Foreign Prices Shocks in a Small Open Economy

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Abstract

Countries specialize in producing goods that they have comparative advantages in producing. This results in a country exporting some goods while it imports other. Hence, there is a reason to expect that changes in the prices of these goods have considerable economic effect and that demand management can be used to improve welfare following such changes. This paper analyses this using a New Open Economy Macro (NOEM) model of a small open economy. Among others, the results indicate that, in a small open economy, a terms of trade appreciation results in increased consumption, labor use and output on impact while consumption increases but labor use and output decrease in future time periods. The results also indicate that the vulnerability of an economy towards such shocks is negatively related to its size. Finally, the results indicate that there exists a welfare improving demand management policy following a terms of trade shock.

JEL classification: E63, F41

Keywords: Open Economy Macroeconomics, New Open Economy Macro Models, small open economy, tradeables, exportables, importables, terms of trade, demand management, stabilization

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I would like to thank Torben M. Andersen for many helpful comments. All remaining errors are, of course, mine.}
1 Introduction

According to economic theory, an economy specializes in producing goods that it has comparative advantage in producing (cf. the Ricardian trade model). The economy then uses a part of the production itself and exports the rest while it imports other goods. Such comparative advantage may be due to availability of natural resources or factor abundance, which results in lower production costs of the goods in question relative to other countries.

As real life examples consider Iceland, Norway and Denmark. Iceland is surrounded by excellent fishing areas at sea, which has resulted in Iceland specializing in producing and exporting fish products while its import of it is negligible. In 2005, export of fish products in Iceland accounted for 52% of goods export while import of it accounted for only 2% of goods import. In Norway, crude oil and natural gas are the main exporting goods. In 2004, those goods accounted for 46% of export of goods and services while it accounted for only 0.5% of its imports. In Denmark, agricultural products have been the main exporting goods, although their share in goods export has decreased during the last 50 years (from around 54% in 1960 to 17% in 2005).

In a small open economy, foreign prices, i.e. world market prices denoted in foreign currency, of exporting and importing goods are exogenous since it can be assumed that a domestic producer has small market share in the world market for his good and that domestic households consume a small part of the world production of a good. However, domestic prices are endogenous since they also depends on factors that are endogenous to the economy. One such factor is the nominal exchange rate.

Given the exogeneity of foreign prices, there is a reason to expect that changes in them have considerable economic effects in a small open economy and that demand management can be used to improve welfare following changes in them. This paper analyses if this is the case. To achieve this a NOEM model of a small open economy is used where incomplete capital markets and incomplete nominal adjustment in the labor market are assumed. The model is a variation of the Obstfeld and Rogoff (1995) and Lane (1997) small open economy macro models where only traded goods are produced and consumed in the economy and nominal wages are fixed one period.

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2 See www.statice.is.
3 See www.statbank.ssb.no.
4 See www.dst.dk/aarbog.
5 Many papers have analysed foreign prices shocks since the oil price shocks in the seventies. See, for example, Macklem (1993). The present paper is, however, the first one to do this in a NOEM setting.
6 Here is referred to the small open economy model introduced in the appendix to the Obstfeld and Rogoff (1995) Redux paper.
ahead due to incomplete nominal adjustment in the labor market. It is an extension to these models in three ways: First, uncertainty is introduced. Second, shocks to foreign prices are introduced to the model. Third, two groups of traded goods are used instead of one.\footnote{In fact, the model further extends these models since it does not constrain the intertemporal elasticity of substitution in consumption to equal 1.}

Since the birth of the NOEM models with the Obstfeld and Rogoff (1995) Redux paper there have been considerable attention paid to modelling small open economies using this framework. However, these papers do not analyze how foreign price shocks affect the economy and how demand management can be used to improve welfare following such shocks.\footnote{See, for example, Kollmann (1997), Rankin (1998), Andersen and Holden (2002), Gali and Monacelli (2005), Faia and Monacelli (2006) and De Paoli (2006).} This paper does exactly this.

The paper is organized as follows: Chapter 2 sets up the model economy. In chapter 3 the model is solved and the results are discussed in chapter 4. Chapter 5 analyses if and how demand management can be used to improve welfare following shocks to foreign prices. The paper then concludes.

## 2 The Model Economy

The model economy consists of firms, households and a government. The economy has comparative advantages in producing certain goods (exportables), which are partly consumed by domestic households and the rest is exported, while it imports other goods (importables) for consumption by domestic households.\footnote{Together, exportables and importables are called tradeables in the paper.} There are infinitely many firms in the economy and each of them takes the domestic price of its good as given while they determine output by profit maximization. Hence, it is assumed that each firm exports only a small share of the total export in the economy and therefore correctly expects that its decisions do not affect the nominal exchange rate, which is a reasonable assumption since there are infinitely many firms producing exportables in the economy.\footnote{Remember that the foreign prices of exportables and importables are exogenous as is discussed in the introduction.} There are infinitely many households in the economy which have infinite time horizons. The households supply labor for production and consume exportables and importables. Each household is assumed to take the domestic price of each good as given when making its consumption decisions, which is a reasonable assumption since there are infinitely many households in the economy. Hence, like each firm, each household correctly expects that its decisions do not affect the nominal exchange rate. Note that although the nominal exchange rate is assumed to be
exogenous to each firm in the economy it is endogenous to the economy as a whole, as is discussed in the introduction to the paper. The economy faces shocks to foreign import and export prices (foreign prices of importables and exportables) while foreign real interest rates are assumed to be constant. The government conducts its demand management policy by controlling nominal transfers to households while the nominal exchange rate is allowed to float freely.

2.1 Firms

Labor is the only variable factor of production in the economy, which is a standard assumption in the NOEM literature.\textsuperscript{11} Since symmetry between firms is assumed when solving the model the analysis below can be simplified by assuming that there is only one firm and one exportable good (exportables) produced in the economy (while the firm still takes the domestic price of its good as given when deciding on how much to produce), which is what is done in what follows.\textsuperscript{12}

Labor use is the following in time $t$:

$$L_t = \left[ \int_{j=0}^{1} L_t(j) \frac{\delta-1}{\phi} dj \right]^{\frac{\phi}{\phi-1}} \tag{2.1}$$

where $L_t(i,j)$ is the use of labor from household $j$ in production in period $t$ and $\phi > 1$ is the elasticity of substitution in production between labor from different households. Heterogenous labor is assumed, which gives market power in the labor market, to provide microeconomic foundations for fixing the nominal wage rate one period ahead (incomplete nominal adjustment). Total production of exportables is the following:

$$Y_t = L_t^\alpha \tag{2.2}$$

where $0 < \alpha < 1$. Hence, diminishing marginal product of labor is assumed.\textsuperscript{13}

The firm chooses the amount of labor it uses from each household: $j \in [0,1]$ by minimizing the cost of producing a certain quantity of the good ($Y_t$). This gives the firm’s demand for labor from household $j$:

$$L_t(j) = \left[ \frac{W_t(j)}{W_t} \right]^{-\phi} Y_t^{\frac{1}{\phi}} \tag{2.3}$$

\textsuperscript{11}See discussion on page 231 in Walsh (2003).

\textsuperscript{12}Assuming an infinite number of firms producing exportables only makes the analysis more complicated due to more notation and does not give different results to the analysis that follows.

\textsuperscript{13}The assumption of diminishing marginal product of labor is necessary to get a unique and finite output level for a given output price and a given nominal wage rate. To justify this one could tell the story that high technology goods are being produced, which requires skilled labor.
where \( W_t(j) \) is the nominal wage paid to labor from household \( j \) and \( W_t \) is the aggregate wage index:

\[
W_t = \left[ \prod_{j=0}^{1} W_t(j)^{1-\phi} \right]^{\frac{1}{1-\phi}} \tag{2.4}
\]

The law of one price is assumed to hold for exportables. Hence, the domestic price of exportables is:

\[
P_{E,t} = S_t P_{EF,t} \tag{2.5}
\]

where \( P_{EF,t} \) is the foreign price of exportables and \( S_t \) is the nominal exchange rate. The firm producing exportables chooses the quantity it produces such that its profits:

\[
\Pi_t = P_{E,t} Y_t - \prod_{j=0}^{1} W_t(j) L_t(j) \tag{2.6}
\]

are maximized given information about the foreign price of the good and the nominal exchange rate. This gives the following solution for the quantity produced:

\[
Y_t = \alpha \frac{\phi}{1-\phi} \left( \frac{P_{E,t}}{W_t} \right)^{\frac{\phi}{1-\phi}} \tag{2.7}
\]

From 2.7 (and 2.5) it is obvious that output and labor use (from 2.2) are increasing in the foreign price of exportables. Further, maximum profits are (i) positive and (ii) increasing in the price of exportables:

\[
\Pi_t > 0 \tag{2.8}
\]

\[
\frac{\partial \Pi_t}{\partial P_{E,t}} > 0 \tag{2.9}
\]

Hence, for a given nominal exchange rate and nominal wage rate, a positive shock to the foreign price of exportables results in increased profits by the firm.

To sum up, an increase in the foreign price of exportables results in increased output, labor use and labor income by the households and profits by the firms, for a given nominal exchange rate and nominal wage rate. This indicates that a positive shock to the foreign price of exportables has economic effects by increasing income (and wealth) in the economy. Note that this is only a partial equilibrium analysis since nominal wages and the nominal exchange rate are held constant, and it is crucial

\[14\text{Note that } P_{EF,t} \text{ is exogenous in the model as is discussed in the introduction.}\]

\[15\text{This is shown in appendix 1.}\]

\[16\text{Note that according to this a decrease in the foreign price of exportables can never result in negative profits for the firm. This is since maximum profits measure the firm’s profits after it has adjusted its labor use (and output) following a change in the price.}\]

\[17\text{This is shown in appendix 2.}\]
for the general equilibrium outcome how the nominal exchange rate and the nominal wage rate develop following a shock to the foreign price of exportables.

2.2 Households

There is a continuum of households: \( j \in [0, 1] \) supplying labor in the economy. No labor mobility between countries is assumed. Each household maximizes the expected present value of the sum of utility now and in the future. Household \( j \)'s instantaneous utility function is the following in time \( t \):

\[
U_t(j) = C_t(j)^{1-\rho} + \lambda \left( \frac{M_t(j)}{P_t} \right)^{1-\varepsilon} - \tau \left( \frac{L_t(j)^{\nu+1}}{\nu+1} \right)
\]

and its budget constraint is:

\[
M_t(j) + P_t C_t(j) + P_t F_t(j) = M_{t-1}(j) + T_t(j) + W_t(j) L_t(j) + \Pi_t(j) + P_t (1 + r) F_{t-1}(j)
\]

where \( \rho \geq 0 \) is the inverse of the intertemporal elasticity of substitution in consumption and \( \varepsilon \geq 0, \nu \geq 0, \lambda > 0 \) and \( \tau > 0 \) are parameters, \( C_t(j) \) is consumption by household \( j \) in period \( t \), \( M_t(j) \) is money holding by household \( j \) at the end of period \( t \), \( P_t \) is the price level in the economy (see the discussion below), \( T_t(j) \) are nominal transfers from the government to household \( j \), \( \Pi_t(j) \) are profits distributed from the firm to household \( j \), \( F_t(j) \) is (net) holding of real foreign bonds by household \( j \) at the end of period \( t \) and \( r \) is the real interest rate on foreign bonds. Since the economy is a small one the foreign, or world, real interest rate \( (r) \) is assumed to be exogenous to the model. It is also assumed that the domestic firm is owned by domestic households and that only domestic households own shares in it and, hence, that there are no trade in stocks between countries. Finally, a household's behavior is constrained by a no-Ponzi game condition:

\[
\lim_{i \to \infty} \frac{F_{t+i}(j)}{(1 + r)^i} = 0
\]

\( ^{18} \)Hence, only domestic labor is used in domestic production and domestic households only supply labor in the domestic labor market.

\( ^{19} \)Note that \( \Pi_t = \int_{j=0}^{1} \Pi_t(j) dj \) where \( \Pi_t \) is given in 2.6.

\( ^{20} \)Note that since the purchasing power parity (PPP) holds in this model, domestic real interest rates equal the foreign ones.

\( ^{21} \)It is also assumed that only domestic households have demand for domestic currency. This can be justified by the fact that this is a small economy and, hence, there is little probability that it is used as money in other countries.
Two points require discussion about the budget constraint in 2.11. First, perfect foreign bonds market is assumed in the sense that domestic households can sell or buy whatever quantity of foreign bonds they wish to at the real interest rate level \(r\). Second, it is assumed that capital markets are incomplete in the way that domestic households do not achieve sufficient diversification compared to what they else would. The empirical evidence presented in Lewis (1999) supports this assumption. The lack of diversification can be reflected in more correlation between domestic consumption and output then there would be if the markets were complete. One simple way to incorporate this in the model is to assume that there is no trade in stocks between countries, like is done here.

Household \(j\) needs to solve two problems. First, since nominal wages are fixed one period ahead, it chooses \(W_t(j)\) before it observes the shocks in period \(t\) and then \(C_t(j), M_t(j)\) and \(F_t(j)\) after it observes the shocks. Hence, the household chooses \(W_t(j)\) such that the following is maximized:

\[
E_{t-1} \sum_{i=0}^{\infty} \delta^i U_{t+i}(j) \tag{2.13}
\]

subject to the budget constraint in 2.11 and the no-Ponzi game condition in 2.12 and taking the firms’ demand for labor as given (by 2.3), where \(E_{t-1}\) is an expectations operator conditional on available information before the shocks are observed, i.e. conditional on information available in period \(t-1\), and \(\delta\) is the subjective discount factor, where \(0 < \delta < 1\). The solution to this maximization problem is the following for household \(j\):\(^{22}\)

\[
W_t(j) = \frac{\phi}{\phi - 1} \frac{E_{t-1} [L_t(j)^{v+1}]}{E_{t-1} [C_t(j)^{-r} L_t(j) / P_t]} \tag{2.14}
\]

The household chooses \(C_t(j), M_t(j)\) and \(F_t(j)\) such that it maximizes:

\[
E_t \sum_{i=0}^{\infty} \delta^i U_{t+i}(j) \tag{2.15}
\]

subject to the budget constraint in 2.11 and the no-Ponzi game condition in 2.12. The solution to this maximization problem gives the following first order conditions (in addition to the budget constraint in 2.11) for household \(j\):

\[
\delta E_t \left[ C_{t+1}(j)^{-\rho} P_t / P_{t+1} \right] + \lambda \left[ M_t(j) / P_t \right]^{-\varepsilon} = C_t(j)^{-\rho} \tag{2.16}
\]

\[
\delta E_t \left[ C_{t+1}(j)^{-\rho} (1 + r) \right] = C_t(j)^{-\rho} \tag{2.17}
\]

\(^{22}\)The calculations for deriving this result are well known in the NOEM literature. Hence, they are not detailed here. The same applies to the results in 2.16, 2.17 and 2.19 - 2.21.
Second, household $j$ needs to decide on its consumption of exportables and importables in period $t$. Its consumption index in period $t$ is defined in the following way:

$$
C_t(j) = \left[ \gamma \frac{\kappa}{\kappa - 1} C_{E,t}(j)^{\frac{\kappa - 1}{\kappa}} + (1 - \gamma) \frac{\kappa}{\kappa - 1} C_{I,t}(j)^{\frac{\kappa - 1}{\kappa}} \right]^{\frac{\kappa}{\kappa - 1}}
$$

(2.18)

where $\kappa$ is the elasticity of substitution between exportables and importables, $\gamma$ is the share of exportables in $C_t(j)$ if the relative price of exportables and importables equals 1, $C_{E,t}(j)$ is the household’s consumption of exportables in time $t$ and $C_{I,t}(j)$ is its consumption of importables. Note that the parameter $\gamma$ can be interpreted as a measure of how large the economy is relative to the rest of the world, where a higher value of $\gamma$ indicates a larger economy. The household’s demand functions for exportables and importables are obtained by minimizing the cost of consuming a certain amount of the consumption composite ($C_t(j)$):

$$
C_{E,t}(j) = \gamma \left( \frac{P_{E,t}}{P_t} \right)^{-\kappa} C_t(j)
$$

(2.19)

$$
C_{I,t}(j) = (1 - \gamma) \left( \frac{P_{I,t}}{P_t} \right)^{-\kappa} C_t(j)
$$

(2.20)

and the price index is:

$$
P_t = \left[ \gamma P_{E,t}^{1-\kappa} + (1 - \gamma) P_{I,t}^{1-\kappa} \right]^{\frac{1}{1-\kappa}}
$$

(2.21)

where $P_{I,t}$ is the domestic price of importables in time $t$. The law of one price is assumed to hold for importables:

$$
P_{I,t} = S_t P_{IF,t}
$$

(2.22)

where $P_{IF,t}$ is the foreign price of importables.

### 2.3 The government

The government distributes transfers to the households and finances them by issuing money through the central bank. The government’s budget constraint is therefore the following:

$$
M_t = M_{t-1} + T_t
$$

(2.23)

where $T_t = \int_{j=0}^{1} T_t(j) dj$ are (total) nominal transfers distributed to the households in period $t$ and $M_t = \int_{j=0}^{1} M_t(j) dj$ is (total) money supply in the end of the period $t$. 

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2.4 Terms of trade and the current account

Terms of trade are defined as the relative domestic price of exportables and importables:

\[
TOT_t = \frac{P_{E,t}}{P_{I,t}} = \frac{P_{EF,t}}{P_{IF,t}}
\]  
(2.24)

where 2.5 and 2.22 are used. Hence, terms of trade are exogenous in the model.

Since the nominal exchange rate is allowed to float freely in the model it must adjust such that the current account \( (CA_t(j)) \) equals the increase in foreign bonds holding minus the interest rate income on foreign bonds:\(^23\)

\[
CA_t(j) = F_t(j) - (1 + r)F_{t-1}(j)
\]  
(2.25)

2.5 Market equilibrium

There are four (groups of) markets in the model: Labor market, goods markets, a foreign bonds market and a money market. In the money market, supply is exogenous to the model and the endogenous variables in the model adjust to set money demand (determined by 2.16) equal to its supply. This holds for all households: \( j \in [0,1] \).

In the foreign bonds market, domestic households are price takers (\( r \) is exogenous) and can buy or sell any quantity they want of foreign bonds at the prevailing real interest rate level (\( r \)). The optimal quantity of foreign bonds is determined by 2.17. This holds for all households: \( j \in [0,1] \).

In the market for exportables, the firm takes the domestic price of the good as given by 2.5 and produces its optimal quantity given that price (determined by 2.7). Note that the quantity produced of exportables does not have to be the same as the quantity demanded by domestic households (determined by 2.19 after integrating over households) since this is an open economy. Exportables that are not consumed by domestic households are exported and sold abroad.\(^24\)

In the market for importables, domestic households take the domestice price as given by 2.22 and import the quantity they demand at that price (determined by 2.20). This holds for all households: \( j \in [0,1] \).

In the market for labor, each household sets nominal wages using 2.14 and supplies the demanded labor at that wage rate. The quantity of labor is determined by 2.3. This holds for all households: \( j \in [0,1] \).

\(^{23}\)In other words, the balance of payments has to equal zero at every point in time. This is implied by the households’ and the government’s budget constraints.

\(^{24}\)Note that it is assumed that the economy has comparative advantage in producing exportables and that its production of it always exceeds domestic demand.
3 The Model Solved

Symmetry among domestic households is assumed when solving the model. The equations that are used to solve the model are summarized in the following table.\textsuperscript{25}

Table 1. Equations used to solve the model

\begin{align*}
\text{(E1)} \quad & \delta E_t \left( C_t^{\rho - 1} \frac{P_t}{F_t + 1} \right) + \lambda \left( \frac{M_t}{P_t} \right)^{-\frac{1}{\gamma}} = C_t^{\rho} \\
\text{(E2)} \quad & \delta E_t \left[ C_t^{\rho - 1} (1 + \tau) \right] = C_t^{\rho} \\
\text{(E3)} \quad & W_t = \tau \frac{E_{t-1}(L_{t+1})}{E_{t}(W)} \\
\text{(E4)} \quad & P_{t,E,t} = S_t P_{EF,t} \\
\text{(E5)} \quad & P_{t,I,t} = S_t P_{IF,t} \\
\text{(E6)} \quad & P_t = \left[ \gamma P_{t,E}^{1-\kappa} + (1 - \gamma) P_{t,I}^{1-\kappa} \right]^{\frac{1}{1-\kappa}} \\
\text{(E7)} \quad & Y_t = \alpha \frac{P_t}{W_t} \left( \frac{P_{t,E}}{W} \right)^{\frac{\alpha}{1-\alpha}} \\
\text{(E8)} \quad & P_t C_t + P_t F_t - (1 + \tau) P_t F_{t-1} = P_{t,E,t} Y_t \\
\text{(E9)} \quad & Y_t = L_t^p \\
\text{(E10)} \quad & M_t = M_{t-1} + T_t
\end{align*}

The equation system contains 10 equations and 10 endogenous variables: \( C_t, Y_t, L_t, W_t, P_t, P_{t,E,t}, P_{t,I,t}, S_t, M_t \) and \( F_t \), which is a necessary condition for a unique solution to the system to exist. The exogenous variables are: \( T_t, P_{EF,t} \) and \( P_{IF,t} \).\textsuperscript{26} The system is, however, non-linear and it is therefore uncertain if a unique solution, or a solution at all, exists to it. The paper therefore continues by solving the system for a steady state and then write it in deviations from steady state to obtain a dynamic solution to the system.

3.1 The steady state

A steady state is derived and discussed in appendix 3. In it, all exogenous variables are assumed to be constant over time and, hence, neither uncertainty nor inflation is assumed in the steady state.\textsuperscript{27} It is, however, distortionary due to market power in the labor market. Further, foreign bonds holding and nominal transfers are assumed to equal zero in steady state, i.e. \( F = 0 \) and \( T = 0 \), which are reasonable assumptions, and that the foreign prices of exportables and importables equal 1 in steady state, i.e. \( P_{EF} = P_{IF} = 1 \).\textsuperscript{28}

\textsuperscript{25}Note that E8 is the economy wide resource constraint which is obtained by plugging 2.23 into 2.11.
\textsuperscript{26}\( T_t \) is made endogenous below when an endogenous demand management policy is defined (see 3.5).
\textsuperscript{27}Steady state of a variable is denoted by not including a time subscript.
\textsuperscript{28}\( T = 0 \) in steady state follows from the assumptions below, i.e. that demand management only involves responding to deviations in foreign prices from their steady state values (see 3.5).
3.2 The dynamic solution

It is well known in the NOEM literature that due to the effects that shocks have on foreign bonds holding, the steady state of the model changes when shocks occur.29 This can cause problems when solving a NOEM model if incomplete capital markets are assumed since unconditional moments of endogenous variables may not be well defined. In this paper, the model is solved assuming incomplete capital markets. This can be justified here since the main interest is in analysing the effects of shocks on economic variables, i.e. their impulse responses, and not the moments of variables. Further, using the results of Schmitt-Grohe and Uribe (2003), assuming complete capital markets in a small open economy model gives virtually the same dynamics on business cycle frequencies following a productivity shock as assuming incomplete capital markets does. The only notable difference is that consumption is smoother when complete capital markets are assumed.

3.2.1 Assumptions

The dynamic solution to the model assumes the following five assumptions:

First, the exogenous variables; $P_{EF,t}$ and $P_{IF,t}$, are ex-ante log-normally distributed. This implies that a logarithmic transformation of the variables gives ex-ante normally distributed variables:

$$x_t \sim N [E_{t-1} x_t, Var_{t-1} (x_t)]$$  \hspace{1cm} (3.1)

where $x_t = \ln(X_t)$, $X_t = P_{EF,t}, P_{IF,t}$, $E_{t-1} x_t$ is the ex-ante expected value of $x_t$ given available information in time $t - 1$ and $Var_{t-1} (x_t)$ is the ex-ante variance of $x_t$ given available information in time $t - 1$. Further, using 3.1 it results that deviations in $x_t$ from the steady state are normally distributed:

$$\tilde{x}_t \sim N [E_{t-1} \tilde{x}_t, Var_{t-1} (x_t)]$$  \hspace{1cm} (3.2)

where $\tilde{x}_t = \ln(X_t) - \ln(X)$ is a deviation in $X_t$ from the steady state.

The second assumption is that the ex-ante variances and covariances of the exogenous variables; $P_{EF,t}$ and $P_{IF,t}$, are constant over time and independent of the information available:

$$Var_{t-1}(x_t) = Var(x) \equiv \sigma_x^2$$  \hspace{1cm} (3.3)

$$Cov_{t-1}(x_t, z_t) = Cov(x, z) \equiv \sigma_{xz}$$  \hspace{1cm} (3.4)

---

29Foreign bonds holding is exogenous in steady state but endogenous in the dynamic solution, which causes changes in it and the steady state when shocks occur.
where $Z_t$ represents the same variables as $X_t$ and the same discussion applies to $Z_t$ as to $X_t$ above. Hence, it is assumed that a shock to foreign prices does not affect their ex-ante variances and covariances.

The third assumption concerns demand management policy:

$$\hat{i}_t = \Psi + \Psi_{E} \hat{p}_{EF,t} + \Psi_{I} \hat{p}_{IF,t}$$

(3.5)

where $\hat{i}_t = \frac{T}{M}$ and $\Psi$ are coefficients which are defined by the exact demand management policy conducted in the economy.\(^{30}\) Hence, the government is assumed to respond linearly to fluctuations in foreign prices around their steady state values. Note that many other forms of demand management policy can be assumed.\(^{31}\) Here, the only objective is to show that there exists a welfare improving demand management policy.\(^{32}\)

The fourth assumption is that:

$$E_{t-1} \hat{x}_t = 0$$

(3.6)

Hence, no deviations in the foreign prices of exportables and importables from the steady state are expected. This implies that shocks are unexpected in the model. This is a reasonable assumption since shocks to foreign prices are assumed to be temporary in the model.

The last assumption is that shocks to the exogenous variables are small enough such that the households can supply enough labor as is needed when the shocks occur.

Since the dynamic solution to the model involves (see the discussion below) writing each endogenous variable (in deviation from the steady state) as a linear function of the exogenous variables and past values of an endogenous variables (in deviations from the steady state), each of the endogenous variables are ex-ante log-normally distributed. Hence, the above discussion concerning the distributions of the exogenous variables; $P_{EF,t}$ and $P_{IF,t}$, also applies to the distributions of the endogenous variables.

3.2.2 The linearized system

Using the assumptions and their results when writing the equation system in table 1 and the endogenous demand management policy (from 3.5) in deviations from the steady state gives the following:\(^{33}\)

\(^{30}\)\(\Psi_E\) and $\Psi_I$ as functions of the parameters of the model for a welfare improving demand management policy are derived in chapter 5.

\(^{31}\)Non-linear policies can, for example, be assumed.

\(^{32}\)Hence, the welfare improving demand management policy derived in chapter 5 may not be the optimal policy.

\(^{33}\)The calculations for deriving these results are well known in the NOEM literature. Hence, I will not go into the details of them here.
Table 2. The equation system in deviation from the steady state

\[(D1) \rho (E_t \hat{c}_{t+1} - \hat{c}_t) + (E_t \hat{p}_{t+1} - \hat{p}_t) + \varepsilon r (\hat{m}_t - \hat{p}_t) - \rho r \hat{c}_t = \Omega_1 \]

\[(D2) \rho (E_t \hat{c}_{t+1} - \hat{c}_t) = \Omega_2 \]

\[(D3) \hat{w}_t = \nu E_{t-1} \hat{I}_t + E_{t-1} \hat{p}_t + \rho E_{t-1} \hat{c}_t + \Omega_3 \]

\[(D4) \hat{p}_{E,t} = \hat{s}_t + \hat{p}_{EF,t} \]

\[(D5) \hat{p}_{I,t} = \hat{s}_t + \hat{p}_{IF,t} \]

\[(D6) \hat{p}_t = \gamma \hat{p}_{E,t} + (1 - \gamma) \hat{p}_{I,t} \]

\[(D7) \hat{y}_t = \frac{\alpha}{1 - \alpha} (\hat{p}_{E,t} - \hat{w}_t) \]

\[(D8) \hat{f}_t = (1 + r) \hat{f}_{t-1} + \hat{y}_t + \hat{p}_{E,t} - \hat{c}_t - \hat{\rho}_t \]

\[(D9) \hat{y}_t = \alpha \hat{I}_t \]

\[(D10) \hat{m}_t = \hat{m}_{t-1} + \hat{I}_t \]

\[(D11) \hat{I}_t = \Psi + \Psi \hat{p}_{EF,t} + \Psi \hat{p}_{IF,t} \]

where

\[\Omega_1 = \ln \left[ r \left( \frac{1}{1+r} \right)^{1+r} \right] + \frac{1}{2} \rho^2 \sigma_c^2 + \frac{1}{2} \sigma_p^2 + \rho \sigma_{cp}, \]

\[\Omega_2 = \frac{1}{2} \rho^2 \sigma_c^2, \]

\[\Omega_3 = \frac{\nu (\nu + 2)}{2} \sigma_l^2 - \frac{1}{2} \sigma_p^2 - \frac{1}{2} \rho^2 \sigma_c^2 + \sigma_{lp} + \rho \sigma_{lc} - \rho \sigma_{pc}, \]

\[\hat{f}_t = \frac{\hat{f}}{1 + r}. \]

By substitution \(D2\) into \(D1\) (and ignoring constants) the demand for real money balances is the following:

\[\hat{m}_t - \hat{\rho}_t = \frac{\rho}{\varepsilon} \hat{c}_t - \frac{1}{\varepsilon r} (E_t \hat{p}_{t+1} - \hat{p}_t) \]  \( (3.7) \)

where the term \((E_t \hat{p}_{t+1} - \hat{p}_t)\) is expected nominal interest rate from holding foreign bonds (in deviations from the steady state nominal interest rates \((r)\)), i.e.

the opportunity cost of holding money. \(D2\) in table 2 is the traditional Euler equation, which determines the optimal allocation of consumption over time (ignoring constants):

\[E_t \hat{c}_{t+1} - \hat{c}_t = 0 \]  \( (3.8) \)

This has to hold since the real interest rates are constant and equal to the subjective real interest rates in the discount factor, i.e.

\[r = \frac{1}{3} - 1 \]  in the model. Note that 3.8 implies that consumption follows a random walk process. Writing 2.25 in deviations from the steady state and using \(D8\) gives:

\[\hat{c}_a_t = \hat{f}_t - (1 + r) \hat{f}_{t-1} = \hat{y}_t + \hat{p}_{E,t} - \hat{c}_t - \hat{\rho}_t \]  \( (3.9) \)

where \(\hat{c}_a_t = \frac{CA_t}{r} \) is deviation in the current account from the steady state. \(D3\) in table 2 is gives nominal wages in the economy; \(D4\) and \(D5\) give domestic prices of exportables and importables, \(D6\) gives the price level in the economy, \(D7\) gives production of exportables, \(D9\) can be used to derive labor use, \(D10\) can be used to calculate the money supply and \(D11\) can be used to calculate the nominal transfers (where all variables are in deviations from the steady state).
3.2.3 The solution

In this paper, the objective is to analyze how shocks to foreign prices affect domestic economic variables and how demand management policy can be used to increase welfare following such shocks. Hence, constants are ignored in what follows.

The dynamic solution is derived in appendix 4. Obtaining it involves solving for each of the endogenous variables in table 2: $c_t$, $y_t$, $lt$, $wt$, $pt$, $p_Et$, $p_It$, $st$, $mt$, $E_t$ and $I_t$ as a function of the exogenous variables; $p_Elt$ and $p_Il_t$, and the predetermined endogenous variables; $f_{t-1}$ and $m_{t-1}$. To do this, the method of undetermined coefficients and assume rational expectations. This gives the following dynamic solution:

Table 3. The dynamic solution

\[ \begin{align*}
\dot{c}_t &= \Gamma_{cf} \dot{f}_{t-1} + \Gamma_{ct} \tilde{p}_{E,t} - \Gamma_{ct} \tilde{p}_{I,t} \\
\dot{s}_t &= -\Gamma_{sf} \dot{f}_{t-1} + \dot{m}_{t-1} - \Gamma_{se} \tilde{p}_{E,t} - \Gamma_{se} \tilde{p}_{I,t} \\
\dot{f}_t &= \dot{f}_{t-1} + \Gamma_{fe} \tilde{p}_{E,t} - \Gamma_{fe} \tilde{p}_{I,t} \\
\dot{p}_{E,t} &= -\Gamma_{pe} \dot{f}_{t-1} + \dot{m}_{t-1} + \Gamma_{pe} \tilde{p}_{E,t} - \Gamma_{pe} \tilde{p}_{I,t} \\
\dot{p}_{I,t} &= -\Gamma_{pi} \dot{f}_{t-1} + \dot{m}_{t-1} - \Gamma_{pi} \tilde{p}_{E,t} + \Gamma_{pi} \tilde{p}_{I,t} \\
\dot{p}_t &= -\Gamma_p \dot{f}_{t-1} + \dot{m}_{t-1} - \Gamma_p \tilde{p}_{E,t} + \Gamma_p \tilde{p}_{I,t} \\
\dot{y}_t &= -\Gamma_y \dot{f}_{t-1} + \Gamma_y \tilde{p}_{E,t} - \Gamma_y \tilde{p}_{I,t} \\
\dot{l}_t &= -\Gamma_l \dot{f}_{t-1} + \Gamma_l \tilde{p}_{E,t} - \Gamma_l \tilde{p}_{I,t} \\
\dot{w}_t &= -\Gamma_w \dot{f}_{t-1} + \dot{m}_{t-1} \\
\dot{E}_t &= \Psi_E \tilde{p}_{E,t} + \Psi_E \tilde{p}_{I,t} \\
\dot{I}_t &= \Psi_I \tilde{p}_{E,t} + \Psi_I \tilde{p}_{I,t} \\
\dot{m}_t &= \dot{m}_{t-1} + \Psi_E \tilde{p}_{E,t} + \Psi_E \tilde{p}_{I,t}
\end{align*} \]

where the \( \Gamma \) are functions of the parameters of the model.\(^{34}\)

3.2.4 Discussion

According to the results in table 3, lagged money supply only affects nominal variables (nominal exchange rate, nominal wage rate, domestic prices of exportables and importables and the price level) and does not affect real variables (consumption, foreign bonds holding, output and labor use). Further, from appendix 4, the \( \Pi_{xe} \) and the \( \Pi_{xi} \) are functions of \( \Psi_E \) and \( \Psi_I \), respectively. Hence, different demand management policies (different values of \( \Psi_E \) and \( \Psi_I \)) result in different effects of a foreign price change on both real and nominal variables in the economy. From this it is clear that demand management policy has both nominal and real effects in the dynamic solution to the model.

It is obvious from table 3 that foreign bonds holding follows a random walk process. Hence, a shock to the foreign prices of exportables and/or importables causes foreign bonds holding to change at the time of the shock (the current period) and to stay at

\(^{34}\)Consult appendix 4 to see how the \( \Gamma \)'s are functions of the parameters of the model.
that level forever (future periods). To make this clear, the following holds from the
results in table 3, where a shock is assumed to occur in period $t$:\textsuperscript{35}

$$
\hat{f}_{t+i} = \Gamma_{fe}\hat{p}_{EF,t} - \Gamma_{fi}\hat{p}_{IF,t} \quad i = 0, 1, 2, \ldots
$$

(3.10)

Further, since $\Gamma_{cf}\Gamma_{fe} = \Gamma_{ce}$ and $\Gamma_{cf}\Gamma_{fi} = \Gamma_{ci}$ as can be verified using the results
in appendix 4, consumption also follows a random walk (as was expected from 3.8). Hence, a shock to the foreign prices of exportables and/or importables causes con-
sumption to change in the current period and to stay at that level in future periods.
The following holds from the results in table 3:\textsuperscript{36}

$$
\hat{c}_{t+i} = \Gamma_{ce}\hat{p}_{EF,t} - \Gamma_{ci}\hat{p}_{IF,t} \quad i = 0, 1, 2, \ldots
$$

(3.11)

The processes for labor use and output are a bit more complicated than those for
foreign bonds holding and consumption. First note that they do not follow random
walk processes since $-\Gamma_{lf}\Gamma_{fe} \neq \Gamma_{le}, \Gamma_{lf}\Gamma_{fi} \neq -\Gamma_{li}, -\Gamma_{yf}\Gamma_{fe} \neq \Gamma_{ye}$ and $\Gamma_{yf}\Gamma_{fi} \neq -\Gamma_{yi}$, as can be verified using the results in appendix 4. In fact, the following holds from
the results in table 3:\textsuperscript{37}

$$
\hat{l}_t = \Gamma_{le}\hat{p}_{EF,t} - \Gamma_{li}\hat{p}_{IF,t}
$$

(3.12)

$$
\hat{l}_{t+i} = -\Gamma_{lf}\Gamma_{fe}\hat{p}_{EF,t} + \Gamma_{lf}\Gamma_{fi}\hat{p}_{IF,t} \quad i = 1, 2, \ldots
$$

(3.13)

$$
\hat{y}_t = \Gamma_{ye}\hat{p}_{EF,t} - \Gamma_{yi}\hat{p}_{IF,t}
$$

(3.14)

$$
\hat{y}_{t+i} = -\Gamma_{yf}\Gamma_{fe}\hat{p}_{EF,t} + \Gamma_{yf}\Gamma_{fi}\hat{p}_{IF,t} \quad i = 1, 2, \ldots
$$

(3.15)

Hence, following a shock, labor use and output change in the current and future
periods. The current and future changes are, however, not the same.

4 The Effects of Foreign Prices Shocks

In this chapter, the effects of shocks to foreign prices of exportables and/or importables
on a few domestic economic variables are discussed. In doing this, passive demand
management policy ($\hat{m}_{t-1} = \Psi_E = \Psi_I = 0$) following shocks is assumed to be able to
focus on the effects that the shocks have in the absense of active demand management
policy.\textsuperscript{38} In the next chapter of the paper, demand management following shocks to
foreign prices of exportables and/or importables is discussed.

\begin{footnotesize}
\begin{enumerate}
\item This is shown in appendix 5.
\item This is shown in appendix 6.
\item This is shown in appendix 7.
\item Since constants are ignored, $\Psi$ is already set to 0.
\end{enumerate}
\end{footnotesize}
4.1 Shocks to the foreign price of exportables vs. shocks to the foreign price of importables

Imposing passive demand management policy on the results in table 3 gives $\Gamma_{x_e} = -\Gamma_{x_i}$ for all the endogenous variables in table 3 except the nominal exchange rate.\(^{39}\) Hence, shocks to foreign prices of exportables and importables have opposite but symmetric effects on all the endogenous variables in the model except the nominal exchange rate. It can therefore be concluded that an increase (decrease) in the foreign price of exportables has exactly opposite effects, compared to an increase (decrease) in the foreign price of importables, on all domestic economic variables except the nominal exchange rate.

These results make it possible to write the solutions for the endogenous variables as functions of a shock to the terms of trade for all the endogenous variables except the nominal exchange rate:

Table 4. The dynamic solution assuming passive demand management policy

<table>
<thead>
<tr>
<th>Endogenous Variable</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{c}_t$</td>
<td>$\Gamma_{cf} \dot{c}<em>{t-1} + \Gamma</em>{ct} \hat{t}_{tot}$</td>
</tr>
<tr>
<td>$\dot{s}_t$</td>
<td>$-\Gamma_{sf} \dot{s}<em>{t-1} - \Gamma</em>{sc} \hat{p}<em>E,</em>{t} - \Gamma_{si} \hat{p}<em>I,</em>{t}$</td>
</tr>
<tr>
<td>$\dot{f}_t$</td>
<td>$\dot{f}<em>{t-1} + \Gamma</em>{fe} \hat{t}_{tot}$</td>
</tr>
<tr>
<td>$\hat{p}<em>E,</em>{t}$</td>
<td>$-\Gamma_{pef} \dot{f}<em>{t-1} + \Gamma</em>{pex} \hat{t}_{tot}$</td>
</tr>
<tr>
<td>$\hat{p}<em>I,</em>{t}$</td>
<td>$-\Gamma_{pif} \dot{f}<em>{t-1} - \Gamma</em>{piz} \hat{t}_{tot}$</td>
</tr>
<tr>
<td>$\hat{y}_t$</td>
<td>$-\Gamma_{yf} \dot{f}<em>{t-1} + \Gamma</em>{ye} \hat{t}_{tot}$</td>
</tr>
<tr>
<td>$\hat{l}_t$</td>
<td>$-\Gamma_{lf} \dot{f}<em>{t-1} + \Gamma</em>{le} \hat{t}_{tot}$</td>
</tr>
<tr>
<td>$\hat{w}_t$</td>
<td>$-\Gamma_{wf} \dot{f}_{t-1}$</td>
</tr>
<tr>
<td>$\hat{m}_t$</td>
<td>0</td>
</tr>
</tbody>
</table>

where $\hat{t}_{tot} \equiv \hat{p}_E,_{t} - \hat{p}_I,_{t}$ is deviation in terms of trade (from 2.24) from its steady state value of 1. It can be concluded from these results that the effects of a terms of trade appreciation are independent of whether they are a result of a higher price of exportables or a lower price of importables for all other variables than the nominal exchange rate.

4.2 Size of an economy and vulnerability towards shocks to the terms of trade

As can be verified from the results in appendix 4, the numerical values of $\Gamma_{ce}$, $\Gamma_{sc}$, $\Gamma_{si}$, $\Gamma_{fe}$, $\Gamma_{pex}$, $\Gamma_{piz}$, $\Gamma_{ye}$ and $\Gamma_{le}$ in table 4 become larger the smaller is the value of $\gamma$,\(^{39}\) This can be verified from the results in appendix 4.
where $\gamma$ is a measure of the size of the economy as is discussed in chapter 2. Hence, a shock to the terms of trade has greater effects on domestic economic variables the smaller the economy is. It can therefore be concluded that the smaller an economy is the more vulnerable it is towards a shock to the terms of trade.

### 4.3 Effects of a terms of trade appreciation

Using the results in table 4 it is possible to analyse the effects that a terms of trade appreciation has on domestic economic variables. Before doing this it is useful to investigate the signs of the $\Gamma$ in table 4.

As can be verified from the results in appendix 4, all the $\Gamma$ in table 4 are positive except $\Gamma_{st}$, $\Gamma_{pec}$, $\Gamma_{ye}$, $\Gamma_{le}$ and $\Gamma_{wf}$. Using the results from appendix 4 the first four of these are positive if the following condition holds:

\[
(1 - \alpha + \nu) [\varepsilon (1 + r) - \rho r] + \alpha \varepsilon \rho > 0
\]

or

\[
\varepsilon (1 + r) > \rho r - \frac{\alpha \varepsilon \rho}{1 - \alpha + \nu}
\] (4.1)

The last one is positive if the following holds:

\[
\nu > (\varepsilon - 1) (1 - \alpha)
\] (4.2)

By looking at parameter values such as in Andersen and Beier (2003), both 4.1 and 4.2 can easily be assumed to hold.\footnote{In their paper, they set $\varepsilon = 9$, $\rho = \frac{1}{3}$, $\nu = 10$ and $\alpha = \frac{2}{5}$. Assuming that $r = 0.05$ this gives: $9.45 > -0.71$ for 4.1 and $10 > 2.67$ for 4.2.} Hence, it will be assumed to hold in what follows. 4.1 is especially important for the following discussion since it ensures that a terms of trade appreciation results in higher domestic price of exportables in the current period. To explain this, suppose that the terms of trade appreciate following an increase in the foreign price of exportables. It is clear that this results in an appreciation of the nominal exchange rate in the current period (since $\Gamma_{se} > 0$). This can outweight the higher foreign price of exportables leaving the domestic price of exportables lower than before. 4.1 prevents this from happening.

### 4.3.1 Resulting paths for some endogenous variables

According to the results in table 4 and for the signs of the $\Gamma$, the following pictures describe how some domestic economic variables develop following a terms of trade appreciation:\footnote{Note that these pictures are mainly supposed to show the directions (increase, decrease etc. compared to the steady state) of movements in variables.}

\[
\begin{align*}
\text{Resulting paths for some endogenous variables}\end{align*}
\]
These results are discussed briefly in the following subchapters:

**Price level and domestic prices of exportables and importables** Since \(-\Gamma_{pf} \Gamma_{fe} = -\Gamma_{pe}\) for the \(\Gamma\)'s in table 4 (this can be verified using the results in appendix 4), the price level in the economy follows a random walk process. Hence, the resulting path for the price level following a terms of trade shock can be written in the following way, where a shock is assumed to occur in period \(t\):\(^{42}\)

\[
\hat{p}_{t+i} = -\Gamma_{pe} \hat{o}_{lt} \quad i = 0, 1, 2, ... 
\]  

(4.3)

Hence, it can be concluded that following a terms of trade appreciation the price level decreases in the current period and stays at that level in future periods. This can be seen from picture 1.

\(^{42}\)This result is derived similarly to 3.11.
However, since $-\Gamma_{pej}\Gamma_{fe} \neq \Gamma_{pee}$ and $-\Gamma_{psj}\Gamma_{fe} \neq -\Gamma_{pie}$ the domestic prices of exportables and importables do not follow random walk processes. This can be seen from picture 1. The resulting path for the domestic price of exportables following a terms of trade shock is the following: \(^{43}\)

\[
\hat{p}_{E; t} = \Gamma_{pee}\hat{ol}_t
\]

\[
\hat{p}_{E; t+i} = -\Gamma_{pej}\Gamma_{fe}\hat{ol}_t \quad i = 1, 2, ...
\]

and the domestic price of importables: \(^{44}\)

\[
\hat{p}_{I; t} = -\Gamma_{pie}\hat{ol}_t
\]

\[
\hat{p}_{I; t+i} = -\Gamma_{psj}\Gamma_{fe}\hat{ol}_t \quad i = 1, 2, ...
\]

Hence, it can be concluded that following a terms of trade appreciation the domestic price of exportables increases and the domestic price of importables decreases in the current period while in future periods the domestic prices of exportables and importables decrease (compared to their steady state values). The future periods decrease in the domestic price of importables is less than the current period one since $\Gamma_{pie} > \Gamma_{psj}\Gamma_{fe}$.

To give an intuition for these results let us assume that the terms of trade appreciates following an increase in the foreign price of exportables. This results in an appreciation of the nominal exchange rate in the current period, as is discussed above, and in future periods since $\Gamma_{sf}\Gamma_{fe} > 0$. Further, since $\Gamma_{se} > \Gamma_{sf}\Gamma_{fe}$, the nominal exchange rate appreciates more in the current period than in future periods following an increase in the foreign price of exportables. Given that 4.1 holds, as is assumed, this results in an increase in the domestic price of exportables in the current period and a decrease in it in future periods while the domestic price of importables decreases more in the current period than in future periods.

**Labor use and output** As is discussed in chapter 3, labor use and output do not follow random walk processes. Imposing the assumption of passive demand management policy on 3.12 - 3.15 in chapter 3 gives the following paths for labor use and output following a terms of trade shock in period $t$:

\[
\hat{l}_t = \Gamma_{lc}\hat{ol}_t
\]

\[
\hat{l}_{t+i} = -\Gamma_{lf}\Gamma_{fe}\hat{ol}_t \quad i = 1, 2, ...
\]

\(^{43}\)This result is derived similarly to 3.12 and 3.13.

\(^{44}\)This result is derived similarly to 3.12 and 3.13.
\[ \dot{y}_t = \Gamma_{ye} \tilde{\nu} \tilde{d}_t \]  
\[ \dot{y}_{t+i} = -\Gamma_{yf} \Gamma_{fe} \tilde{\nu} \tilde{d}_t \quad i = 1, 2, \ldots \]

Hence, it can be concluded that following a terms of trade appreciation labor use and output increase in the current period and decrease in future period (compared to the steady state). This is shown in picture 1.

To give an intuition for these results let us, like above, assume that the terms of trade appreciates following an increase in the foreign price of exportables. From the results above, this result in an increase in the domestic price of exportables which results in increased labor use and increased output in the current period. In future periods, the domestic price of exportables decreases (compared to the steady state) since the foreign price of exportables returns to its steady state value and the nominal exchange rate appreciates in current and future periods, as is discussed above, which results in less output and labor use in future periods.

**Real wages** Using the results in table 4, the path for nominal wages can be written in the following way where a terms of trade shocks is assumed to occur in period \( t \):\(^{45}\)

\[ \hat{w}_t = 0 \]  
\[ \hat{w}_{t+i} = -\Gamma_{wf} \Gamma_{fe} \tilde{\nu} \tilde{d}_t \quad i = 1, 2, \ldots \]

The path for real wages is then obtained by subtracting 4.3 from 4.12 and 4.13:

\[ \hat{w}_t - \hat{p}_t = \Gamma_{pe} \tilde{\nu} \tilde{d}_t \]  
\[ \hat{w}_{t+i} - \hat{p}_{t+i} = (\Gamma_{pe} - \Gamma_{wf} \Gamma_{fe}) \tilde{\nu} \tilde{d}_t \quad i = 1, 2, \ldots \]

Using the results in appendix 4, it can be verified that \( \Gamma_{pe} - \Gamma_{wf} \Gamma_{fe} > 0 \). Hence, it can be concluded that following a terms of trade appreciation real wages increase in the current period and in future periods. The increase in the current period is, however, greater then in future periods. This is shown in picture 1.

**Foreign bonds holding, current account and consumption** As is discussed in chapter 3, both foreign bonds holding and consumption follow random walk processes. Imposing the assumption of passive demand management policy on 3.10 and 3.11 in chapter 3 gives the following paths for foreign bonds holding and consumption following a terms of trade shock in period \( t \):

\[ \hat{f}_{t+i} = \Gamma_{fe} \tilde{\nu} \tilde{d}_t \quad i = 0, 1, 2, \ldots \]

\(^{45}\)This result is derived similarly to 3.12 and 3.13.
\[
\hat{c}_{t+i} = \Gamma_0 \hat{t} \tilde{\omega}_t \quad i = 0, 1, 2, \ldots \quad (4.17)
\]

Hence, it can be concluded that following a terms of trade appreciation both consumption and foreign bonds holding increase in the current period and stay at those levels in future periods. This is shown in picture 1. Further, it can be concluded that the current account becomes positive in the current period and negative in future periods, as can be seen using 3.9 and 4.16:

\[
\hat{c}_t = \Gamma_f \hat{t} \tilde{\omega}_t \quad (4.18)
\]

\[
\hat{c}_{t+i} = -r \Gamma_f \hat{t} \tilde{\omega}_t \quad i = 1, 2, \ldots \quad (4.19)
\]

This is shown in picture 1.

To gain an intuition into these results note that the effects of a terms of trade appreciation on consumption, foreign bonds holding and the current account can be devited into (i) price effects and (ii) income effects. The price effects are due to the fact that following a terms of trade appreciation current and future price level decreases equally which results in increased lifetime purchasing power of the households. This causes equal increase in consumption in the current period and in future periods. The income effects are due to the fact that following a terms of trade appreciation the domestic price of exportables increases in the current period, which results in increased labor income and profits of the firm distributed to the households. Following these increased income the households smooth consumption over the current and future periods which results in a current account surplus and increased foreign bonds holding in the current period. The increase in foreign bonds holding results in increased interest rate income for the households in future periods which, despite of less income from labor and profits of the firm in future periods due to lower domestic price of exportables, allows the households to maintain higher consumption level over future periods (as compared to the steady state).

5 A Welfare Improving Demand Management Policy

The government decides on its demand management policy by setting the values of the coefficients in 3.5 conditional on the solution to the model in table 3. The question here is how the government can responds to shocks to foreign prices, i.e. what the values of \( \Psi_E \) and \( \Psi_I \) should be, such that welfare is increased following a terms of trade shock. The discussion in this chapter shows that there exits a welfare improving demand management policy following a shock to foreign prices.
In macroeconomic modelling there are two main sources of distortions arising as a result of deviations in exogenous variables from their steady state levels.\footnote{In this paper, these exogenous variables are foreign prices of exportables and importables.} On one hand, there is incomplete adjustment in labor and/or output markets and incomplete capital markets, on the other. In the model presented in this paper, both of these are present. Hence, given \ref{eq:3.5}, there is a reason to expect that demand management policy can be used to improve welfare following a shock to foreign prices in this model.

Identifying the welfare maximizing demand management policy given \ref{eq:3.5} and showing that such a policy implies that at least one of $\Psi_E$ and $\Psi_I$ in is non-zero, is sufficient for showing that there exists a welfare improving demand management policy (other than the passive one) following a shock to foreign prices. Hence, this chapter starts by discussing an appropriate welfare measure and then goes on to solving for the welfare maximizing demand management policy given \ref{eq:3.5}.

\section{The welfare measure}

Following what is a usual practice in the NOEM literature,\footnote{See, for example, Obstfeld and Rogoff (2000).} the real part of a households’ utility function (from \ref{eq:2.10}) is chosen here to represent welfare at every point in time. Assuming symmetry among households and noting that a foreign price shock has effects in current and future periods, the relevant welfare measure following a foreign price shock in time $t$ is the following:

$$ V_t = \sum_{i=0}^{\infty} \left( \frac{1}{1 + r} \right)^i \left( \frac{C_{t+i}^{1-\rho} - \tau L_{t+i}^{\nu+1}}{1 - \rho} \right) \quad (5.1) $$

To get $C_{t+i}$ and $L_{t+i}$ in \ref{eq:5.1} in deviations from the steady state, I perform a second order Taylor approximation of \ref{eq:5.1} around the steady state. This gives the following:\footnote{Note that to be accurate, an error term capturing the error from not including third and higher derivatives terms should be included here. Whether this done or not does, however, not affect the calculations below. Hence, it is ignored here.}

$$ dV_t = \sum_{i=0}^{\infty} \left( \frac{1}{1 + r} \right)^i \left( C^{1-\rho} c_{t+i} - \tau L^{\nu+1} \hat{c}_{t+i} - \frac{\rho C^{1-\rho}}{2} c_{t+i}^2 - \frac{\tau \nu L^{\nu+1}}{2} \hat{c}_{t+i}^2 \right) \quad (5.2) $$

where $C$ and $L$ are the steady state values of consumption and labor from appendix 3.

The first two terms in the second bracket in \ref{eq:5.2} only give information about the coefficient $\Psi$ in \ref{eq:3.5} when \ref{eq:5.2} is used to obtain the welfare maximizing demand management policy conditional on the solution to the model in table 3 after constants have been included. In fact, ignoring these terms when maximizing \ref{eq:5.2} conditional on the
solution to the model in table 3 does not affect the solutions for the coefficients $\Psi_E$ and $\Psi_I$. Hence, the optimal demand management policy following a shock to foreign prices can be obtained by minimizing the following function conditional on the solution to the model in table 3:\footnote{This is obtained by realizing that maximizing a function gives the same result as minimizing the same function multiplied by $-1$.}

$$
\tilde{d}V_t = \sum_{i=0}^{\infty} \left( \frac{1}{1+r} \right)^i \left( \frac{\rho C^{1-\rho}}{2} \tilde{c}_{t+i}^2 + \frac{\tau \nu L^{\nu+1}}{2} \tilde{l}_{t+i}^2 \right) \tag{5.3}
$$

From 5.3 it can be concluded that, given 3.5, the demand management policy that maximizes welfare following a shock to foreign prices is the one that minimizes the present value of weighted average of deviations in consumption and labor use from the steady state. Hence, demand management policy should not only be aimed at stabilizing output (labor use), but also consumption. Hence, it can be concluded that an example of a welfare improving demand management policy following a shock to foreign prices is a policy that is constructed in such a way that nominal transfers are used as a stabilizing tool.

### 5.2 The demand management policy

From the discussion in chapter 3, it is clear that following a shock to foreign prices in time $t$ consumption and labor use develop such that: $\hat{c}_t = \hat{c}_{t+1} = \hat{c}_{t+2} = \ldots$ and $\hat{l}_t \neq \hat{l}_{t+1} = \hat{l}_{t+2} = \ldots$. Hence, it is possible to write 5.3 in the following way:

$$
\tilde{d}V_t = \frac{1 + r}{r} \frac{\rho C^{1-\rho}}{2} \hat{c}_t^2 + \frac{\tau \nu L^{\nu+1}}{2} \hat{l}_t^2 + \frac{1}{r} \frac{\tau \nu L^{\nu+1}}{2} \hat{l}_{t+1}^2 \tag{5.4}
$$

Further $\hat{c}_t$, $\hat{l}_t$ and $\hat{l}_{t+1}$ can be written in the following way:\footnote{This is shown in appendix 8.}

$$
\hat{c}_t = \tilde{\Gamma}_{ctot} \hat{c}_{tot} + \tilde{\Gamma}_{ct} \hat{c}_t \tag{5.5}
$$

$$
\hat{l}_t = \tilde{\Gamma}_{ltot} \hat{c}_{tot} + \tilde{\Gamma}_{lt} \hat{l}_t \tag{5.6}
$$

$$
\hat{l}_{t+1} = -\tilde{\Gamma}_{ltot} \hat{c}_{tot} - \tilde{\Gamma}_{lt} \hat{l}_t \tag{5.7}
$$

where the $\tilde{\Gamma}$ are functions of the parameters of the model.\footnote{Consult appendix 8 to see how the $\tilde{\Gamma}$ are functions of the parameters of the model.} All the $\tilde{\Gamma}$ are positive, as can be verified using the results from appendix 8.\footnote{The positivity of $\tilde{\Gamma}_{ltot}$ is dependent on 4.1 holding, as is done here.}

The welfare maximizing demand management policy, given 3.5, can be obtained by minimizing 5.4, given 5.5 - 5.7, with respect to $\hat{l}_t$. This gives the following result
for $\hat{t}_t$: \(^{53}\)

$$
\hat{t}_t = \left\{ \frac{(1 + r) \rho C^{1-\rho} \Gamma_{cl} \Gamma_{clot} + \tau \nu L_{\nu+1} \left( r \Gamma_{lt} \Gamma_{lilot} + \Gamma_{lt} \Gamma_{lilot} \right)}{(1 + r) \rho C^{1-\rho} \Gamma_{cl}^2 + \tau \nu L_{\nu+1} \left( r \Gamma_{lt}^2 + \Gamma_{lilot}^2 \right)} \right\} \text{told}_t \tag{5.8}
$$

Since all the $\Gamma$ are positive, 5.8 implies that the welfare maximizing demand management policy given 3.5 is such that nominal transfers should be decreased (increased) when there is a terms of trade appreciation (depreciation) as compared to what they would be in the absence of a shock to foreign prices.

From 5.8 the welfare maximizing demand management policy following a shock to foreign prices, given 3.5, is such that $\Psi_E = -\Psi_I \neq 0$. Hence, it can be concluded that there exists a welfare improving demand management policy following a shock to foreign prices. An example of such a policy is the one in 5.8. Further, the policy is such that opposite but symmetric responses in nominal transfers are to be used depending on whether there is a shock to the foreign price of exportables or the foreign price of importables. Hence, the policy should be aimed at responding to deviations of the terms of trade from its steady state value of 1. According to these results, the policy following a terms of trade appreciation (depreciation) is independent of whether it is a result of a higher (lower) price of exportables or a lower (higher) price of importables.

### 6 Conclusions

The results in this paper indicate that a terms of trade shock has permanent real effects in small open economies and that there exists a demand management policy that improves welfare following such shocks.

According to the results in chapters 3 and 4, increases (decreases) in foreign prices of exportables have opposite but symmetric real effects compared to increases (decreases) in foreign prices of importables in small open economies. A terms of trade appreciation results in an even permanent increase in consumption and foreign bonds holding since households spend their increased income from labor and profits distributed from the firms to present and future consumption (consumption smoothing). The current account becomes positive at the time of the appreciation due to increased savings but negative in all future periods when households fund a part of their consumption by interest income on foreign bonds. Labor use and output increase at the time of the appreciation while they decrease in all future periods since the domestic price of exportables decreases in future periods due to an appreciation of the nominal exchange rate. Exactly the opposite applies following a terms of trade depreciation.

\(^{53}\)This is shown in appendix 9.
Further, according to the results in chapters 3 and 4, a small open economy is more vulnerable towards shocks to the terms of trade the smaller the economy is. Hence, there is a reason to expect that there are more fluctuations in real variables the smaller an economy is.

The results in chapter 5 indicate that there exists a welfare improving demand management policy following a shock to foreign prices. The policy is such that nominal transfers respond symmetrically but opposite depending on whether foreign prices of exportables or foreign prices of importables increase (or decrease). This means that the demand management policy should be aimed at responding to fluctuations in the terms of trade. Further, the policy is such that a terms of trade appreciation (depreciation) should be followed by a decrease (an increase) in nominal transfers to the households as compared to what it would be in the absence of a shock to foreign prices.

7 References


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8 Appendixes

8.1 Appendix 1

Plugging 2.3 into 2.6 and using 2.4 gives the profit function:

$$\Pi_t = P_{E,t} Y_t - W_t Y_t^{\frac{1}{\alpha}}$$

which is a strictly concave function in $Y_t$. Hence, a necessary and sufficient condition for profit maximization is:

$$\frac{\partial \Pi_t}{\partial Y_t} = P_{E,t} - \frac{1}{\alpha} W_t Y_t^{\frac{1}{\alpha} - 1} = 0$$

Solving this equation for $Y_t$ gives 2.7.

8.2 Appendix 2

By plugging 2.7 into 2.3 and the resulting equation and 2.7 into 2.6 gives:

$$\Pi_t = P_{E,t} \alpha^{\frac{\alpha}{1-\alpha}} \left( \frac{P_{E,t}}{W_t} \right) \frac{1}{\alpha} - \int_{j=0}^{1} W_t(j) \left[ \frac{W_t(j)}{W_t} \right]^{-\phi} \alpha^{\frac{1-\alpha}{1-\alpha}} \left( \frac{P_{E,t}}{W_t} \right) \frac{1}{\alpha} dj$$

Rearranging and using 2.4 gives:

$$\Pi_t = (1 - \alpha) \alpha^{\frac{\alpha}{1-\alpha}} \left( \frac{P_{E,t}}{W_t} \right) \frac{1}{\alpha} W_t > 0$$

Using this result gives:

$$\frac{\partial \Pi_t}{\partial P_{E,t}} = \alpha^{\frac{\alpha}{1-\alpha}} \left( \frac{P_{E,t}}{W_t} \right)^{\frac{\alpha}{1-\alpha}} > 0$$
8.3 Appendix 3

Assuming no uncertainty and constant exogenous variables in steady state gives the following steady state version of $E^2$ in table 1:

$$\delta = \frac{1}{1 + r}$$

Hence, the subjective discount factor ($\delta$) is determined by the foreign real interest rate level ($r$) in steady state. There are therefore only 9 equations available to solve for the 10 endogenous steady state variables. As is done in Obstfeld and Rogoff (1995), the current account ($CA$) is assumed to be exogenous in steady state here to get an equal number of endogenous variables and equations. Further, it is assumed that $F = 0$ holds in steady state, which implies that the $CA = 0$ in steady state (from 2.25), and that $T = 0$ holds in steady state. Finally, it is assumed that foreign prices of importables and exportables equal 1 in steady state ($P_{EF} = P_{IF} = 1$). The steady state solution is given in the following table:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$C = Y = \left(\frac{a}{\beta}\right)^{1-\alpha(1+\alpha)}$</td>
</tr>
<tr>
<td>$L$</td>
<td>$L = \left(\frac{a}{\beta}\right)^{1-\alpha(1+\alpha)}$</td>
</tr>
<tr>
<td>$S$</td>
<td>$S = P = P_E = P_I = \left[\frac{r}{(1+r)\lambda}\right]^\frac{1}{2} \left(\frac{a}{\beta}\right)^{-\frac{\alpha p}{\alpha + \epsilon (1-\alpha+p-r)}} M$</td>
</tr>
<tr>
<td>$W$</td>
<td>$W = \left[\frac{r}{(1+r)\lambda}\right]^\frac{1}{2} \left(\frac{a p + \epsilon (1-\alpha + p - r)}{\alpha + \epsilon (1-\alpha + p - r)}\right) M$</td>
</tr>
</tbody>
</table>

where $\beta = \frac{\phi}{\phi - 1}$ is a measure of the effects of market power in the labor market on the steady state solution, which is the main reason for a distortionary steady state in this model. Note that $\phi \rightarrow \infty$ implies no market power in the labor market and $\beta = \tau$ while $\phi < \infty$ implies market power in the labor market and $\beta > \tau$. Hence, market power in the labor market results in less labor use, output and consumption than is optimal. Note further that money is neutral in steady state, i.e. money has no real effects in the model (i.e. it has no effects on $C$, $Y$ and $L$)- it only inflates prices ($S$, $P$, $P_E$, $P_I$ and $W$).

8.4 Appendix 4

Obtaining a dynamic solution to the model involves solving for each of the endogenous variables in table 2: $c_t$, $y_t$, $l_t$, $w_t$, $p_t$, $p_{EF,t}$, $p_{IF,t}$, $s_t$, $m_t$, $i_t$ and $f_t$ as a function of the exogenous variables; $p_{EF,t}$ and $p_{IF,t}$, and the predetermined endogenous variables; $f_{t-1}$ and $m_{t-1}$. To do this, the method of undetermined coefficients is used. Using
this method involves taking the following steps: (i) Guess a solution for each of the endogenous variables, (ii) check if the equations in table 2 support such solutions and (iii) calculate the coefficient values for the solutions as functions of the parameters of the equations in table 2.

The guess is the following:

\[ \hat{x}_t = \Phi_x + \Phi_{xf} \hat{f}_{t-1} + \Phi_{xm} \hat{m}_{t-1} + \Phi_{xe} \hat{p}_{EF,t} + \Phi_{xI} \hat{p}_{IF,t} \]

where \( \hat{x}_t \) is an endogenous variable and the \( \Phi \)'s are the (undetermined) coefficients which are functions of the parameters of the equation system in table 2. In this paper, the objective is to analyze how shocks to foreign prices affect domestic economic variables and how demand management policy can be used to increase welfare following such shock. Hence, in this paper there is only interest in the values of the coefficients: \( \Phi_{xf}, \Phi_{xm}, \Phi_{xe} \) and \( \Phi_{xI} \) in the equation above. Using the fact that ignoring constants in the equation above and in D1 – D11 in table 2 does not affect the values of \( \Phi_{xf}, \Phi_{xm}, \Phi_{xe} \) and \( \Phi_{xI} \), the dynamic solution is derived below by setting these constants equal to 0, i.e. \( \Phi_x = \Omega_1 = \Omega_2 = \Omega_3 = \Psi = 0 \) is imposed.

The system can be shrunked down to 3 equations in 3 endogenous variables: \( \hat{c}_t, \hat{s}_t \) and \( \hat{f}_t \), where rational expectations are assumed:

\[
\begin{align*}
(D1) & \quad E_t \hat{c}_{t+1} - \left(1 + \varepsilon r \right) \hat{c}_t - \rho \varepsilon \hat{m}_{t-1} + \left[ \varepsilon r \Psi_E - \gamma \left(1 + \varepsilon r \right) \right] \hat{p}_{EF,t} \\
& \quad + \left[ \varepsilon r \Psi_I - \left(1 - \gamma \right) \right] \left(1 + \varepsilon r \right) \hat{p}_{IF,t} = 0 \\
(D2) & \quad E_t \hat{c}_{t+1} - \hat{c}_t = 0 \\
(D3) & \quad \hat{f}_t - \left(1 + r \right) \hat{f}_{t-1} - \frac{\alpha}{1 - \alpha} \hat{s}_t + \hat{c}_t - \frac{1 - \gamma (1 - \alpha)}{1 - \alpha} \hat{p}_{EF,t} + \left(1 - \gamma \right) \hat{p}_{IF,t} + \frac{\alpha}{1 - \alpha} E_{t-1} \hat{s}_t = 0
\end{align*}
\]

The following solution is assumed for each endogenous variable:

\[ \hat{x}_t = \Phi_{xf} \hat{f}_{t-1} + \Phi_{xm} \hat{m}_{t-1} + \Phi_{xe} \hat{p}_{EF,t} + \Phi_{xI} \hat{p}_{IF,t} \]

Substituting these into D1 – D3, assuming rational expectations and using 3.6 gives:

D1:

\[
\begin{align*}
\Phi_{xf} \Phi_{ff} \hat{f}_{t-1} + & \quad \Phi_{xf} \Phi_{fm} \hat{m}_{t-1} + \Phi_{xf} \Phi_{fc} \hat{p}_{EF,t} + \Phi_{xf} \Phi_{fI} \hat{p}_{IF,t} \\
& + \Phi_{xm} \hat{m}_{t-1} + \Phi_{xm} \Psi_E \hat{p}_{EF,t} + \Phi_{xm} \Psi_I \hat{p}_{IF,t} - \left(1 + \varepsilon r \right) \Phi_{xf} \hat{f}_{t-1} \\
& - \left(1 + \varepsilon r \right) \Phi_{xm} \hat{m}_{t-1} - \left(1 + \varepsilon r \right) \Phi_{xe} \hat{p}_{EF,t} - \left(1 + \varepsilon r \right) \Phi_{xI} \hat{p}_{IF,t} \\
& - \rho \varepsilon \Phi_{cf} \hat{f}_{t-1} - \rho \varepsilon \Phi_{cm} \hat{m}_{t-1} - \rho \varepsilon \Phi_{ce} \hat{p}_{EF,t} - \rho \varepsilon \Phi_{cI} \hat{p}_{IF,t} + \varepsilon r \hat{m}_{t-1} \\
& + \left[ \varepsilon r \Psi_E - \gamma \left(1 + \varepsilon r \right) \right] \hat{p}_{EF,t} + \left[ \varepsilon r \Psi_I - \left(1 - \gamma \right) \right] \left(1 + \varepsilon r \right) \hat{p}_{IF,t} = 0
\end{align*}
\]

D2:

\[
\begin{align*}
\Phi_{cf} \Phi_{ff} \hat{f}_{t-1} + & \quad \Phi_{cf} \Phi_{fm} \hat{m}_{t-1} + \Phi_{cf} \Phi_{fc} \hat{p}_{EF,t} + \Phi_{cf} \Phi_{fI} \hat{p}_{IF,t} \\
& + \Phi_{cm} \hat{m}_{t-1} + \Phi_{cm} \Psi_E \hat{p}_{EF,t} + \Phi_{cm} \Psi_I \hat{p}_{IF,t} \\
& - \Phi_{cf} \hat{f}_{t-1} - \Phi_{cm} \hat{m}_t - \Phi_{ce} \hat{p}_{EF,t} - \Phi_{cI} \hat{p}_{IF,t} = 0
\end{align*}
\]
D3:

\[ \Phi_{ff} \hat{f}_{t-1} + \Phi_{fm} \hat{m}_{t-1} + \Phi_{fe} \hat{p}_{EF,t} + \Phi_{ji} \hat{p}_{IF,t} - (1 + r) \hat{f}_{t-1} - \frac{\alpha}{1-\alpha} \Phi_{sf} \hat{f}_{t-1} - \frac{\alpha}{1-\alpha} \Phi_{sm} \hat{m}_{t-1} - \frac{\alpha}{1-\alpha} \Phi_{se} \hat{p}_{EF,t} - \frac{\alpha}{1-\alpha} \Phi_{si} \hat{p}_{IF,t} + \Phi_{cj} \hat{f}_{t-1} + \frac{\alpha}{1-\alpha} \Phi_{cm} \hat{m}_{t-1} + \frac{\alpha}{1-\alpha} \Phi_{ce} \hat{p}_{EF,t} + \frac{\alpha}{1-\alpha} \Phi_{ci} \hat{p}_{IF,t} \]

These give the following necessary conditions for a solution:

**Necessary condition 1:**

\[ \Phi_{sf} \Phi_{ff} \hat{f}_{t-1} - (1 + \epsilon r) \Phi_{sf} \hat{f}_{t-1} - \rho r \Phi_{cf} \hat{f}_{t-1} = 0 \]

\[ \Phi_{ff} \hat{f}_{t-1} - (1 + r) \hat{f}_{t-1} + \Phi_{cf} \hat{f}_{t-1} + \frac{\alpha}{1-\alpha} \Phi_{sf} \hat{f}_{t-1} + \frac{\alpha}{1-\alpha} \Phi_{cf} \hat{f}_{t-1} = 0 \]

**Necessary condition 2:**

\[ \Phi_{sf} \Phi_{fm} \hat{m}_{t-1} + \Phi_{sm} \hat{m}_{t-1} - (1 + \epsilon r) \Phi_{sm} \hat{m}_{t-1} - \rho r \Phi_{cm} \hat{m}_{t-1} + \epsilon r \hat{m}_{t-1} = 0 \]

\[ \Phi_{cf} \Phi_{fm} \hat{m}_{t-1} + \Phi_{cm} \hat{m}_{t-1} - \Phi_{cm} \hat{m}_{t-1} = 0 \]

\[ \Phi_{fm} \hat{m}_{t-1} - \frac{\alpha}{1-\alpha} \Phi_{sm} \hat{m}_{t-1} + \Phi_{cm} \hat{m}_{t-1} + \frac{\alpha}{1-\alpha} \Phi_{sm} \hat{m}_{t-1} + \frac{\alpha}{1-\alpha} \Phi_{cm} \hat{m}_{t-1} = 0 \]

**Necessary condition 3:**

\[ \Phi_{sf} \Phi_{fe} \hat{p}_{EF,t} + \Phi_{sm} \Psi_{E} \hat{p}_{EF,t} - (1 + \epsilon r) \Phi_{se} \hat{p}_{EF,t} - \rho r \Phi_{cf} \hat{p}_{EF,t} + [\epsilon r \Psi_{E} - \gamma (1 + \epsilon r)] \hat{p}_{EF,t} = 0 \]

\[ \Phi_{cf} \Phi_{fe} \hat{p}_{EF,t} + \Phi_{cm} \Psi_{E} \hat{p}_{EF,t} - \Phi_{ce} \hat{p}_{EF,t} = 0 \]

\[ \Phi_{fe} \hat{p}_{EF,t} - \frac{\alpha}{1-\alpha} \Phi_{se} \hat{p}_{EF,t} + \Phi_{cf} \hat{p}_{EF,t} - \frac{1-\gamma (1-\alpha)}{1-\alpha} \hat{p}_{EF,t} = 0 \]

**Necessary condition 4:**

\[ \Phi_{sf} \Phi_{fi} \hat{p}_{IF,t} + \Phi_{sm} \Psi_{I} \hat{p}_{IF,t} - (1 + \epsilon r) \Phi_{si} \hat{p}_{IF,t} - \rho r \Phi_{ci} \hat{p}_{IF,t} + [\epsilon r \Psi_{I} - (1 - \gamma) (1 + \epsilon r)] \hat{p}_{IF,t} = 0 \]

\[ \Phi_{cf} \Phi_{fi} \hat{p}_{IF,t} + \Phi_{cm} \Psi_{I} \hat{p}_{IF,t} - \Phi_{ci} \hat{p}_{IF,t} = 0 \]

\[ \Phi_{fi} \hat{p}_{IF,t} - \frac{\alpha}{1-\alpha} \Phi_{si} \hat{p}_{IF,t} + \Phi_{cf} \hat{p}_{IF,t} + (1 - \gamma) \hat{p}_{IF,t} = 0 \]

Using that \( \hat{f}_{t-1} \neq 0 \) in general, necessary condition 1 gives:

\[ \Phi_{ff} = \frac{1}{1 - \alpha} \]

\[ \Phi_{sf} = \frac{\rho r (1-\alpha+\nu)}{\epsilon (1-\alpha+\nu + \alpha \rho)} \]

\[ \Phi_{cf} = \frac{\nu (1-\alpha+\nu)}{1-\alpha+\nu + \alpha \rho} \]

Plugging these results into necessary condition 2 and using that \( \hat{m}_{t-1} \neq 0 \) in general, necessary condition 2 gives:

\[ \Phi_{fm} = 0 \]

\[ \Phi_{sm} = 1 \]

\[ \Phi_{cm} = 0 \]

Plugging the results from necessary condition 1 into necessary condition 3 and using that \( \hat{p}_{EF,t} \neq 0 \) in general, necessary condition 3 gives:
\[
\Phi_{fe} = \frac{\varepsilon(1-\gamma+\alpha \Psi_f)(1-\alpha+\nu+\alpha \rho)}{\alpha \rho (1-\alpha+\nu+\varepsilon(1-\alpha))[(1+r)(1-\alpha+\nu)+\alpha \rho]}
\]
\[
\Phi_{se} = -\frac{\alpha \rho (1-\alpha+\nu+\varepsilon(1-\alpha))[(1+r)(1-\alpha+\nu)+\alpha \rho]}{\varepsilon(1-\gamma+\alpha \Psi_f)(1-\alpha+\nu)}
\]
\[
\Phi_{ce} = \frac{\alpha \rho (1-\alpha+\nu+\varepsilon(1-\alpha))[(1+r)(1-\alpha+\nu)+\alpha \rho]}{\varepsilon(1-\gamma+\alpha \Psi_f)(1-\alpha+\nu)}
\]

Plugging the results from necessary condition 1 into necessary condition 4 and using that \(\hat{p}_{t+1} \neq 0\) in general, necessary condition 4 gives:

\[
\Phi_{fs} = \frac{\varepsilon(1-\gamma+\alpha \Psi_f)(1-\alpha+\nu+\alpha \rho)}{\alpha \rho (1-\alpha+\nu+\varepsilon(1-\alpha))[(1+r)(1-\alpha+\nu)+\alpha \rho]}
\]
\[
\Phi_{si} = \frac{\alpha \rho (1-\alpha+\nu+\varepsilon(1-\alpha))[(1+r)(1-\alpha+\nu)+\alpha \rho]}{\varepsilon(1-\gamma+\alpha \Psi_f)(1-\alpha+\nu)}
\]
\[
\Phi_{ci} = -\frac{\alpha \rho (1-\alpha+\nu+\varepsilon(1-\alpha))[(1+r)(1-\alpha+\nu)+\alpha \rho]}{\varepsilon(1-\gamma+\alpha \Psi_f)(1-\alpha+\nu)}
\]

Plugging the results from necessary conditions 1 - 4 into the assumed solutions for \(\hat{c}_t\), \(\hat{s}_t\) and \(\hat{f}_t\) above gives the solutions for these variables. The solutions for the remaining variables are then obtained by using the relationships for the linearized system in table 2 (after setting the constants to zero: \(\Omega_1 = \Omega_2 = \Omega_3 = 0\) and assuming rational expectations). The complete dynamic solution to the model is given in the following table:

The dynamic solution

29
\[
\begin{align*}
\hat{c}_t &= \left\{ \frac{r(1-\alpha+\nu)}{1-\alpha+\nu+\rho} \right\} \hat{f}_{t-1} + \left\{ \frac{\sigma(1-\gamma+\alpha \Psi_E)(1-\alpha+\nu)}{\Sigma} \right\} \hat{p}_{EF,t} - \left\{ \frac{\sigma(1-\gamma-\alpha \Psi_E)(1-\alpha+\nu)}{\Sigma} \right\} \hat{p}_{IF,t} \\
\hat{s}_t &= -\left\{ \frac{\rho r(1-\alpha+\nu)}{\varepsilon(1-\alpha+\nu+\rho)} \right\} \hat{f}_{t-1} + \hat{m}_{t-1} - \left\{ \frac{\rho r[1-\gamma+\varepsilon(\gamma-\Psi_E)(1-\alpha)](1+r)(1-\alpha+\nu)+\rho}{\Sigma} \right\} \hat{p}_{EF,t} - \left\{ (1-\alpha)(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)-\rho+\alpha \rho \right\} - \left\{ (1-\alpha)(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)+\alpha \rho \right\} \hat{p}_{IF,t} \\
\hat{f}_t &= \hat{f}_{t-1} - \left\{ \frac{\rho r(1-\alpha+\nu)}{\varepsilon(1-\alpha+\nu+\rho)} \right\} \hat{f}_{t-1} + \hat{m}_{t-1} - \hat{p}_{EF,t} - \left\{ \frac{\rho r(1-\alpha+\nu)}{\varepsilon(1-\alpha+\nu+\rho)} \right\} \hat{f}_{t-1} - \left\{ \frac{\rho r[1-\gamma+\varepsilon(\gamma-\Psi_E)(1-\alpha)](1+r)(1-\alpha+\nu)+\rho}{\Sigma} \right\} \hat{p}_{EF,t} + \left\{ (1-\alpha)(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)-\rho+\alpha \rho \right\} \hat{p}_{IF,t} \\
\hat{p}_t &= -\left\{ \frac{\rho r(1-\alpha+\nu)}{\varepsilon(1-\alpha+\nu+\rho)} \right\} \hat{f}_{t-1} + \hat{m}_{t-1} - \hat{p}_{EF,t} - \left\{ \frac{\rho r(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)+\alpha \rho}{\Sigma} \right\} \hat{p}_{IF,t} + \left\{ \frac{\rho r[1-\gamma+\varepsilon(\gamma-\Psi_E)(1-\alpha)](1+r)(1-\alpha+\nu)+\rho}{\Sigma} \right\} \hat{p}_{EF,t} - \left\{ (1-\alpha)(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)-\rho+\alpha \rho \right\} \hat{p}_{IF,t} \\
\hat{y}_t &= -\left\{ \frac{\alpha \rho r}{1-\alpha+\nu+\rho} \right\} \hat{f}_{t-1} + \left\{ \frac{\alpha(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)-\rho+\alpha \rho}{\Sigma} \right\} \hat{p}_{EF,t} - \left\{ \frac{\alpha(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)+\alpha \rho}{\Sigma} \right\} \hat{p}_{IF,t} - \left\{ \frac{\alpha(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)-\rho+\alpha \rho}{\Sigma} \right\} \hat{p}_{EF,t} - \left\{ \frac{\alpha(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)+\alpha \rho}{\Sigma} \right\} \hat{p}_{IF,t} \\
\hat{l}_t &= -\left\{ \frac{\rho r}{1-\alpha+\nu+\rho} \right\} \hat{f}_{t-1} + \left\{ \frac{(1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)-\rho+\alpha \rho}{\Sigma} \right\} \hat{p}_{EF,t} - \left\{ (1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)-\rho+\alpha \rho \right\} \hat{p}_{IF,t} - \left\{ (1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)+\alpha \rho \right\} \hat{p}_{EF,t} - \left\{ (1-\gamma)(1-\alpha+\nu)\varepsilon(1+r)+\alpha \rho \right\} \hat{p}_{IF,t} \\
\hat{w}_t &= -\left\{ \frac{\rho r(1-\alpha+\nu)+\varepsilon(1-\gamma)(1+r)(1-\alpha+\nu)+\alpha \rho}{\Sigma} \right\} \hat{f}_{t-1} + \hat{m}_{t-1} - \hat{p}_{EF,t} + \hat{p}_{IF,t} + \hat{P}_{EF,t} + \hat{P}_{IF,t} + \hat{P}_{IF,t} + \hat{P}_{IF,t} \\
\hat{t}_t &= \hat{t}_{t-1} + \hat{m}_{t-1} + \hat{P}_{EF,t} + \hat{P}_{IF,t}
\end{align*}
\]

where \( \Sigma \equiv \alpha \rho r (1 - \alpha + \nu) + \varepsilon (1 - \gamma) [(1 + r)(1 - \alpha + \nu) + \alpha \rho] > 0. \)

### 8.5 Appendix 5

The equation for \( \hat{f}_t \) in table 3 can be written in the following way:

\[
\hat{f}_{t+i} = \hat{f}_{t+i-1} + \Pi_{f \varepsilon \hat{p}_{EF,t+i-1}} - \Pi_{f \varepsilon \hat{p}_{IF,t+i}}
\]

By repeatedly substituting in for \( \hat{f}_{t+i-1} \), using that \( \hat{f}_{t-1} = 0 \) (the steady state holding of foreign bonds) and assuming that a shock occurs at time \( t \) gives for \( i \geq 0 \):

\[
\hat{f}_{t+i} = \Pi_{f \varepsilon \hat{p}_{EF,t}} - \Pi_{f \varepsilon \hat{p}_{IF,t}}
\]

which is identical to 3.10.
8.6 Appendix 6

The equation for $c_t$ in table 3 can be written in the following way:

$$
\dot{c}_{t+i} = \Pi_{cf} \hat{f}_{t+i-1} + \Pi_{ce} \hat{p}_{EF,t+i} - \Pi_{ca} \hat{p}_{IF,t+i}
$$

By repeatedly substituting in for $\hat{f}_{t+i-1}$ from appendix 5, using that $\hat{f}_{t-1} = 0$ (the steady state holding of foreign bonds) and assuming that a shock occurs at time $t$ gives (for $i \geq 0$):

$$
\dot{c}_{t+i} = \Pi_{ce} \hat{p}_{EF,t} - \Pi_{ca} \hat{p}_{IF,t}
$$

which is identical to 3.11.

8.7 Appendix 7

The equation for $l_t$ in table 3 can be written in the following way:

$$
\dot{l}_{t+i} = -\Pi_{lf} \hat{f}_{t+i-1} + \Pi_{le} \hat{p}_{EF,t+i} - \Pi_{li} \hat{p}_{IF,t+i}
$$

Assuming that a shock occurs in time $t$ and using that $\hat{f}_{t-1} = 0$ (the steady state holding of foreign bonds) gives 3.12:

$$
\dot{l}_t = \Pi_{le} \hat{p}_{EF,t} - \Pi_{li} \hat{p}_{IF,t}
$$

3.14 is derived similarly.

By repeatedly substituting into the $\dot{l}_t$ equation in table 3 for $\hat{f}_{t+i-1}$ from appendix 5 and assuming that a shock occurs at time $t$ gives (for $i \geq 1$):

$$
\dot{l}_{t+i} = -\Pi_{lf} \Pi_{fe} \hat{p}_{EF,t} + \Pi_{lf} \Pi_{fi} \hat{p}_{IF,t}
$$

which is identical to 3.13. 3.15 is derived similarly.

8.8 Appendix 8

Using 3.11 and that $\dot{c}_t = \dot{c}_{t+1} = \dot{c}_{t+2} = \ldots$ gives:

$$
\dot{c}_t = \Pi_{ce} \hat{p}_{EF,t} - \Pi_{ca} \hat{p}_{IF,t}
$$

Using the results from appendix 4 to plug in for $\Pi_{ce}$ and $\Pi_{ca}$:

$$
\dot{c}_t = \left\{ \frac{\varepsilon r (1 - \gamma + \alpha \Psi_E) (1 - \alpha + \nu)}{\Sigma} \right\} \hat{p}_{EF,t} - \left\{ \frac{\varepsilon r (1 - \gamma - \alpha \Psi_I) (1 - \alpha + \nu)}{\Sigma} \right\} \hat{p}_{IF,t}
$$

Rewriting gives:

$$
\dot{c}_t = \left\{ \frac{\varepsilon r (1 - \gamma) (1 - \alpha + \nu)}{\Sigma} \right\} (\hat{p}_{EF,t} - \hat{p}_{IF,t}) + \left\{ \frac{\alpha \varepsilon r (1 - \alpha + \nu)}{\Sigma} \right\} (\Psi_E \hat{p}_{EF,t} + \Psi_I \hat{p}_{IF,t})
$$
Using that $\tilde{t}^\alpha_{\ell} = \hat{p}_{EF,\ell} - \hat{p}_{IF,\ell}$ and 3.5 (ignoring constants) then gives:

$$\hat{t}_t = \left\{ \frac{1}{\Sigma} \left[ (1 - \gamma) \left( \alpha + \nu \right) - \rho r + \alpha \varepsilon_\ell \left( 1 + r \right) + \varepsilon \Psi_E \left[ (1 + r) \left( 1 - \alpha + \nu \right) + \alpha r \right] \right] \right\} \hat{p}_{EF,\ell} + \left\{ \frac{1}{\Sigma} \left[ \alpha \varepsilon_\ell \left( 1 + r \right) + \varepsilon \Psi_E \left[ (1 + r) \left( 1 - \alpha + \nu \right) + \alpha r \right] \right] \right\} \hat{p}_{IF,\ell}
$$

Using 3.12 and the results from appendix 4 to plug in for $\Pi_{\ell}$ and $\Pi_r$:

$$\hat{t}_t = \left\{ \frac{1}{\Sigma} \left[ (1 - \gamma) \left( \alpha + \nu \right) - \rho r + \alpha \varepsilon_\ell \left( 1 + r \right) + \varepsilon \Psi_E \left[ (1 + r) \left( 1 - \alpha + \nu \right) + \alpha r \right] \right] \right\} \hat{p}_{EF,\ell} - \hat{p}_{IF,\ell}
$$

Rewriting gives:

$$\hat{t}_t = \left\{ \frac{1}{\Sigma} \left[ (1 - \gamma) \left( \alpha + \nu \right) - \rho r + \alpha \varepsilon_\ell \left( 1 + r \right) + \varepsilon \Psi_E \left[ (1 + r) \left( 1 - \alpha + \nu \right) + \alpha r \right] \right] \right\} \hat{t} + \left\{ \frac{1}{\Sigma} \left[ \alpha \varepsilon_\ell \left( 1 + r \right) + \varepsilon \Psi_E \left[ (1 + r) \left( 1 - \alpha + \nu \right) + \alpha r \right] \right] \right\} \hat{t}_t$$

Using that $\tilde{t}^\alpha_{\ell} = \hat{p}_{EF,\ell} - \hat{p}_{IF,\ell}$ and 3.5 (ignoring constants) then gives:

$$\hat{t}_t = \left\{ \frac{1}{\Sigma} \left[ (1 - \gamma) \left( \alpha + \nu \right) - \rho r + \alpha \varepsilon_\ell \left( 1 + r \right) + \varepsilon \Psi_E \left[ (1 + r) \left( 1 - \alpha + \nu \right) + \alpha r \right] \right] \right\} \tilde{t}_t + \left\{ \frac{1}{\Sigma} \left[ \alpha \varepsilon_\ell \left( 1 + r \right) + \varepsilon \Psi_E \left[ (1 + r) \left( 1 - \alpha + \nu \right) + \alpha r \right] \right] \right\} \hat{t}_t$$

Using 3.13 and that $\hat{t}_{t+1} = \hat{t}_{t+2} = \ldots$ gives:

$$\hat{t}_{t+1} = -\Pi_{\ell} \Pi_{\ell_c} \hat{p}_{EF,\ell} + \Pi_{\ell} \Pi_{\ell_c} \hat{p}_{IF,\ell}
$$

Using the results from appendix 4 to plug in for $\Pi_{\ell_f}, \Pi_{\ell_e}$ and $\Pi_{\ell_i}$:

$$\hat{t}_{t+1} = -\left\{ \frac{\varepsilon \rho r \left( 1 - \gamma + \alpha \Psi_E \right)}{\Sigma} \right\} \hat{p}_{EF,\ell} + \left\{ \frac{\varepsilon \rho r \left( 1 - \gamma - \alpha \Psi_I \right)}{\Sigma} \right\} \hat{p}_{IF,\ell}
$$

Rewriting gives:

$$\hat{t}_{t+1} = -\left\{ \frac{\varepsilon \rho r \left( 1 - \gamma \right)}{\Sigma} \right\} \left( \hat{p}_{EF,\ell} - \hat{p}_{IF,\ell} \right) - \left\{ \frac{\alpha \varepsilon \rho r \left( 1 - \gamma \right)}{\Sigma} \right\} \left( \Psi_E \hat{p}_{EF,\ell} + \Psi_I \hat{p}_{IF,\ell} \right)
$$

Using that $\tilde{t}^\alpha_{\ell} = \hat{p}_{EF,\ell} - \hat{p}_{IF,\ell}$ and 3.5 (ignoring constants) then gives:

$$\hat{t}_{t+1} = -\left\{ \frac{\varepsilon \rho r \left( 1 - \gamma \right)}{\Sigma} \right\} \tilde{t}_t - \left\{ \frac{\alpha \varepsilon \rho r \left( 1 - \gamma \right)}{\Sigma} \right\} \hat{t}_t$$

### 8.9 Appendix 9

Differentiating 5.4, given 5.5 - 5.7, with respect to $\hat{t}_t$ gives:

$$\frac{\partial \tilde{V}_t}{\partial \hat{t}_t} = \frac{1 + \rho C}{r} \rho C^1 - \rho \tilde{\Gamma}_C \left( \tilde{\Gamma}_{t_0 \alpha} \tilde{t}_t + \tilde{\Gamma}_{C t} \hat{t}_t \right)
$$

$$+ \tau \nu L^{\nu+1} \tilde{\Gamma}_{t} \left( \tilde{\Gamma}_{t_0 \alpha} \tilde{t}_t + \tilde{\Gamma}_{t \alpha} \hat{t}_t \right)
$$

$$+ \frac{1}{r} \tau \nu L^{\nu+1} \tilde{\Gamma}_{t} \left( \tilde{\Gamma}_{t_0 \alpha} \tilde{t}_t + \tilde{\Gamma}_{t \alpha} \hat{t}_t \right)$$

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The second derivative is the following:

\[
\frac{\partial^2 d\hat{V}_t}{\partial \hat{t}_t^2} = \frac{1 + r}{r} \rho C^{1-\rho} \hat{\Gamma}_t^2
+ \tau \nu L^{\nu+1} \hat{\Gamma}_t^{2}
+ \frac{1}{r} \tau \nu L^{\nu+1} \hat{\Gamma}_{lt}^2 > 0
\]

Since the second derivative is positive then 5.4, given 5.5 - 5.7, is strictly convex in \( \hat{t}_t \).
Hence, \( \frac{\partial d\hat{V}_t}{\partial \hat{t}_t} = 0 \) is a necessary and sufficient condition for a solution. Solving this for \( \hat{t}_t \) gives 5.8.
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