of defaults to business cycle fluctuations. Over the different recession periods in our sample, the frailty factors may induce higher as well as lower predictions of defaults compared to models that use macro variables only.

The remainder of this paper is organized as follows. Section 2 introduces our general methodological framework. Section 3 presents our core empirical results, in particular a decomposition of total systematic default risk into its latent constituents. We comment on implications for portfolio credit risk management in Section 4. Section 5 concludes.
2 A joint model for default, macro, and industry risk

The key challenge in decomposing systematic credit risk is to define a factor model structure that can simultaneously handle normally distributed (macro variables) and non-normally distributed (default counts) data, as well as linear and non-linear factor dependence. The factor model we introduce for this purpose is a Mixed Measurement Dynamic Factor Model, or in short, MiMe DFM. In the development of our new model, we focus on the decomposition of systematic default risk. However, the model may also find relevant applications in other areas of finance. The model is applicable to any setting where different distributions have to be mixed in a factor structure.

In our analysis we consider the vector of observations given by

\[ y_t = (y_{1t}, \ldots, y_{jt}, y_{J+1,t}, \ldots, y_{J+N,t})' \]

for \( t = 1, \ldots, T \), where the first \( J \) elements of \( y_t \) are default counts. We count defaults for different ratings and industries. As a consequence, the first \( J \) elements of \( y_t \) are discrete-valued. The remaining \( N \) elements of \( y_t \) contain macro and financial variables which take continuous values. We assume that both the default counts and the macro and financial time series data are driven by a set of dynamic factors. Some of these factors may be common to all variables in \( y_t \). Other factors may only affect a subset of the elements in \( y_t \).

In our study, we distinguish macro, frailty, and industry (or contagion) factors. The common factors are denoted as \( f_{mt} \), \( f_{dt} \), and \( f_{it} \), respectively. The factors \( f_{mt} \) capture shared business cycle dynamics in macroeconomic data and default counts. Therefore, factors \( f_{mt} \) are common to all data. Frailty factors \( f_{dt} \) are default-specific, i.e., common to default data \((y_{1t}, \ldots, y_{jt})\) and independent of observed macroeconomic and financial data by construction. By not allowing the frailty factors to impact the macro series \( y_{jt} \) for \( j = J + 1, \ldots, J + N \), we effectively restrict \( f_{dt} \) to pick up any default clustering above and beyond that implied by macroeconomic and financial factors \( f_{mt} \). The third set of factors \( f_{it} \) considered in this paper affects firms in the same industry. Such factors may arise as a result of default contagion through up- and downstream business links. Alternatively, they may be interpreted as industry-specific frailty factors. Disentangling these two interpretations is empirically
impossible unless detailed information at the firm-level is available on firm interlinkages at the trade and institutional level. Such data are not available for our current analysis.

We gather all factors into the vector \( f_t' = (f_t^m, f_t^d, f_t^i) \). Note that we only observe the default counts and macro variables \( y_t \). The factors \( f_t \) themselves are unobserved or latent. We assume the following simple autoregressive dynamics for the latent factors,

\[
f_t = \Phi f_{t-1} + \eta_t, \quad t = 1, 2, \ldots,
\]

with the coefficient matrix \( \Phi \) diagonal, and with \( \eta_t \sim N(0, \Sigma_\eta) \). More complex dynamics than (2) can be considered as well. The autoregressive structure allows the components of \( f_t \) to be sticky. For example, it allows the macroeconomic factors \( f_t^m \) to evolve slowly over time and capture the business cycle component in both macro and default data. Similarly, the credit climate and industry default conditions can be captured by persistent processes for \( f_t^d \) and \( f_t^i \), such that they can capture the clustering of high-default years. To complete the specification of the factor process, we specify the initial condition \( f_1 \sim N(0, \Sigma_0) \). We assume stationarity of the factor dynamics by insisting that all \( m \) eigenvalues of \( \Phi \) lie inside the unit circle. The \( m \times 1 \) disturbance vectors \( \eta_t \) are serially uncorrelated.

To combine the normally and non-normally distributed elements in \( y_t \), we use our new MiMe DFM (Mixed Measurement Dynamic Factor Model) approach. The MiMe DFM is based on the standard factor model assumption: conditional on the factors \( f_t \), the measurements in \( y_t \) are independent. In our specific case, we assume that conditional on \( f_t \), the first \( J \) elements of \( y_t \) have a binomial distribution with parameters \( k_{jt} \) and \( \pi_{jt} \), \( j = 1, \ldots, J \). Here, \( k_{jt} \) denotes the number of firms in a specific rating and industry bucket \( j \) at time \( t \), and \( \pi_{jt} \) denotes the conditional (on \( f_t \)) probability of default. For more details on the conditionally binomial model see e.g. McNeil, Frey, and Embrechts (2005, Chapter 9). The remaining \( N \) elements of \( y_t \) conditional on \( f_t \) follow a normal distribution with mean \( \mu_{jt} \) and variance \( \sigma_{jt}^2 \) for \( j = J + 1, \ldots, J + N \).

2.1 The mixed measurement dynamic factor model

Both the binomial and the normal distribution are members of the exponential family of distributions. The exponential family contains both discrete and continuous-valued distri-
butions, including normal, binomial, Bernoulli, Poisson, and Gamma distributions. In this paper, we formulate the MiMe DFM for random variables from the exponential family. This restriction is not strictly necessary, and the model can easily be extended to handle distributions outside this class. The estimation methodology presented in this paper applies to the general case as well.

The link between the factors $f_t$ and the observations $y_t$ relies on time-varying location parameters, such as the default probability $\pi_{jt}$ for default data, and the mean $\mu_{jt}$ for Gaussian data. In general, let each variable $y_{jt}$ follow the distribution

$$y_{jt}|\mathcal{F}_t \sim p_j(y_{jt}; \mathcal{F}_t, \psi),$$

where $\mathcal{F}_t = \{f_t, f_{t-1}, \ldots\}$, and $\psi$ is a vector of fixed and unknown parameters, including for example the elements of $\Phi$ and $\Sigma_\eta$ in (2). The index $j$ of the density $p_j(\cdot)$ indicates that the type of measurement $y_{jt}$ (discrete versus continuous) may vary across $j$. We assume that the information from past factors $\mathcal{F}_t = \{f_t, f_{t-1}, \ldots\}$ which is relevant for the distribution of $y_{jt}$ is contained in an unobserved signal $\theta_{jt}$. For exponential family data, $\theta_{jt}$ is the so-called canonical parameter, see also the Appendix A1.

We assume that the signal $\theta_{jt}$ is a linear function of unobserved factors, $f_t$, such that

$$\theta_{jt} = \alpha_j + \lambda_j'f_t,$$

with $\alpha_j$ an unknown constant, and $\lambda_j$ an $m \times 1$ loading vector with unknown coefficients. It is conceptually straightforward to let $\theta_{jt}$ also depend on past values of the factors $f_t$. We emphasize that $y_t$ may depend linearly as well as non-linearly on the common factors $f_t$. This follows from the dependence of the distribution of $y_{jt}$ on the non-linear function $b_{jt}(\cdot)$ of $\theta_{jt}$.

As the key question in this paper concerns the relative contributions of macro, frailty, and contagion (or industry) risk to general default risk, we introduce further restrictions on
the general form of (4). In particular, we specify the signals by

\[
\theta_{jt} = \lambda_{0j}^j + \beta_j^m f^m_t + \gamma_j^d f^d_t + \delta_j^i f^i_t \quad \text{for} \quad j = 1, \ldots, J, \tag{5}
\]

\[
\theta_{jt} = \lambda_{0j}^j + \beta_j^m f^m_t, \quad \text{for} \quad j = J + 1, \ldots, J + N. \tag{6}
\]

The signal specification in (6) implies that the means of the macro variables depend linearly on the macro factors \( f^m_t \). The components of \( f^m_t \) capture general developments in business cycle activity, lending conditions, financial markets, etc. The log-odds ratios in (5) partly depend on macro factors, but also depend on frailty risk \( f^d_t \) and industry \( f^i_t \) factors. The specification of the signals in (5) and (6) is key to our empirical analysis where we focus on studying whether macro dynamics explain all systematic default rate variation, or whether and to which extent frailty and industry factors also are important.

For model identification, we impose the restriction \( \Sigma = I - \Phi \Phi' \). This implies that the factor processes in (2) have an autoregressive structure with unconditional unit variance. It also implies that factor loadings in \( \beta_j^0, \gamma_j^0, \) and \( \delta_j^0 \) can be interpreted as factor standard deviations (volatilities) for firms of type \( j = 1, \ldots, J \).

As mentioned, all model parameters that need to be estimated are collected in a parameter vector \( \psi \). This includes the factor loadings \( \beta_j^0, \gamma_j^0, \delta_j^0 \), but also the coefficients in the autoregressive matrix \( \Phi \) in (2). We aim to estimate \( \psi \) by maximum likelihood. For this purpose, we numerically maximize the likelihood function as given by

\[
p(y; \psi) = \int p(y, f; \psi)df = \int p(y|f; \psi)p(f; \psi)df, \tag{7}
\]

where \( p(y, f; \psi) \) is the joint density of the observation vector \( y' = (y'_1, \ldots, y'_T) \) and the factors \( f' = (f'_1, \ldots, f'_T) \). The integral in (7) is not known analytically, and we therefore rely on numerical methods. The likelihood function (7) can be evaluated efficiently via Monte Carlo integration and using the method of importance sampling, see Durbin and Koopman (2001). Maximizing the Monte Carlo estimate of the likelihood function is feasible using standard computers. Once maximum likelihood estimates of \( \psi \) are obtained, (smoothed) estimates of the unobserved macro, frailty and industry factors \( f_t \) and their standard errors can be obtained using the same Monte Carlo methods. This methodology has a number of
interesting features in the current setting, but we defer all details on the estimation procedure to the Appendix.

2.2 Decomposition of mixed measurement variation

Once the model is estimated, we need to assess which share of variation in mixed-measurement data \( y_t \) is captured by the different latent factors. Obviously, this cannot be achieved by a standard \( R^2 \) measure. We therefore adopt a pseudo-\( R^2 \) measure that is similar to those discussed in Cameron and Windmeijer (1997). The pseudo-\( R^2 \) measure is based on a distance measure between two distributions. Our distance measure is the Kullback-Leibler (KL) divergence and is defined as

\[
KL(\theta_1, \theta_2) = 2 \int \log \left[ \frac{p_{\theta_1}(y)}{p_{\theta_2}(y)} \right] p_{\theta_1}(y) dy. \tag{8}
\]

It measures the distance between the two log-densities \( \log p_{\theta_1} \) and \( \log p_{\theta_2} \). These densities are specified by competing parameter vectors \( \theta_1 \) and \( \theta_2 \), respectively. For the case of the normal regression model, scaled reductions in (8) reduce to the standard goodness-of-fit measure \( R^2 \). For the binary choice model, the McFadden pseudo-\( R^2 \) is recovered. Similarly, in a more general context where normal and binomial distributions are mixed, scaled reductions in (8) provide a natural measure of fit. The contribution of each of the separate factors can be computed. We refer to Hastie (1987) and Vos (1991) for further details.

By taking the expectation over \( y \) in (8), all values in the support of \( p \) are taken into account when calculating the divergence. As definition (8) compares marginal distributions, the unobserved factors \( f_t \) have to be integrated out. For this purpose, we use the same Monte Carlo integration techniques based on Importance Sampling as used for the estimation stage, see (7).

Figure 1 illustrates our idea for assessing the contribution of common factors to default risk. We distinguish several alternative model specifications indicated by \( M^{na}, M^{m}, M^{md}, \) and \( M^{mdi} \). These models contain an increasing collection of latent factors. Model \( M^{na} \) does not contain any factors, while models \( M^{m}, M^{md}, \) and \( M^{mdi} \) cumulatively add the macro, frailty, and industry (contagion) factors, respectively. More factors imply a better fit to observed data.
Model $M^{\text{max}}$ provides the maximum possible fit by considering a model with a separate dummy variable for each observation. The model contains as many parameters as observations. While useless for practical purposes, the unrestricted model provides a natural benchmark for what is the maximum possible amount of recoverable information in the data. In the linear regression model, $M^{\text{max}}$ would be the model with a perfect fit ($R^2 = 1.0$).

To decompose systematic credit risk, we consider the improvements in the KL divergence when moving from $M^{\text{na}}$ to $M^{m}$, $M^{md}$, $M^{mdi}$, and ultimately to $M^{\text{max}}$. We scale the KL improvements by the total distance between models $M^{\text{na}}$ and $M^{\text{max}}$, given by $KL(\theta_{\text{na}}, \theta_{\text{max}})$. In this way, we obtain a natural scaling for the contribution of each set of factors to explaining default rate variation. Similar to the standard $R^2$, all values lie between zero and one. Moreover, summing the contributions due to the different factors and the residual risk component produces one by definition.\(^1\) We thus interpret the improvements in these pseudo-$R^2$ values as the percentage of default risk variation captured by a specific set of systematic risk factors. More details and discussion are given by Cameron and Windmeijer (1997).

3 Empirical findings for U.S. default data

We study the quarterly default and exposure counts obtained from the Moody’s corporate default research database for the period 1971Q1 to 2009Q1. We distinguish seven industry groups (financials and insurance; transportation; media, hotels, and leisure; utilities and energy; industrials; technology; and retail and consumer products) and four rating groups (investment grade $\text{Aaa} - \text{Baa}$, and the speculative grade groups $\text{Ba}$, $\text{B}$, $\text{Caa} - \text{C}$). We have pooled the investment grade firms because defaults are rare for this segment. Figure 2 presents aggregate default fractions and disaggregated default data. We observe a considerable time variation of about a factor of five in aggregate default fractions. The disaggregated data reveals that default clustering around recession periods is present for firms from both the investment grade and speculative grade rating groups.

\(^1\)This is true for exponential family data models which use the so-called canonical link function, see Cameron and Windmeijer (1997), Thm. 2. This is the case for our signal specification and data. The decomposition property may fail in more general settings.
Macroeconomic and financial data are obtained from the St. Louis Fed online database FRED, see Table 1 for a listing of macroeconomic and financial data. This data enters the analysis in the form of annual growth rates, see Figure 3 for time series plots.

### 3.1 Parameter estimation

Parameter estimates associated with the default counts are presented in Table 2. Estimated coefficients refer to a model specification with macroeconomic, frailty, and industry-specific factors. Parameter estimates in the first column combine to fixed effects for each cross-section, according to $\lambda_{0,j} = \lambda_0 + \lambda_{1,d_j} + \lambda_{2,s_j}$, where the common intercept $\lambda_0$ is adjusted by specific coefficients indicating industry sector $(s_j)$ and rating group $(d_j)$, respectively. The two middle columns report the autoregressive parameters $(\phi)$ and factor loadings $\beta$ associated with four common factors $f_t^m$. Loading coefficients differ across rating groups. The loadings tend to be higher for investment grade firms. This confirms the notion that financially healthy firms are more sensitive to business cycle risk, see e.g. Basel Committee on Banking Supervision (2004).

Factor loadings $\gamma$ and $\delta$ associated respectively with one common frailty factor $f_t^d$ and six orthogonal industry (or contagion) factors $f_t^i$ are given in the last column of Table 2. The frailty risk factor $f_t^d$ is, by construction, common to all firms but unrelated to the included macroeconomic data. Frailty risk is relatively large for all firms, but particularly pronounced for speculative grade firms. Industry sector loadings are highest for the financial, transportation, and energy and utilities sector.
The estimated factors $f^m_t$ displayed in Figure 4 show clear business cycle dynamics. The factors are ordered row-wise from top-left to bottom-right according to their share of explained variation for the macro and financial data listed in Table 1.

[Insert Figure 5 around here]

Figure 5 presents the shares of variation in each macroeconomic time series that can be attributed to the common macroeconomic factors. The first two macroeconomic factors load mostly on labor market, production, and interest rate data. The last two factors displayed in the bottom panels of Figure 5 load mostly from survey sentiment data and changes in price level indicators. The macroeconomic factors capture 24.7%, 22.4%, 11.0%, and 8.0% of the total variation in the macro data panel, respectively (66.1% in total). The range of explained variation ranges from about 30% (S&P 500 index returns, fuel prices) to more than 90% (unemployment rate, average weekly hours index, total non-farm payrolls). All four common factors $f^m_t$ tend to load more on default probabilities of firms rated investment grade rather than speculative grade, see Table 2.

[Insert Figure 6 around here]

Figure 6 presents smoothed estimates of the frailty and industry-specific factors. The frailty factor is high before and during the recession years 1991 and 2001. As a result, the frailty factor implies additional default clustering in these times of stress. On the other hand, the large negative values before the 2007-09 credit crisis imply defaults that are systematically ‘too low’ compared to what is implied by macroeconomic and financial data. The frailty factor reverts to its mean level during the 2007-09 credit crisis. Apparently, the extreme realizations during 2008-09 in macroeconomic and financial variables are sufficient to account for the levels of observed defaults.

Industry factors $f^i_t$ capture deviations of industry-specific dynamics from common variation. For example, we observe industry-specific default stress for financial firms during the US savings and loan crises from 1986-1990, and during the current crisis of 2007-09. Similarly, we observe considerably higher default stress for the technology sector following the 2000/01 asset bubble burst, or for the transportation industry following the 9/11 attack.

[Insert Figure 7 around here]
Figure 7 shows the model-implied economy-wide default rate against the aggregate observed rates. We distinguish four specifications with (a) no factors, (b) \( f_t^m \) only, (c) \( f_t^m, f_t^d \), and (d) all factors \( f_t^m, f_t^d, f_t^i \). This allows to assess the goodness of fit achieved at the aggregate level when adding latent factors. The static model fails to capture the observed default clustering around recession periods. The changes in the default rate for the static model are due to changes in the composition and quality of the rated universe. Such changes are captured by the rating and industry specific intercepts in the model. The upper-right panel indicates that the inclusion of macro variables helps to explain default rate variation. The latent frailty dynamics given by \( f_t^d \), however, are clearly required for a good model fit. This holds both in low default periods such as 2002-2007, as well as in high default years such as 1991. The bottom graphs of Figure 7 indicate that industry-specific developments cancel out in the cross-section to some extent and can thus be diversified. As a result, they may matter less from a portfolio perspective.

3.2 Total default risk: a decomposition

We use the pseudo-\( R^2 \) measure as explained in Section 2.2 to assess which share of default rate volatility is captured by an increasing set of systematic risk factors. The earlier literature on default modeling in the presence of explanatory variables has not addressed this issue in detail.

Table 3 reports the estimated risk shares. By pooling over rating and industry groups, and by taking into account default and macroeconomic data for more than 35 years, we find that approximately 69% of a firm’s total default risk is idiosyncratic. The idiosyncratic risk can be eliminated in a large credit portfolio through diversification. The remaining share of risk, approximately 31%, does not average out in the cross section and is referred to as systematic risk. We find that the largest share of systematic default risk is due to the common exposure to macroeconomic and financial time series data. This common exposure can be regarded as the business cycle component, and constitutes approximately 17% of total default risk, or 54% of systematic risk. The business cycle variation is not sufficient to account for all default rate variability in the data. Specifically, our results indicate that approximately 6% of total default risk, or 20% of systematic risk, is due to an unobserved
frailty factor. Finally, approximately 8% of total default risk, or 26% of systematic risk, can be attributed to industry-specific developments such as default contagion through business links.

[Insert Table 3 around here]

The dynamic behavior in defaults differs considerably across industries. For example, Table 3 reveals that firms from the utilities and energy, and from the transportation sector are less affected by common macroeconomic and frailty risk. These industries require an industry-specific factor to capture sectoral dynamics. We also find that firms with a speculative grade rating do not appear to have less systematic default risk than firms with an investment grade rating. The lower sensitivity towards macroeconomic risk for the latter group is compensated by a higher sensitivity to latent frailty risk.

Figure 8 presents time series of estimated risk shares over a rolling window of eight quarters. These estimated risk shares vary considerably over time. While common variation with the business cycle explains approximately 17% of total variation on average, this share may be as high as 40% before and during times of crisis, for example in 1991 and 2007. Similarly, the frailty factor captures a higher share of systematic default risk before and during times of crisis such as 1990-1991 and 2006-2007. In the former case, positive values of the frailty factor imply higher default rates that go beyond those implied by macroeconomic data. In the latter case, the significant negative values of the frailty factor during 2006-2007 imply lower default rates than expected from macroeconomic data only. High absolute values of the frailty factor imply times when systematic default risk diverges from business cycle developments as represented by the common factors. Industry specific effects have been important mostly during the late 1980s and 2008. These are periods when e.g. banking specific risk is captured through an industry-specific factor.

The bottom right graph of Figure 8 presents the share of idiosyncratic risk over time. We observe a gradual decrease in idiosyncratic risk building up to the 2007-2009 crisis. Defaults become more systematic between 2001 and 2007 due to both macro and frailty effects. Negative values of the frailty risk factor during these years indicate that default rates were ‘systematically lower’ than what would be expected from macroeconomic developments. The rapid correction of this phenomenon over the financial crisis is striking. The eight-quarter
rolling $R^2$ for the macro factors decreases by a factor 2 from 40% to 20% over 2008Q3-2009Q1. Given the rolling window approach, the instantaneous effect may be even higher. The effect is offset by an increased explanatory power of industry effects (from 2% to 12%) and idiosyncratic risk (from roughly 40% to 50%). Both of these are diversifiable to a lesser or greater extent. The explanatory power of the frailty factor remains high over the entire crisis period and only decreases towards the end of our sample. Again, this underlines the need for default risk models that include other risk factors above and beyond standard observed macroeconomic and financial time series. Such factors pick up rapid changes in the credit climate that might not be captured sufficiently well by observed risk factors. We address the economic impact of frailty and industry factors in the next section.

[Insert Figure 8 around here]

4 Implications for risk management

Many default risk models that are employed in day-to-day risk management rely on the assumption of conditionally independent defaults, or doubly stochastic default times, see Das et al. (2007). At the same time, most models do not allow for unobserved (frailty) risk factors and intra-industry (contagion) dynamics to capture excess default clustering. We have reported in Section 3.2 that frailty and industry factors account for approximately 45% of systematic default risk. In this section we explore the consequences for portfolio credit risk when frailty and industry factors are not accounted for in explaining default variation. This is of key importance for internal risk assessment as well as external supervision.

4.1 The frailty factor

The frailty factor captures approximately 20% of the common variation in disaggregated default rates at the industry and rating level, see Section 3.2. The presence of a frailty factor may increase default rate volatility compared to a model without latent risk dynamics. As a result it may shift probability mass of the portfolio credit loss distribution towards more extreme values. This would increase the capital buffers prescribed by the model. To explore this issue we conduct the following stylized credit risk experiment.
We consider a financial institution with a portfolio of short-term (rolling) loans to all Moody’s rated US firms. Loans are extended at the beginning of each quarter during 1981Q1 and 2008Q4 at no interest. A non-defaulting loan is re-extended after three months. The loan exposure to each firm at time $t$ is given by the inverse of the total number of firms at that time, $\left(\sum_j k_{jt}\right)^{-1}$. This implies that the total credit portfolio value is 1$ at all times.

In case of a default only a certain percentage of the principal is recovered. Rather than using an average recovery rate of around 60%, we assume a stressed recovery rate of 20%. This substantially lower recovery rate accounts for the possible empirical correlation between the probability of default and the recovery rate, see for example Altman (2006). Since we are interested in the tail of the loss distribution, the clustering of defaults during periods of low recovery rates is important.

The institution uses the reduced form model of Section 3 to set its capital buffers against future losses at a high percentile of the predictive loss distribution. Future risk factor realizations are obtained from our empirical model and are forecast out-of-sample. The model-based forecasts can be computed by filtering methods introduced in the Appendix A2.

Our example portfolio is stylized in many regards. Nevertheless, it allows us to investigate the importance of macroeconomic, frailty-, and industry-specific dynamics for the out-of-sample risk measurement of a diversified loan or bond portfolio.

Panel (a) of Figure 9 is the credit portfolio loss distribution implied by actual default data. This distribution can be compared with the loss distribution implied by three different specifications of our econometric model of Section 2, see panels (b) to (d). Portfolio credit loss distributions for actual loan portfolios are known to be skewed to the right and leptokurtic, see e.g. McNeil, Frey, and Embrechts (2005, Chapter 8). Flat segments or bi-modality may arise due to the discontinuity in recovered principal in case of default. These qualitative features are confirmed in panel (a) of Figure 9.

By comparing the loss distributions in the top panels of Figure 9, we find that the common variation obtained from macroeconomic data is in general not sufficient to reproduce the thick right-hand tail implied by actual default data. The shape of the tail and extreme tail of the empirical distribution (a) is not well reproduced by in panel (b) if only macroeconomic
factors are included. The additional frailty factor shifts some of the probability mass into the right tail, see panel (c). The loss distribution implied by the complete model, (d), however, is close to the actual distribution. The full model is able to reproduce the distributional characteristics of default rates, such as the positive skewness, excess kurtosis, and an irregular shape in the upper tail. Industry-specific variation in default rates may cancel to some extent in a diversified portfolio, but not entirely, explaining the differences between the bottom two graphs.

4.2 Industry specific risk dynamics

Section 3.2 shows that industry-specific variation accounts for about 25% of default rate volatility at the rating and industry level. Industry-specific factors capture the differential impact of each crisis on a given sector. For example, default stress for the banking industry has been high before and during the 1991 and 2008 recessions, but negligible during the 2001 recession. Similarly, while the 2007-2009 crisis is particularly stressful for firms from the financial, manufacturing, and media, hotels, and leisure sector, it is relatively benign on the technology, energy, and transportation sectors.

A specific case illustrates how macro, frailty, and industry-specific dynamics combine to capture industry-level variation in default rates. Figure 10 plots the observed quarterly default rate for the financial sector subsample of the entire Moody's data base. The rates are computed as the percentage of financial sector defaults over the number of firms rated in the financial industry. We compare the observed fractions to the corresponding model-implied rates. Again we distinguish three model specifications, i.e., common variation with macroeconomic factors only, macro and frailty factors, and macro, frailty, and industry-specific factors.

Common variation of defaults with macroeconomic and financial data already implies substantial time-variation in implied default rates. The frailty factor captures the general pattern that defaults are higher before and during the 1991 and 2001 recessions than implied by macroeconomic data alone, see Figure 6 for the respective peaks of the frailty factor during these times. A high degree of default clustering in the financial industry, however, is only
observed during the former 1991 recession, not the later 2001 recession. The industry factor corrects the common dynamics to capture the sector-specific stress during the banking crisis periods of 1986-1990 and 2007-09. It also adjusts the industry default rate to the observed low rates during the 2001 recession. Finally, over the 2008 crisis, the industry factors have a lower impact compared to what is captured by the macroeconomic factors.

We conclude that industry factors are important to capture default rates at the industry level. The bottom graphs of Figure 9 indicate that industry-specific developments may cancel to some extent, at least in a large diversified loan portfolio. If a portfolio is not well diversified, however, but exhibits clear industry concentrations, industry-specific effects may form a dominant cause for default clustering.

5 Conclusion

We have presented a new decomposition of systematic default risk based on a novel methodological framework. Our mixed measurement dynamic factor model allows us to jointly model discrete and continuous economic variables. We used this framework to model default counts together with continuous-valued macroeconomic and financial data. Based on the factor decomposition, we can quantify the contribution of macro, frailty, and contagion or industry risk to overall default rate volatility. In the empirical study for US data, we found that approximately 30% of default rate volatility at the industry and rating level is systematic. Approximately half of systematic risk variation is caused by macroeconomic and financial activity, while the other half is caused by frailty and industry factors, in approximately equal proportions. Therefore, credit risk management at the portfolio level should account for the three sources of risk. In particular, typical industry models that account for macroeconomic dependence only, do not account for about half of the systematic risk. This is a warning signal from a financial stability perspective.

We have also given evidence that the composition of systematic risk varies over time. In particular, there has been a gradual build-up of systematic risk over the period 2002-2007. Such patterns can be used as early warning signals for financial institutions and supervising agencies. If the degree of systematic comovements between credits exposures increases through time, the fragility of the financial system may increase and prompt for
adequate (re)action.

Finally, the contributions of the frailty and industry components to systematic credit risk volatility change through time. The effects turned out to be most important before and during high-default years in our sample. The direction of the effect, however, differs over time. Whereas frailty helps to capture rapid rises in defaults around the 1991 and 2001 recessions, the component is needed to explain the low default frequencies before the 2007-2008 financial crisis. During the more recent crisis, large changes in macro indicators suffice to pick up the rises in defaults over 2008.

Our results have a clear bearing for risk management at financial institutions. When conducting risk analysis at the portfolio level, the frailty and industry components cannot be discarded. This is confirmed in a risk management experiment using a stylized loan portfolios. The extreme tail clustering in defaults cannot be captured using macro variables alone. Additional, unknown sources of default volatility such as frailty and contagion are needed to capture the patterns in the empirical data.

References


Appendix A1: factors for exponential family data

We assume throughout that each \( p_j(\cdot) \) is a member of the exponential family. This means that we can write

\[
p_j(y_{jt}; \mathcal{F}, \psi) = \exp \left[ y_{jt} \theta_j - b_j(\theta_j) + c_j(y_{jt}) \right].
\]

The exponential family in (A.9) is defined in terms of a signal \( \theta_j \) and two functions \( b_j(\cdot) \) and \( c_j(\cdot) \). The function \( b_j(\cdot) \) depends only on the signal \( \theta_j \), and \( c_j(\cdot) \) depends on the observed data \( y_{jt} \) and not on \( \theta_j \). For example, the normal distribution for given \( \sigma_{jt} \) is recovered by setting

\[
\theta_j = \frac{\mu_{jt}}{\sigma^2_{jt}}, \quad b_j(\theta_j) = \theta_j^2 \sigma^2_{jt} + \ln(2\pi \sigma^2_{jt}), \quad c_j(y_{jt}) = \frac{-y^2_{jt}}{2\sigma^2_{jt}}.
\]

The binomial distribution can also be seen to be a special case by setting

\[
\theta_j = \ln \left( \frac{\pi_{jt}}{1 - \pi_{jt}} \right), \quad b_j(\theta_j) = \ln(1 + \exp(\theta_j)), \quad c_j(y_{jt}) = \ln \left( \frac{k_{jt}^l}{y_{jt}(k_{jt} - y_{jt})} \right).
\]

One can prove that the mean and variance of (A.9) are given by \( \partial b(\theta_j)/\partial \theta_j \) and \( \partial^2 b(\theta_j)/\partial \theta^2_j \), respectively, see McCullagh and Nelder (1989, Ch. 2.2).

Note that the signal \( \theta_j \) has a natural interpretation in both (A.10) and (A.11). In (A.10), it is the re-scaled mean of the distribution, while in (A.11) it is the log-odds ratio given the probability of default. By the transformations defined in (A.10) and (A.11), the parameters of the corresponding distributions remain in the appropriate interval. For example, the inverse log-odds transformation \( \pi_{jt} = (1 + \exp(-\theta_j))/\exp(\theta_j) \) ensures that \( \pi_{jt} \) lies in the unit interval even though \( \theta_j \) may take values over the entire real line. Location parameters \( \theta_j \) are therefore our quantity of interest for the modeling of factor dependence, such as \( \theta_{jt} = \alpha_j + \lambda_j f_t \).

Appendix A2: estimation via importance sampling

An analytical expression for the the maximum likelihood (ML) estimate of parameter vector \( \psi \) for the MiMe DFM is not available. A feasible approach to the ML estimation of \( \psi \) given the likelihood in (7) is provided by importance sampling. The parameters are estimated via the direct maximization of the likelihood function that is evaluated by Monte Carlo integration. A short description of this approach is given below.

The observation density function of \( y = (y'_1, \ldots, y'_T)' \) can be expressed by the joint density of \( y \) and \( f = (f'_1, \ldots, f'_T)' \) where \( f \) is integrated out, that is

\[
p(y; \psi) = \int p(y, f; \psi)df = \int p(y|f; \psi)p(f; \psi)df,
\]

where \( p(y|f; \psi) \) is the density of \( y \) conditional on \( f \) and \( p(f; \psi) \) is the density of \( f \). A Monte Carlo estimator
of \( p(y; \psi) \) can be obtained by

\[
\hat{p}(y; \psi) = M^{-1} \sum_{k=1}^{M} p(y|f^{(k)}; \psi), \quad f^{(k)} \sim p(f; \psi),
\]

for some large integer \( M \). The estimator \( \hat{p}(y; \psi) \) is however numerically inefficient since most draws \( f^{(k)} \) will not contribute substantially to \( p(y|f; \psi) \) for any \( \psi \) and \( k = 1, \ldots, K \). Importance sampling improves the Monte Carlo estimation of \( p(y; \psi) \) by sampling \( f \) from the Gaussian importance density \( g(f|y; \psi) \). We can express the observation density function \( p(y; \psi) \) by

\[
p(y; \psi) = \int \frac{p(y, f; \psi)}{g(f|y; \psi)} g(f|y; \psi) df = g(y; \psi) \int \frac{p(y|f; \psi)}{g(y|f; \psi)} g(f|y; \psi) df,
\]

(A.13)

since \( f \) is from a Gaussian density such that \( g(f; \psi) = p(f; \psi) \) and therefore we have \( g(y; \psi) = g(y, f; \psi) / g(f|y; \psi) \).

In case \( g(f|y; \psi) \) is close to \( p(f|y; \psi) \) and in case simulation from \( g(f|y; \psi) \) is feasible, the Monte Carlo estimator implied by (A.13) and given by

\[
\tilde{p}(y; \psi) = g(y; \psi) M^{-1} \sum_{k=1}^{M} \frac{p(y|f^{(k)}; \psi)}{g(y|f^{(k)}; \psi)}, \quad f^{(k)} \sim g(f|y; \psi),
\]

(A.14)

is numerically much more efficient, see Kloek and van Dijk (1978), Geweke (1989) and Durbin and Koopman (2001).

For a practical implementation, the importance density \( g(f|y; \psi) \) can be based on the linear Gaussian approximating model

\[
y_{jt} = \mu_{jt} + \theta_{jt} + \varepsilon_{jt}, \quad \varepsilon_{jt} \sim N(0, \sigma_{jt}^2),
\]

(A.15)

where mean correction \( \mu_{jt} \) and variance \( \sigma_{jt}^2 \) are determined in such a way that \( g(f|y; \psi) \) is sufficiently close to \( p(f|y; \psi) \). It is argued by Shephard and Pitt (1997) and Durbin and Koopman (1997) that \( \mu_{jt} \) and \( \sigma_{jt} \) can be uniquely chosen such that the modes of \( p(f|y; \psi) \) and \( g(f|y; \psi) \) with respect to \( f \) are equal, for a given value of \( \psi \).

To simulate values from the importance density \( g(f|y; \psi) \), the simulation smoothing method of Durbin and Koopman (2002) can be applied to the approximating model (A.15). For a set of \( M \) draws of \( g(f|y; \psi) \), the evaluation of (A.14) relies on the computation of \( p(y|f; \psi) \), \( g(y|f; \psi) \) and \( g(y; \psi) \). Density \( p(y|f; \psi) \) is based on (3), density \( g(y|f; \psi) \) is based on the Gaussian density for \( y_{jt} - \mu_{jt} - \theta_{jt} \sim N(0, \sigma_{jt}^2) \) (A.15) and \( g(y; \psi) \) can be computed by Kalman filter applied to (A.15), see Harvey (1989).

Once the likelihood function can be evaluated for any value of \( \psi \) and for a given set of random numbers from which factors are simulated from \( g(f|y; \psi) \), we can maximize the likelihood function with respect to \( \psi \).

Once the ML estimate of \( \psi \) is available, we can estimate the latent factors \( f_{jt} \). This is also based on importance sampling. We have

\[
E(f|y; \psi) = \int f \cdot p(f|y; \psi) df = \int f \cdot \frac{p(y|f; \psi)}{g(y|f; \psi)} df.
\]

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After some minor algebra, it can be shown that the estimation of \(E(f|y; \psi)\) via importance sampling can be achieved by

\[
\hat{f} = \frac{\sum_{k=1}^{M} w_k \cdot f^{(k)}}{\sum_{k=1}^{M} w_k},
\]

with \(w_k = p(y|f^{(k)}; \psi)/g(y|f^{(k)}; \psi)\), and \(f^{(k)} \sim g(f|y; \psi)\). Similarly, the standard errors \(s_t\) of \(\hat{f}_t\) can be estimated by

\[
s^2_t = \left( \frac{\sum_{k=1}^{M} w_k \cdot (f^{(k)})^2}{\sum_{k=1}^{M} w_k} \right) - \hat{f}_t^2.
\]
Table 1: Macroeconomic Time Series Data

The table gives a full listing of included macroeconomic time series data $x_t$ and binary indicators $b_t$. All time series are obtained from the St. Louis Fed online database, http://research.stlouisfed.org/fred2/.

<table>
<thead>
<tr>
<th>Category</th>
<th>Summary of time series in category</th>
<th>Total no</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Macro indicators, and business cycle conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial production index</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disposable personal income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ISM Manufacturing index</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uni Michigan consumer sentiment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New housing permits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) Labour market conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Civilian unemployment rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median duration of unemployment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average weekly hours index</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total non-farm payrolls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) Monetary policy and financing conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Federal funds rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moody’s seasoned Baa corporate bond yield</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mortgage rates, 30 year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 year treasury rate, constant maturity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit spread corporates over treasuries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government bond term structure spread</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d) Bank lending</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Consumer Credit Outstanding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Real Estate Loans, all banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e) Cost of resources</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPI Fuels and related Energy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPI Finished Goods</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trade-weighted US dollar exchange rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f) Stock market returns</td>
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<td></td>
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<tr>
<td>S&amp;P 500 yearly returns</td>
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<td></td>
</tr>
<tr>
<td>S&amp;P 500 return volatility</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Table 2: Parameter estimates, binomial part
We report parameter estimates associated with the binomial data. The coefficients in the first column combine to fixed effects for each cross section, according to rating group and industry sector. The middle columns refer to common factors $f^m_t$ and give the respective factor loadings. The last column gives the factor loadings for frailty factor $f^d_t$ and industry-specific factors, $f^i_t$, respectively. Estimation sample is 1971Q1 to 2009Q1.

<table>
<thead>
<tr>
<th>Intercepts $\lambda_j$</th>
<th>Loadings $f^m_t$</th>
<th>Loadings $f^d_t$</th>
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</thead>
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<tr>
<td><em>par</em> <em>val</em></td>
<td><em>par</em> <em>val</em></td>
<td><em>par</em> <em>val</em></td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>-2.52</td>
<td>$\phi_{m,1}$</td>
</tr>
<tr>
<td>$\lambda_{fin}$</td>
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<td>$\beta_{1,IG}$</td>
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<tr>
<td>$\lambda_{tra}$</td>
<td>-0.16</td>
<td>$\beta_{1,Ba}$</td>
</tr>
<tr>
<td>$\lambda_{lei}$</td>
<td>-0.17</td>
<td>$\beta_{1,B}$</td>
</tr>
<tr>
<td>$\lambda_{utl}$</td>
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<td>$\beta_{1,C}$</td>
</tr>
<tr>
<td>$\lambda_{tec}$</td>
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<td>$\phi_{m,2}$</td>
</tr>
<tr>
<td>$\lambda_{ref}$</td>
<td>-0.33</td>
<td>$\beta_{2,IG}$</td>
</tr>
<tr>
<td>$\lambda_{IG}$</td>
<td>-7.13</td>
<td>$\beta_{2,Ba}$</td>
</tr>
<tr>
<td>$\lambda_{BB}$</td>
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<td>$\beta_{2,B}$</td>
</tr>
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<td>$\lambda_{B}$</td>
<td>-2.12</td>
<td>$\beta_{2,C}$</td>
</tr>
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<td></td>
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</tr>
</tbody>
</table>
Table 3: A decomposition of total default risk
The table decomposes total, i.e. systematic and idiosyncratic, default risk into four unobserved constituents. We distinguish (i) common variation in defaults with observed macroeconomic and financial data, (ii) latent default-specific (frailty) risk, (iii) latent industry-sector dynamics, and (iv) non-systematic, and therefore diversifiable risk. The decomposition is based on data from 1971Q1 to 2009Q1.

<table>
<thead>
<tr>
<th>Data</th>
<th>Business cycle</th>
<th>Frailty risk</th>
<th>Industry-level</th>
<th>Idiosyncratic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f^m_t$</td>
<td>$f^d_t$</td>
<td>$f^i_t$</td>
<td>distr.</td>
</tr>
<tr>
<td>Pooled</td>
<td>17.0%</td>
<td>6.3%</td>
<td>8.0%</td>
<td>68.7%</td>
</tr>
<tr>
<td>Rating groups:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aaa-Baa</td>
<td>10.5%</td>
<td>0.7%</td>
<td>4.5%</td>
<td>84.3%</td>
</tr>
<tr>
<td>Ba</td>
<td>7.5%</td>
<td>5.8%</td>
<td>7.7%</td>
<td>79.1%</td>
</tr>
<tr>
<td>B</td>
<td>24.7%</td>
<td>6.8%</td>
<td>7.6%</td>
<td>61.0%</td>
</tr>
<tr>
<td>Caa-C</td>
<td>13.4%</td>
<td>7.0%</td>
<td>9.5%</td>
<td>70.0%</td>
</tr>
<tr>
<td>Industry sectors:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank</td>
<td>10.2%</td>
<td>7.1%</td>
<td>13.7%</td>
<td>68.9%</td>
</tr>
<tr>
<td>Financial non-Bank</td>
<td>6.5%</td>
<td>3.2%</td>
<td>8.0%</td>
<td>82.3%</td>
</tr>
<tr>
<td>Transportation</td>
<td>11.9%</td>
<td>6.5%</td>
<td>12.1%</td>
<td>69.5%</td>
</tr>
<tr>
<td>Media</td>
<td>21.9%</td>
<td>8.1%</td>
<td>6.8%</td>
<td>63.2%</td>
</tr>
<tr>
<td>Leisure</td>
<td>22.4%</td>
<td>4.3%</td>
<td>4.1%</td>
<td>69.2%</td>
</tr>
<tr>
<td>Utilities</td>
<td>8.0%</td>
<td>2.8%</td>
<td>3.3%</td>
<td>85.9%</td>
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<tr>
<td>Energy</td>
<td>13.1%</td>
<td>8.7%</td>
<td>23.0%</td>
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<tr>
<td>Industrial</td>
<td>28.5%</td>
<td>8.9%</td>
<td>1.9%</td>
<td>60.7%</td>
</tr>
<tr>
<td>High Tech</td>
<td>19.6%</td>
<td>6.9%</td>
<td>9.4%</td>
<td>64.0%</td>
</tr>
<tr>
<td>Retail</td>
<td>9.7%</td>
<td>3.9%</td>
<td>8.2%</td>
<td>78.2%</td>
</tr>
<tr>
<td>Consumer Goods</td>
<td>17.8%</td>
<td>4.0%</td>
<td>4.4%</td>
<td>73.9%</td>
</tr>
<tr>
<td>Misc</td>
<td>10.8%</td>
<td>4.4%</td>
<td>2.6%</td>
<td>82.2%</td>
</tr>
</tbody>
</table>
Figure 1: Models and reductions in the Kullback-Leibler divergence
The graph shows how reductions in the estimated $KL$ divergence are used to decompose the total variation in non-Gaussian default counts into risk shares corresponding to increasing sets of latent factors.
Figure 2: Clustering in default data

The top graph plots (i) the total number of defaults in the Moody’s database $\sum_j y_{jt}$, (ii) the total number of exposures $\sum_j k_{jt}$, and (iii) the aggregate default rate for all Moody’s rated US firms, $\sum_j y_{jt} / \sum_j k_{jt}$.

The bottom graph plots time series of default fractions $y_{jt}/k_{jt}$ over time. We distinguish four broad rating groups, i.e., Aaa − Baa, Ba, B, and Caa − C, where each plot contains 12 time series of industry-specific default fractions.
Figure 3: Macroeconomic and financial time series data
The graph contains times series plots of yearly growth rates in macroeconomic and financial data. For a listing of the data we refer to Table 1.
Figure 4: Smoothed Macroeconomic Risk Factors
The figure plots conditional mean estimates for four latent risk factors. These factors are common to all mixed measurement data, i.e. macros and default counts. We also plot approximate standard error bands at a 0.05 significance level.
Figure 5: Shares of explained variation in macro and financial time series data

The figure indicates which share of variation in each time series listed in Table 1 can be attributed to each factor $f^m$. Factors $f^m$ are common to the (continuous) macro and financial as well as the (discrete) default count data.
Figure 6: Smoothed Frailty Risk Factor and Industry-group dynamics
The top graph shows the estimated frailty risk factor, which is assumed common to all default counts. The second graph plots six industry-specific risk factors along with asymptotic standard error bands at a 0.05 significance level. High risk factor values imply higher expected default rates.
**Figure 7: Model fit to observed aggregate default rate**

Each panel plots the observed quarterly default rate for all rated firms against the default rate implied by different model specifications. The models feature either (a) no factors, (b) only macro factors $f^m$, (c) macro factors and a frailty component $f^m, f^d$, and (d) all factors $f^m, f^d, f^i$, respectively.
Figure 8: Time variation in risk shares

We plot risk shares estimated over a rolling window of eight quarters from 1971Q1 to 2009Q1. Shaded areas correspond to recession periods as dated by the NBER.
Figure 9: Real vs. model-implied credit portfolio loss distribution

The distribution plots refer to a credit portfolio with uniform loan exposures. The first graph shows the portfolio loss distribution as implied by actual defaults and exposures in the database. The horizontal axis measures quarterly loan losses as a fraction of the total portfolio value. The second to fourth panel plot the portfolio loss distribution as implied by an econometric model with macro factors $f^m_t$, macro factors and a frailty component $f^m_t, f^d_t$, and all factors $f^m_t, f^d_t, f^i_t$, respectively. The considered sample is 1981Q1 to 2008Q4.
We plot smoothed estimates of quarterly time-varying default rate for the financial sector. We distinguish a model with (i) common variation with macro data only, (ii) macro factors and a frailty component, and (iii) macro factors, frailty component, and industry-specific factors, respectively. The model-implied quarterly rates are graphed against the observed default fractions for financial firms.

Figure 10: Quarterly time-varying default intensities for financial firms